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Comments on "New Method of Performance Analysis for Diversity Reception With Correlated Rayleigh-Fading Signals"

S. Loyka, C. Tellambura, A. Kouki, A. Annamalai, and F. Gagnon

***Abstract*—The purpose of this comment is to provide a critical discussion of the new method of performance analysis of diversity systems proposed in the abovementioned paper. It is shown that this method provides incorrect results for equal-gain and selective combining.**

A new method of performance analysis of two-branch receive diversity systems has been presented in the paper mentioned above.[1] The main idea of this method is to transform the two received correlated signals into uncorrelated ones and to then apply the existing techniques of uncorrelated signal-combining analysis[1]: "Based on the conversion process proposed in Section III, the performance analysis can be conducted by the following steps for the dual-branch diversity reception in correlated Rayleigh-fading environments.

Step 1) Convert two correlated signals into two independent ones with the proposed transformation.

Step 2) Conduct the performance analysis of diversity reception with new independent Rayleigh-fading signals by using existing analysis methods reported in the literature.

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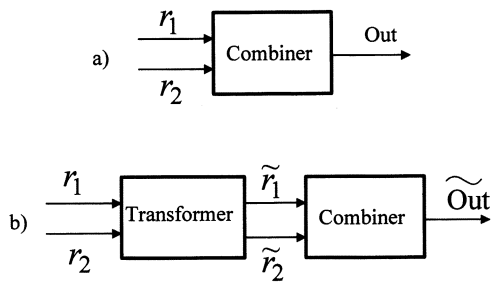

Fig. 1. (a) Block diagram of a diversity-combining system and (b) the transformed system. r_1 and r_2 are the received correlated signals, $\tilde{r}_1$ and $\tilde{r}_2$ are the transformed uncorrelated signals.

The analysis result achieved in Step 2 is the performance of diversity reception with original correlated Rayleigh-fading signals." The implicit assumption behind this approach, which leads to the statement in the last sentence, is that the transformation proposed in Section III[1] does not change the performance of the diversity-combining system. However, the paper[1] fails to provide a proof for this crucial assumption. As the detailed analysis below shows, this assumption is true for maximal-ratio combining (MRC), but not true for equal-gain combining (EGC) and selection combining (SC). Thus, the method above cannot be applied in the latter two cases.

Let us now consider the method proposed in the paper[1] in more detail. Without going into a complex statistical analysis, we examine the basic equations that describe the operation of a diversity-combining system. Fig. 1(a) shows the original diversity-combining system under the analysis, which can be MRC, EGC, or SC. Fig. 1(b) shows the modified system by using the transformation in the paper.[1] The instantaneous signal-to-noise power ratio (SNR) at the output of the original system is

$$\gamma = \frac{F(\mathbf{r})}{2N}, \quad \mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \tag{1}$$

where $\mathbf{r}$ is the received complex envelope-signal vector (without noise), F is the signal-combining function, and N is the noise power per branch (for simplicity, we consider the case of equal noise powers). For MRC, EGC, and SC, correspondingly, it takes the following form [2]:

$$\begin{aligned} F_{\mathrm{MRC}}(\mathbf{r}) &= \mathbf{r}^{+}\mathbf{r} \\ F_{\mathrm{EGC}}(\mathbf{r}) &= \frac{1}{2}\left(|r_1| + |r_2|\right)^2 \\ F_{\mathrm{SC}}(\mathbf{r}) &= \max\left[|r_1|^2, |r_2|^2\right] \end{aligned} \tag{2}$$

where $+$ denotes transpose conjugate. The SNR at the output of the transformed system[1] is

$$\tilde{\gamma} = \frac{F(\tilde{\mathbf{r}})}{2N}, \quad \tilde{\mathbf{r}} = \mathbf{T} \cdot \mathbf{r} \tag{3}$$

where the transformation matrix $\mathbf{T}$ is[1]

$$\mathbf{T} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}. \tag{4}$$

Note that the combining function is the same for both the original and transformed systems. Obviously, the method above holds true if $\tilde{\gamma} = \gamma$. Using (1)–(4), we conclude that this is true for MRC because $\mathbf{T}^{+}\mathbf{T} = \mathbf{I}$ and, consequently, $(\tilde{\mathbf{r}})^{+}\tilde{\mathbf{r}} = \mathbf{r}^{+}\mathbf{r}$, where $\mathbf{I}$ is the identity matrix. However, this is not true for EGC and SC. Consider, for example, the case of fully correlated signals, $r_1 = r_2 = r$. Then $\tilde{r}_1 = \sqrt{2r}$ and $\tilde{r}_2 = 0$. For EGC, one obtains

$$\tilde{\gamma} = \frac{|r|^2}{2N} \tag{5}$$

which is not the same as for the original system

$$\gamma = \frac{|r|^2}{N}. \tag{6}$$

The SNR of a one-branch system $\gamma_1 = |r|^2/2N$. As (6) shows, EGC provides a 3-dB improvement over the one-branch system in this case, which is a physically reasonable conclusion (see also [2, p. 372]). On the contrary, as (5) shows, the method proposed in the paper[1] does not predict any improvement. Thus, this method underestimates the performance of EGC by 3 dB for fully correlated signals.

Let us now consider the SC system for the same case of $r_1 = r_2 = r$. Using (33),[1] we obtain the cumulative probability distribution function of $\tilde{\gamma}$ (normalized correlation $\rho = 1$)

$$P(\tilde{\gamma} \le x) = 1 - \exp\left(-\frac{x}{2 \cdot \Gamma}\right) \tag{7}$$

where Γ is the average per-branch SNR (before combining). For the one-branch system (no diversity) one obtains the following [1]:

$$P_1(\gamma \le x) = 1 - \exp\left(-\frac{x}{\Gamma}\right). \tag{8}$$

Comparing (7) and (8), we conclude that, according to the method proposed in the abovementioned paper,[1] the SC system provides a 3-dB improvement over the one-branch system in this case [the same conclusion can be obtained using (2) and (3). But this is a physically absurd conclusion, because selecting one of the two identical signals (i.e., $r_1 = r_2 = r$) gives the same signal (and, consequently, the same instantaneous SNR) and provides no advantage at all. (Strictly speaking, the total branch signals are not identical due to the noise, but they have the same instantaneous SNR.)

It should be emphasized that the performance of SC in a correlated Rayleigh channel was analyzed several decades ago. The correct results (cumulative probability distribution function or probability density function) have been reported as early as in 1956 [1], then repeated in a modern form in [2, pp. 470–471], in the now classical book by Jakes [3, pp. 324–325], later on in [4, pp. 356–357], and in numerous journal articles and conference papers.

We should also stress that the method in the paper[1] produces incorrect results not only for $\rho = 1$, but also for the other values of the correlation coefficient as well. For example, Fig. 2 shows the diversity gain (defined as the ratio of the diversity-combined signal to the one-branch [no diversity] signal at a given outage probability P [4, p. 573]) of the SC system at the outage probability $P = 0.5$, evaluated using the correct expressions for the cumulative probability distribution function (e.g., [2, eq. 10–10-8] and the [3, eq. 5.2–21]), and using (33).[1] Obviously, the method in paper[1] predicts physically absurd behavior of the SC diversity gain for the entire range of ρ, i.e., its increase with correlation. Extensive numerical Monte Carlo simulations of the cumulative probability distribution function of SC and EGC systems that use the traditional approach (without any signal transformation) and the method proposed in the paper[1] confirm that the latter produces incorrect results.

Finally, we would like to emphasize that the idea of transforming correlated random variables (RV) into independent ones has been well known for a long time. A vector $\mathbf{z}$ of n jointly distributed complex Gaussian RVs with a correlation matrix $\mathbf{R}$ can be transformed using a decomposition of $\mathbf{R} = \mathbf{UDU}^H$, where $\mathbf{D}$ is a diagonal matrix consisting of the eigenvalues of $\mathbf{R}$ and $\mathbf{U}$ consists of the corresponding eigenvectors. The transformed independent variables $\mathbf{U}^H\mathbf{z}$ will have variances equal to the eigenvalues of $\mathbf{R}$[3, Appendix B]. In fact, the transformation introduced in the paper[1] is a special case of this general transformation. For example, $\mathbf{R}$ in the paper[1] is a 2×2 correlation matrix with eigenvalues $(1 \pm \rho)\sigma^2$ and corresponding normalized eigenvectors are $(1/\sqrt{2})[1 \quad 1]^T$ and $(1/\sqrt{2})[1 \quad -1]^T$. This immediately explains the authors' transformation matrix $\mathbf{T}$ and the average SNRs of the equivalent independent branches (28) and (29).

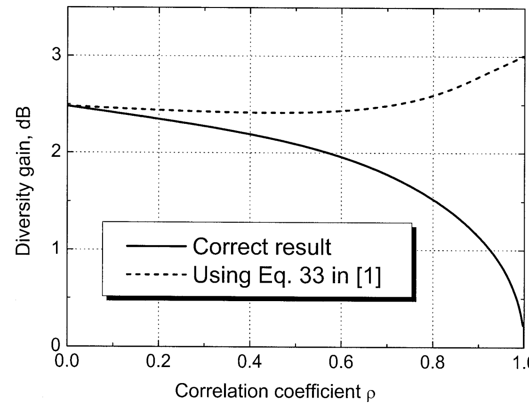

Fig. 2. SC diversity gain versus correlation coefficient at $P = 0.5$.

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Reply to "Comments on 'New Method of Performance Analysis for Diversity Reception With Correlated Rayleigh-Fading Signals'"

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The paper in [1] attempted to simplify the procedures of performance analysis for maximal ratio combining (MRC), selection combining (SC) , and equal gain combining (EGC) diversity systems of two correlated Rayleigh-fading channels by using a known transformation technique. It was then found by Loyka *et al.* [2] that this transformation technique is correct for MRC, but leads to some performance-prediction deviations for SC and EGC diversity systems compared to the results produced by other reported approaches. Because the transformation process redistributes the signal energy among different antennas,

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