ELG6108: Introduction to Convex Optimization

Instructor: Dr. Sergey Loyka (CBY A608)
Lectures: Tue. 14:30-17:20, via MS Teams
Office hours: MS Teams.
Course web page: http://www.site.uottawa.ca/~sloyka/
Pre-(Co) requisites: solid knowledge of calculus and linear algebra. Matrix theory is a plus. Basic probability theory.

Marking scheme:
- Assignments 10%
- Course project + presentation 40%
- Final exam 50%

Lots of bonus points to everybody who takes active part in the course.

Final exam: will be scheduled by the university; 3h + 1h extra, open book.

Pre-requisites:
- Calculus
- Linear algebra/matrices
- Basic probability

**Week-by-week Description (approximate):**


**I Theory**


5. **Duality:** The Lagrange dual function and problem. Geometric and saddle-point interpretations. Optimality conditions. Sub-
optimal solutions via duality. Perturbation and sensitivity
analysis. Examples. Theorems of alternatives. Generalized
inequalities.

6. Application of convex optimization techniques to non-convex
problems.

II Applications

6. Approximation and fitting: Norm approximation. Least-norm
problems. Regularized approximation. Robust approximation.
Function fitting and interpolation.

7. Statistical estimation: Parametric and nonparametric
distribution estimation. Optimal detector design and
hypothesis testing. Chebyshev and Chernoff bounds.
Experiment design.

8. Geometric problems: Projection on a set. Distance between
sets. Euclidean distance and angle problems. Extremal volume
Floor planning.

9. Adaptive antennas (beamforming): minimum variance,
MMSE and max. SNR beamformers. Adaptive equalizers.
Diagonal loading and robustness. MIMO systems: optimal
precoding and decoding, water-filling algorithm.

III Algorithms

10. Unconstrained minimization (gradient and steepest decent
methods). Equality constrained minimization. Interior-point
methods (the barrier method).
References: Main Book
1. S. Boyd, L. Vandenberghe, Convex Optimization, Cambridge University Press, 2004. - *this is an exceptionally well-written book. It is strongly recommended that you study it carefully to learn not only how to recognize, formulate and solve convex optimization problems, but also how to write well in technical English. Solving end-of-chapter problems is essential for deep understanding of the material.*

Additional Books (available in the library, some in pdf)

Some more references (deep but mathematically demanding)

Matrices/linear algebra

15. A.J. Laub, Matrix Analysis for Scientists and Engineers, SIAM, 2005. - this is a good introductory book with discussion of basic techniques and results in linear algebra and matrix theory.
16. F. Zhang, Matrix Theory, Springer, 1999. – this is a more comprehensive textbook of matrix theory, with many end-of-chapter problems.
17. R.A. Horn, C.R. Johnson, Matrix Analysis, Cambridge University Press. – this and the 2nd volume (next) is a comprehensive book, which treats in detail all important methods and results in matrix theory; it is very well written and end-of-chapter problems are well-selected. Strongly recommended, especially if you use matrix theory in your research.
Useful links:

- http://www.stanford.edu/~boyd/cvxbook/
- http://www.stanford.edu/class/ee364a/courseinfo.html
  - there are many useful things at this page, including the videos of lectures by Prof. Boyd.

See the course web page for more.
Rationale:

Most problems in engineering can be formulated as optimization problems. Thus, solid knowledge of optimization techniques is essential for both engineering design and research. In the past two decades, a significant amount of new analytical results and techniques have been accumulated in the area of convex optimization. Keeping in mind that convex problems are numerous in many areas of engineering (and many that are not, can be re-formulated or approximated as convex ones), knowledge of these results and techniques can open new horizons in research for our graduate students and faculty. A quotation from the suggested textbook is in order: “We think that convex optimization is an important enough topic that everyone who uses computational mathematics should know at least a little bit about it. In our opinion, convex optimization is a natural next topic after advanced linear algebra ... and linear programming. ...Our main goal is to help the reader develop a working knowledge of convex optimization, i.e. to develop the skills and background needed to recognize, formulate and solve convex optimization problems. Developing a working knowledge of convex optimization can be mathematically demanding, especially for the reader interested primarily in applications. In our experience (mostly with graduate students in electrical engineering and computer science), the investment often pays well, and sometimes very well.” It should be mentioned that the book was written based on courses taught by the authors at Stanford and UCLA for a number of years, and is now used world-wide for similar courses.

This course will concentrate on analytical structure of convex problems and analytical tools to solve them, with detailed discussion of applications in communications, signal processing, statistical estimation, approximation and fitting etc.

At the end, the students will have developed a working knowledge of convex optimization, i.e. an ability to formulate and solve engineering design problems as convex optimization problems.
How to Study: Learning Efficiency Pyramid

![Learning Pyramid Diagram]


“Tell me and I’ll forget; show me and I may remember; involve me and I’ll understand.” – old Chinese proverb.

“No pain, no gain” – common wisdom.
How to Study

Learning efficiency pyramid is a good guideline

- **Reading** is necessary, but taken alone is not efficient
- **Solving problems** (“practice by doing”) is much more efficient
  - examples, assignments, end-of-chapter problems
- **Group discussions**
  - help provided you contribute something
- **Systematic study** during the semester
  - is a key to a success.
  - do not leave everything to the last day/night before exams!
- **Lectures**
  - should be supplemented by the items above
- **There is no substitute for active learning!** “Seat and watch” approach does not work!