Assignment #4

Due: by 4pm, Mar. 24, in MNO C211 (lecture). Late entries will not be accepted. Only hard copies, no email submissions.

Reading: Chapter 5 of the course textbook (S. Boyd, L. Vandenberghe, Convex Optimization, Cambridge University Press, 2004). Study carefully all examples, make sure you understand them and can repeat them with the book closed. You are encouraged to at least read all end-of-chapter problems and attempt to solve more than actually asked below. Remember the learning efficiency pyramid!

1) Problem 5.1.

2) Problem 5.26. Does Slater’s condition hold?

3) In the class, we derived the WF algorithm using the KKT conditions under the equality total power constraint, i.e. \( \sum_i p_i = P_T \). Give a derivation under the inequality constraint, \( \sum_i p_i \leq P_T \). Prove (from the KKT conditions) that it is always optimal to use the full available power, i.e. that the inequality always holds with equality at optimal point.

4) Consider a multi-stream transmission system with 3 parallel channels and independent AWGN noise on each channel, as discussed in the class. The noise powers are \( \alpha_1 = 1, \alpha_2 = 2, \alpha_3 = 1 \).
   a. Sketch a block diagram of this system, clearly indicate signals at its key points and explain what they are. Write down the equations for the rate on each channel and the total rate, for given Tx signal powers \( p_i, i = 1,2,3 \).
   b. State the optimization problem to maximize the total (sum) rate subject to the total power constraint (\( \leq P_T \)). Is it a convex problem?
   c. Write down the water-filling solution to this problem; explain how the dual variable is determined.
   d. Implement the WF algorithm in a software code (of your choice, Matlab is a good choice). Plot the optimal power allocation \( p_i^* (P_T) \), \( i = 1,2,3 \), vs. \( P_T \) (on a single graph), and, on a separate graph, the total rate \( R(P_T) \) vs. \( P_T \), for \( 0 \leq P_T \leq 100 \). Explain what you see. Hint: a bisection algorithm can be used to efficiently find the dual variable for a given \( P_T \).

5) Problem 5.31.

6) Problem 5.32.

Important rules (deviation will be penalized):

Please give your solutions in the order indicated above. Start each new problem on a new page (no 2 problems on the same page). Staple the sheets.

Please include in your solutions all the intermediate results and their numerical values (if applicable). Detailed solutions are required, not just the final answers.

Make sure your handwriting is readable and is sufficiently large so it can be read without a microscope; otherwise, it will be ignored.

Plagiarism (i.e. “cut-and-paste” from a student to a student, other forms of “borrowing” the material for the assignment) is absolutely unacceptable and will be penalized. Each student is expected to submit his own solutions. If two (or more) identical or almost identical sets of solutions are found, each student involved receives 0 (zero) for that particular assignment. If this happens twice, the students involved receive 0 (zero) for the entire assignment component of the course in the marking scheme and the case will be send to the Dean’s office for further investigation.