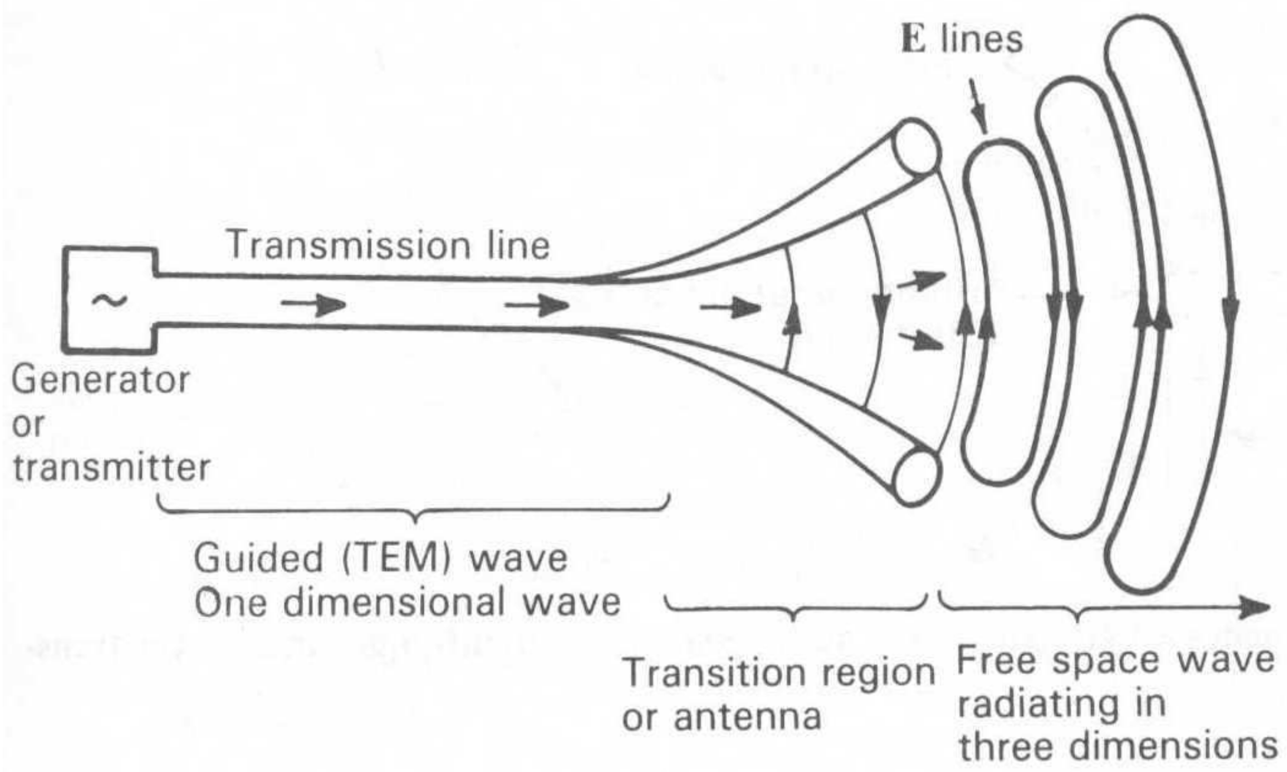
Introduction to Antennas & Arrays

Antenna – transition region (structure) between guided eaves (i.e. coaxial cable) and free space waves.

On transmission, antenna accepts energy from TL and radiates it into space.



J.D. Kraus, Antennas, McGraw Hill, 1988

On reception, antenna gathers energy from incident free-space wave and sends it into TL.

Reciprocity theorem: almost all antenna properties are the same on transmission and reception. Hence, no need to study them twice.

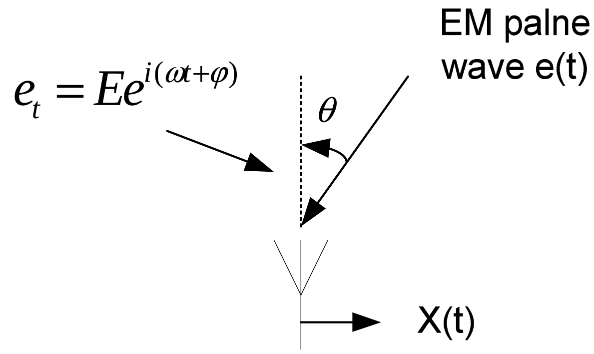
Antenna pattern

Antenna pattern - describes directional properties of an antenna.

Assume a plane wave incident upon the antenna from a particular direction

The output signal is

$$x(t) = A(\theta)e^{j(\omega t+\varphi(\theta))} \tag{4.1}$$



The amplitude pattern is

$$F(\theta) = A(\theta)/A_{\max} \tag{4.2}$$

Similarly, power pattern

$$F^2(\theta) \tag{4.3}$$

Antenna directivity: ratio of maximum radiation intensity (power per unit solid angle) to the average radiation intensity.

Gain = directivity if the antenna is lossless (assumed below).

Radiation intensity:

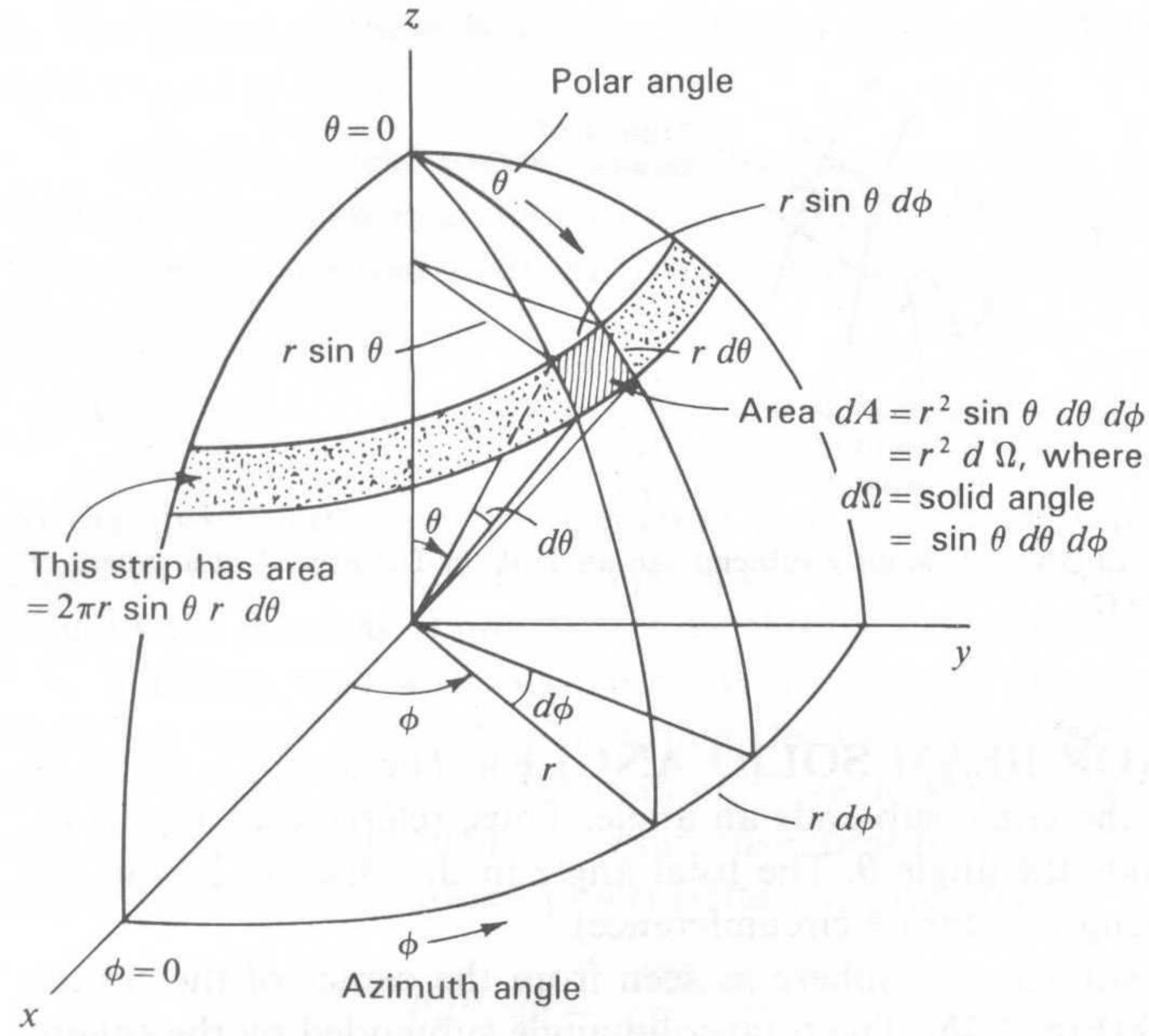
$$\Phi(\theta) = c \cdot F^2(\theta) \tag{4.4}$$

c – constant.

Reciprocity: the Tx and Rx patterns are the same.

θ - incidence angle or angle-of-arrival (AoA).

Solid Angle and Spherical Coordinate System



J.D. Kraus, Antennas, McGraw Hill, 1988.

Gain expressions:

$$\Phi_{max} = c; \qquad \overline{\Phi} = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} \Phi(\theta, \varphi) \sin\theta \, d\theta \, d\varphi \tag{4.5}$$

$$\text{Gain}: \qquad G = \frac{\Phi_{max}}{\overline{\Phi}} = \frac{4\pi}{\int_0^{2\pi} \int_0^{\pi} F^2(\theta, \varphi) \sin\theta \, d\theta \, d\varphi} \tag{4.6}$$

θ - elevation, φ- azimuth

Antenna gain compares max. radiation intensity of a given antenna with that of an isotropic antenna for the fixed total power radiated.

In receive mode, antenna gain tells us how much more power is received by given antenna as compared to the isotropic one

Omnidirectional antenna:

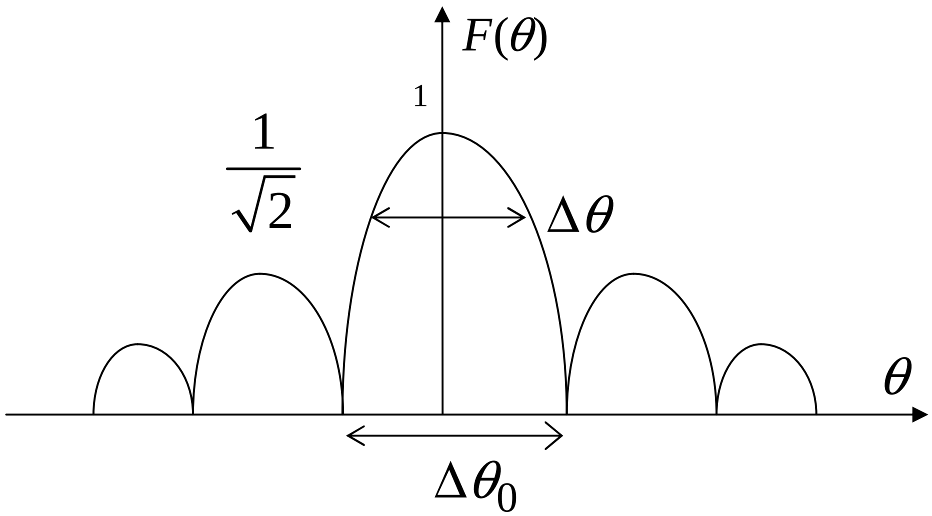
$$F(\theta, \varphi) = F(\theta) \tag{4.7}$$

$$G = \frac{2}{\int_0^{\pi} F^2(\theta) \sin\theta \, d\theta} \tag{4.8}$$

Isotropic antenna:

$$F(\theta, \varphi) = 1 \tag{4.9}$$

Antenna Beamwidth



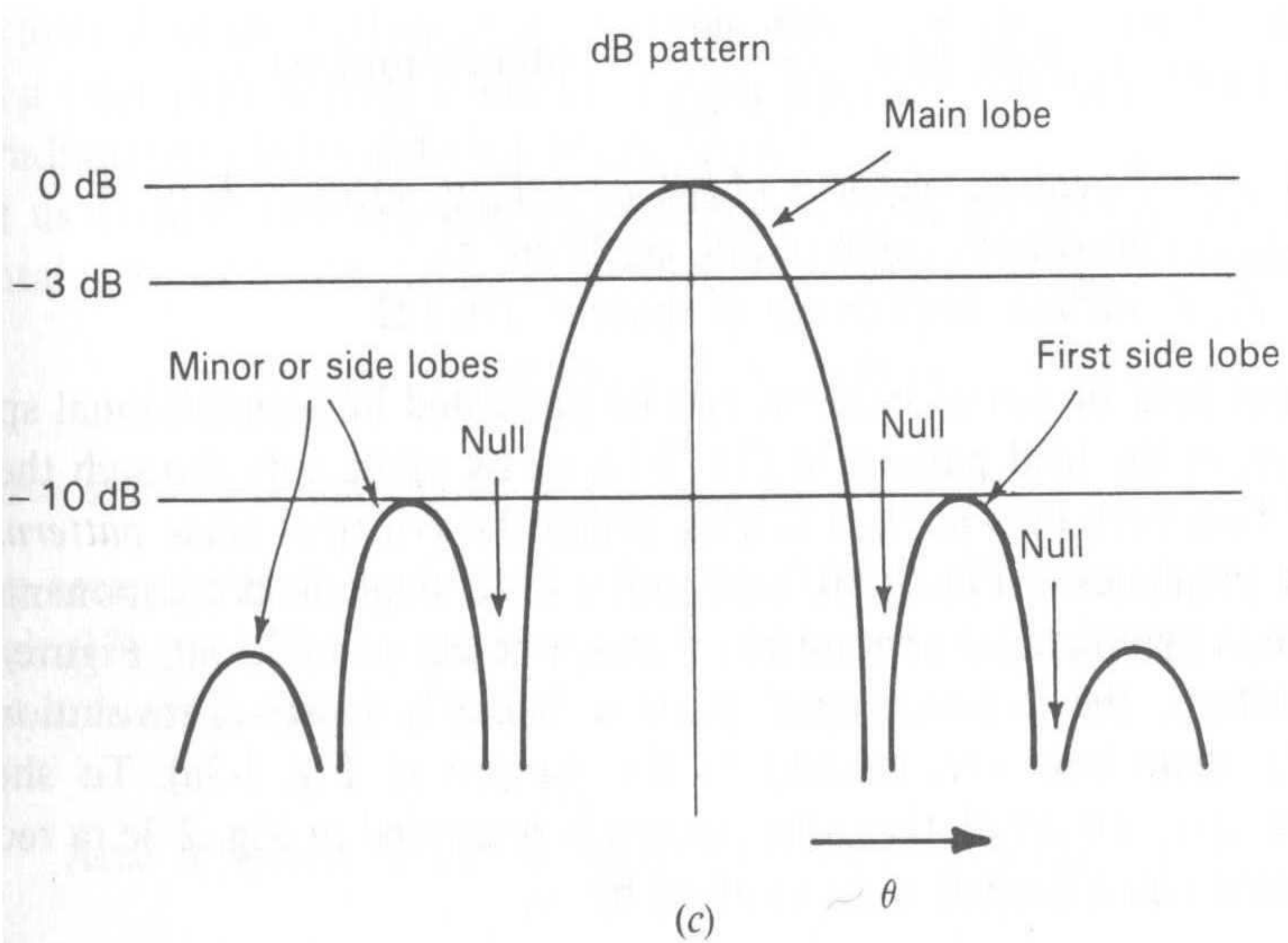
Antenna beamwidth:

3dB beam width $\Delta\theta$ - distance between the points of

$$F(\theta) = 1/\sqrt{2} \quad (-3dB) \tag{4.10}$$

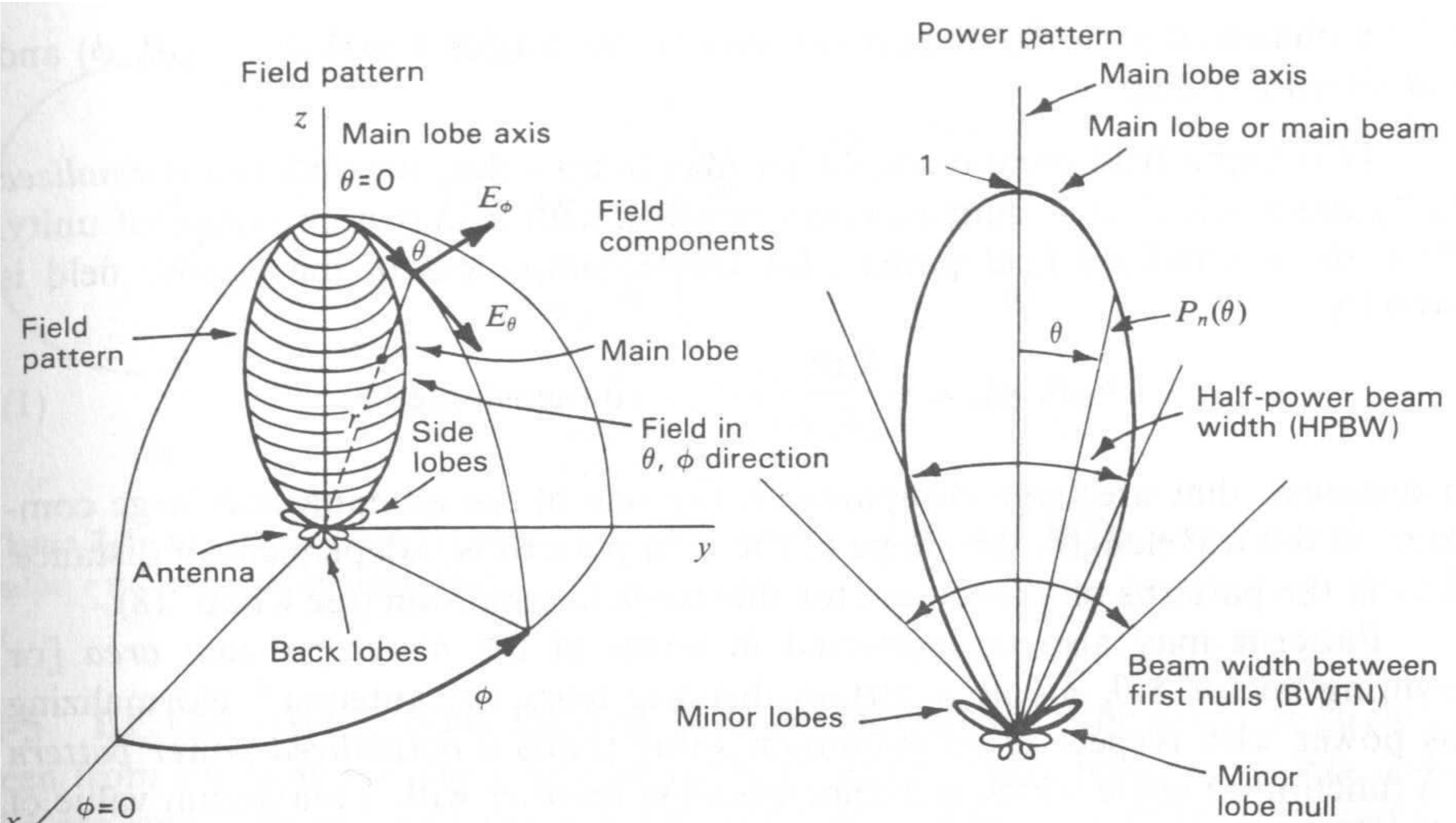
Zero-to-zero beamwidth, $\Delta\theta_0$, is distance between the points of $F(\theta) = 0$.

Antenna Pattern Components



J.D. Kraus, Antennas, McGraw Hill, 1988.

Antenna Pattern in 3-D

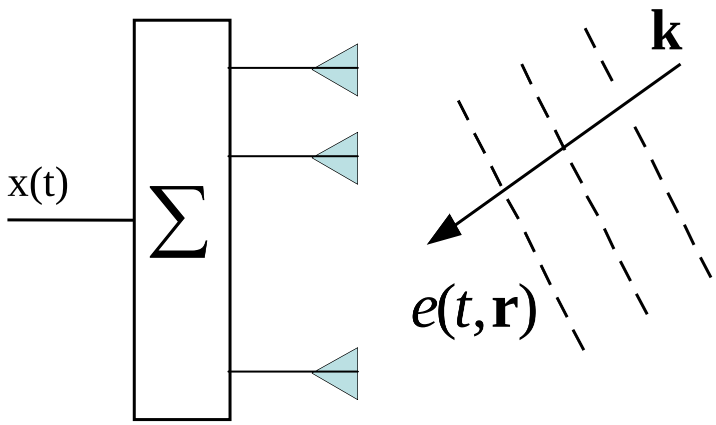


(a) (b)

Uniform Linear Array (of isotropic elements)

Plane wave: input signal (field) is a plane wave or a combination of plane waves (function of space and time) and output signal is a conventional signal (function of time only).

Plane wave signal



$$e(t,\mathbf{r}) = \mathrm{E} \cdot e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})} \qquad (4.11)$$

E – wave amplitude

ω – frequency (radial)

$\mathbf{k}$ - wave vector (sometimes called "number") [1]

$\mathbf{r}$ - position vector (identifies a point in space)

$|\mathbf{k}| = \frac{2\pi}{\lambda} = \frac{\omega}{c}$; λ – wavelength

Direction of $\mathbf{k}$ is the same as the direction of the wave front, the surface of constant amplitude and phase is a plane (for a given moment of time) -> $\mathbf{kr} = \mathrm{const}$.

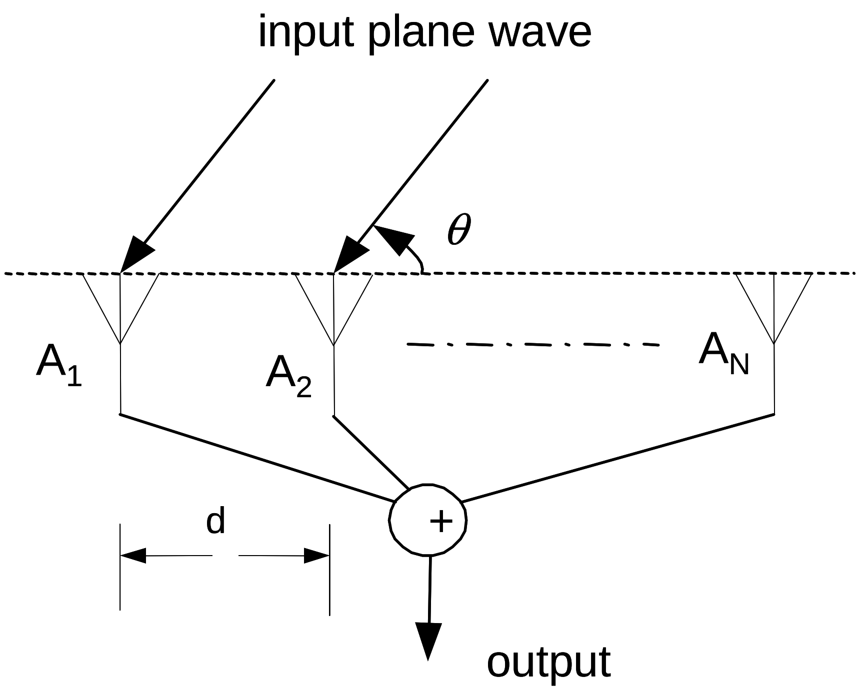
[1] Notations: bold capital (**K**) – matrices; bold lower case (**k**) – vectors; lower case regular (k) – scalars; $\mathbf{k}_i$ - i-th column of **K**.

Why plane waves are important?

- Almost any reasonable (physical) field can be decomposed into plane waves (Fourier transform in 3-D).
- EM field from a distant source (far zone) looks locally like a plane wave.

Note: if $\mathbf{k} \| \mathbf{r}$, then $\Delta\varphi = -kr = -\frac{2\pi}{\lambda} r$ -> phase shift due to propagation along distance *r*.

ULA response to a plane wave



$$x_i(\mathrm{t}) = \mathrm{a}e^{i(\omega t+\varphi_i)} \qquad (4.12)$$

$x_i(\mathrm{t})$ - output signal of i-th antenna element (assumed to be istropic)

a - signal amplitude (the same for all elements)

φ_i – signal phase

Drop $e^{j\omega t}$ -> it is everywhere, further work with complex amplitude,

$$A_i = \mathrm{a}e^{j\varphi_i}, \text{ assume } \varphi_1 = 0: \quad \varphi_2 = \frac{2\pi}{\lambda} d\cos\theta \qquad (4.13)$$

$$\varphi_i = \frac{2\pi}{\lambda} d(i-1)\cos\theta \qquad (4.14)$$

ULA Pattern (factor)

Total signal amplitude at the output,

$$y = \sum_{i=1}^{N} A_i = a \sum_{i=1}^{N} e^{j(i-1)\psi} = a \frac{\sin \frac{N\psi}{2}}{\sin \frac{\psi}{2}} e^{j\frac{(N-1)\psi}{2}} \qquad (4.15)$$

$$\psi = \frac{2\pi}{\lambda} d \cos\theta \qquad (4.16)$$

Array pattern = normalized response to a plane wave versus AoA,

$$F(\theta) = \frac{\sin \frac{N\psi}{2}}{N \sin \frac{\psi}{2}} \qquad (4.17)$$

Assuming array is located along oz,

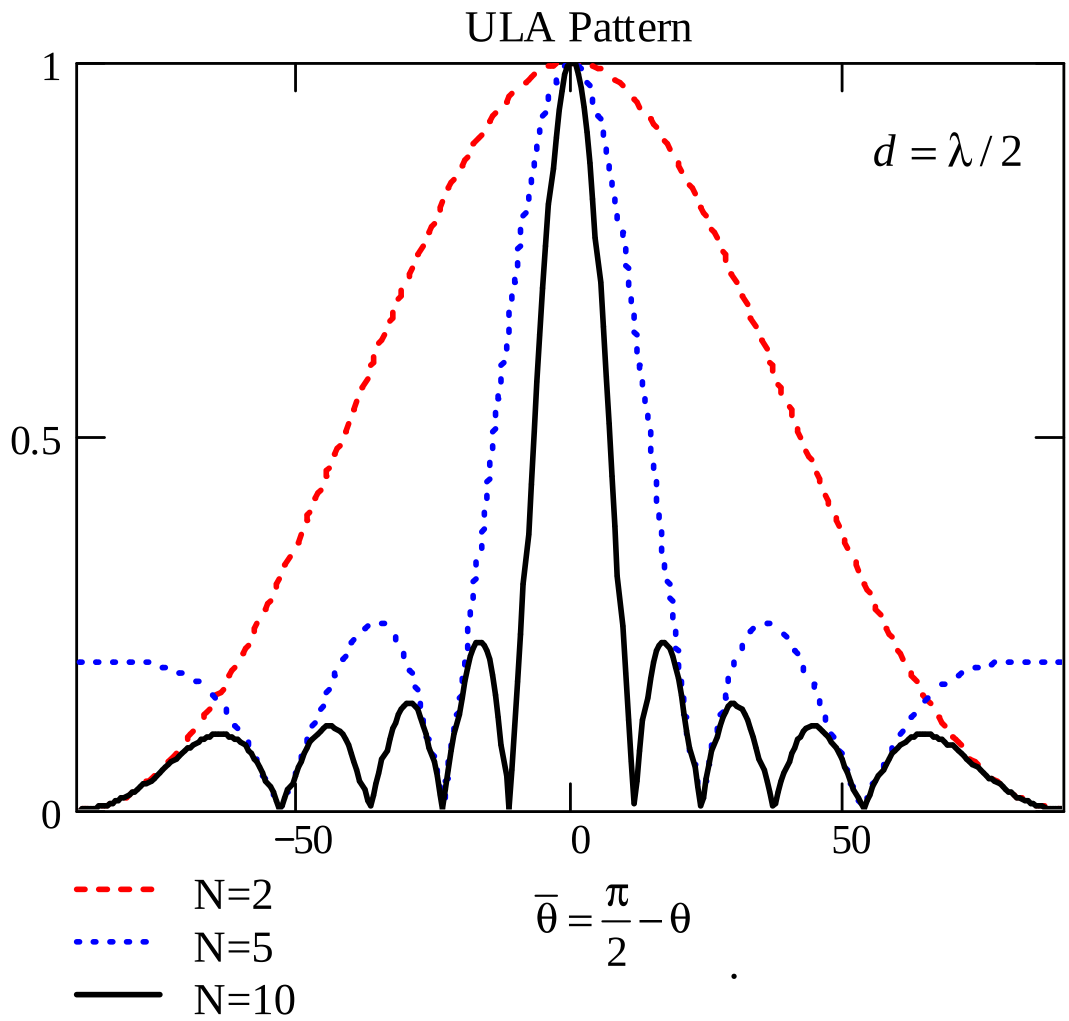
$$F(\mathbf{k}) = \frac{\sin(d \cdot k_z \frac{N}{2})}{N \sin(\frac{d \cdot k_z}{2})} \qquad (4.18)$$

Note:

$$\psi_{\max} = \frac{2\pi}{\lambda} d; \quad \psi_{\min} = -\frac{2\pi}{\lambda} d \qquad (4.19)$$

We started with isotropic elements and managed to obtain a highly-selective pattern!

ULA Pattern



ULA Pattern Parameters

3 dB beamwidth -> $|F(\theta)|^2 = 0.5$ -> no closed-form solution

For large N (N>>1),

$$\Delta\theta_0 \approx 51\frac{\lambda}{L};\ L = Nd \text{ = array aperture (length)} \qquad (4.20)$$

Null-to-null beamwidth

$$\Delta\theta_0 = 2\frac{\lambda}{L}[rad] = 114\frac{\lambda}{L}[\deg]; \qquad (4.21)$$

First sidelobe = -13.2dB

Gain (directivity): if $d = \frac{\lambda}{2}, \quad G = N$

Generic Representation of Array Response

Array elements spatially sample incoming wave

$$\mathbf{x}(t) = \left[e(t,\mathbf{p_0}) \quad e(t,\mathbf{p_1}) \quad \ldots \quad e(t,\mathbf{p_{N\text{-}1}}) \right]^T \qquad (4.22)$$

where $\mathbf{p_0} \ldots \mathbf{p_{N\text{-}1}}$ – position vectors of array elements.

Using plane-wave model of $e(t,\mathrm{p}) = \mathrm{E}e^{i(\omega t - \mathrm{kp})}$,

$$\mathbf{x}(t) = E \cdot e^{i\omega t} \begin{bmatrix} e^{-j\mathbf{k}\mathbf{p}_0} & e^{-j\mathbf{k}\mathbf{p}_1} & \ldots & e^{-j\mathbf{k}\mathbf{p}_{N-1}} \end{bmatrix}^T \qquad (4.23)$$

Array manifold vector is

$$\mathbf{v}(\mathbf{k}) = \begin{bmatrix} e^{-j\mathbf{k}\mathbf{p}_0} & e^{-j\mathbf{k}\mathbf{p}_1} & \ldots & e^{-j\mathbf{k}\mathbf{p}_{N-1}} \end{bmatrix}^T \qquad (4.24)$$

It gives us a phase at each array element.

Array response to a plane wave can be expressed as

$$y(t) = \sum_{i=1}^{N-1} x_i(t) \qquad (4.25)$$

$$= E \cdot e^{i\omega t} \sum_{k=0}^{N-1} v_i(\mathbf{k}) \qquad (4.26)$$

Array pattern (factor) is

$$F(\mathbf{k}) \sim \sum_{k=0}^{N-1} v_i(\mathbf{k}) \qquad (4.27)$$

Generic case: assume that each elements has impulse response of the form $\mathrm{h}_i(\tau)$, introduce vector impulse response of the array

$$\mathbf{h}(\tau) = [\mathrm{h}_0(\tau) \quad \mathrm{h}_1(\tau) \quad \ldots \quad \mathrm{h}_{N\text{-}1}(\tau)]^T \tag{4.28}$$

$e(t,\mathbf{p})$ may be a generic wave, the array output can be expressed as

$$y(t) = \int_{-\infty}^{t} \mathbf{h}(\mathrm{t} - \tau)^{\mathrm{T}} \mathbf{e}(\tau) d\tau \tag{4.29}$$

where

$$\mathbf{e}(\tau) = [e(\tau,\mathbf{p}_0) \quad e(\tau,\mathbf{p}_1) \quad \ldots \quad e(\tau,\mathbf{p}_{N-1})]^T \tag{4.30}$$

Very similar to conventional response (convolution integral) of an LTI system.

Fourier transform to frequency domain,

$$y(\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt = \mathbf{h}(\omega)^T \cdot \mathbf{e}(\omega) \tag{4.31}$$

$$\mathbf{h}(\omega) = FT\{\mathbf{h}(\mathrm{t})\}, \mathbf{e}(\omega) = FT\{\mathbf{e}(\mathrm{t})\} \tag{4.32}$$

$\mathbf{h}(\omega)$ = vector frequency response of the array, $\mathbf{e}(\omega)$ = spectrum of incoming field at element locations,

$$\mathbf{e}(\omega) = e(\omega) \cdot \mathbf{v}(\mathbf{k}) \tag{4.33}$$

Array scalar frequency response is

$$h(\omega) = y(\omega)/e(\omega) = \mathbf{h}(\omega)^T \mathbf{v}(\mathbf{k}) \tag{4.34}$$

It depends on the frequency response of individual elements ($\mathbf{h}(\omega)$) and on the array geometry ($\mathbf{v}(\mathbf{k})$)

Varying $\mathbf{h}(\omega)$ may perform beamforming, e.g. to steer the beam in desired direction.

Example: delay-and-sum beamformer

$$h_i(\tau) = \delta(\tau + \tau_i) \tag{4.35}$$

$$\tau_i = \frac{\mathbf{k}_0 \mathbf{p}_i}{\omega} \tag{4.36}$$

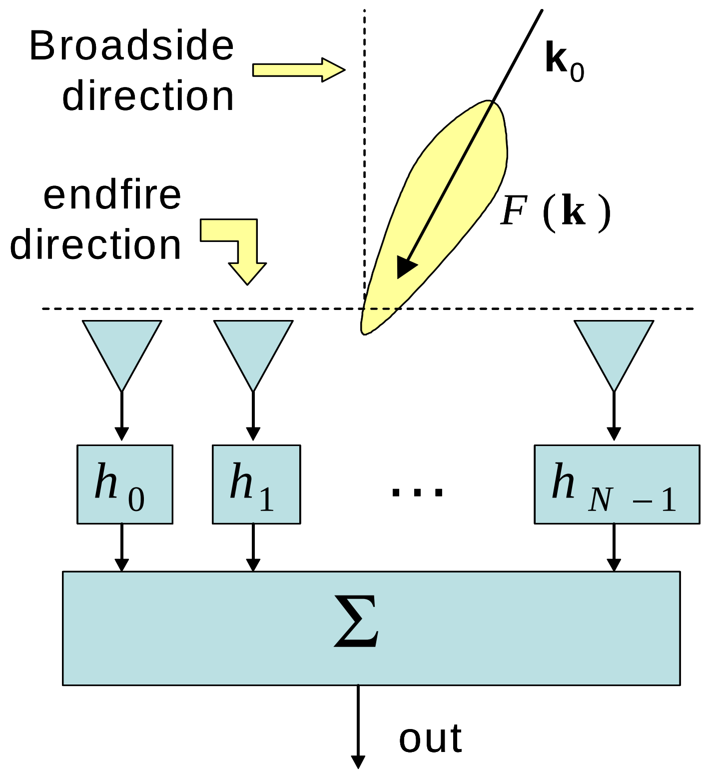
$$h_i(\omega) = e^{j\omega\tau_i} = e^{j\mathbf{k}_0 \mathbf{p}_i} \tag{4.37}$$

and the array pattern (factor)

$$F(\mathbf{k}) = \frac{1}{N} \mathbf{h}^T \mathbf{v}(\mathbf{k}) = \frac{1}{N} \sum_{i=0}^{N-1} e^{j\mathbf{k}_0 \mathbf{p}_i} e^{-j\mathbf{k}\mathbf{p}_i} \tag{4.38}$$

$$= \frac{1}{N} \sum_{i=0}^{N-1} v_i(\mathbf{k} - \mathbf{k}_0) \tag{4.39}$$

Block Diagram of Antenna Array



$$F(\mathbf{k}) = \frac{1}{N}\mathbf{h}^T \mathbf{v}(\mathbf{k})$$

$$= \frac{1}{N}\sum_{i=0}^{N-1} e^{j\mathbf{k}_0\mathbf{p}_i} e^{-j\mathbf{k}\mathbf{p}_i}$$

$$= \frac{1}{N}\sum_{i=0}^{N-1} v_i(\mathbf{k} - \mathbf{k}_0) \qquad (4.40)$$

$\mathbf{k}_0$ = steering direction.

Beam steering: the beam is directed towards $\mathbf{k}_0$!

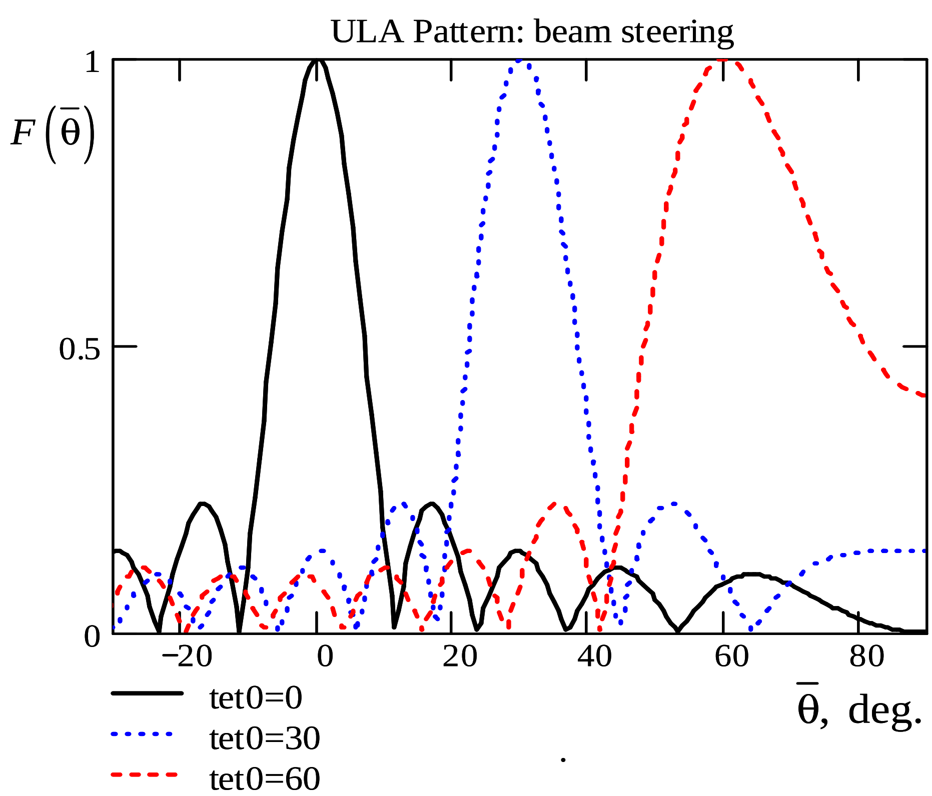
In terms of angles:

$$F(\theta) = \frac{\sin \frac{N(\psi - \psi_0)}{2}}{N \sin \frac{\psi - \psi_0}{2}} \qquad (4.41)$$

where $\psi = \frac{2\pi}{\lambda} d \cos\theta, \qquad \psi_0 = \frac{2\pi}{\lambda} d \cos\theta_0$

Half-power beamwidth,

$$\bar{\theta}_0 = \frac{\pi}{2} - \theta_0, \qquad \bar{\theta}_0 < \frac{\pi}{2}, \ \Delta\theta \approx \frac{51^\circ \lambda}{N d \cos\bar{\theta}_0} \qquad (4.42)$$



Summary

Introduction to antenna arrays.

Antenna pattern. Main beam and sidelobes. Nulls.

Half-power beamwidth and zero-to-zero beamwidth. Sidelobe level. Antenna gain (directivity).

Uniform linear array. Array pattern, beamwidth, sidelobe level.

Beam steering. Pattern parameters.

References

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R.C. Hansen, Phased Array Antennas, Wiley, New York, 1998.

A.W. Rudge et al (Eds.), The Handbook of Antenna Design, IEE & Peter Peregrinus, Norflok, 1986.