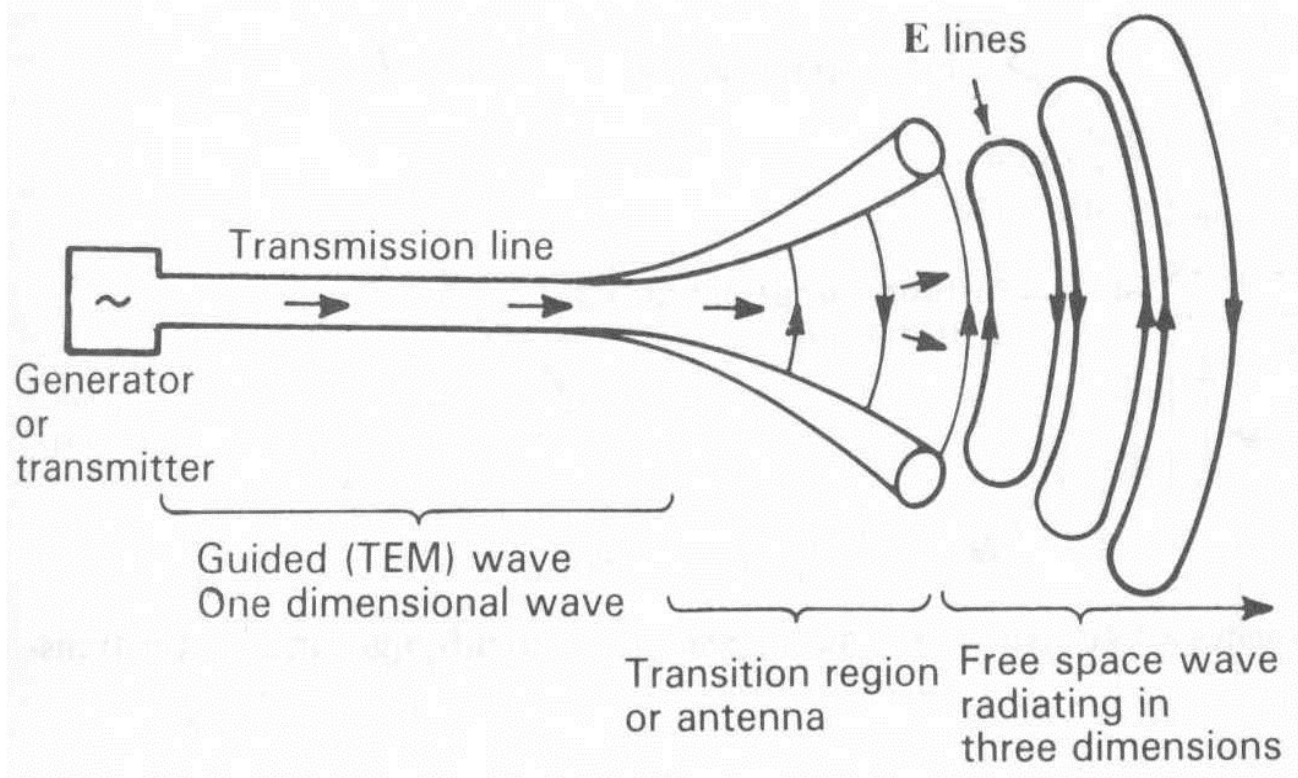


An Introduction to Antennas & Arrays

Antenna – transition region (structure) between guided waves (i.e. coaxial cable) and free space waves.

On transmission, antenna accepts energy from TL and radiates it into space.

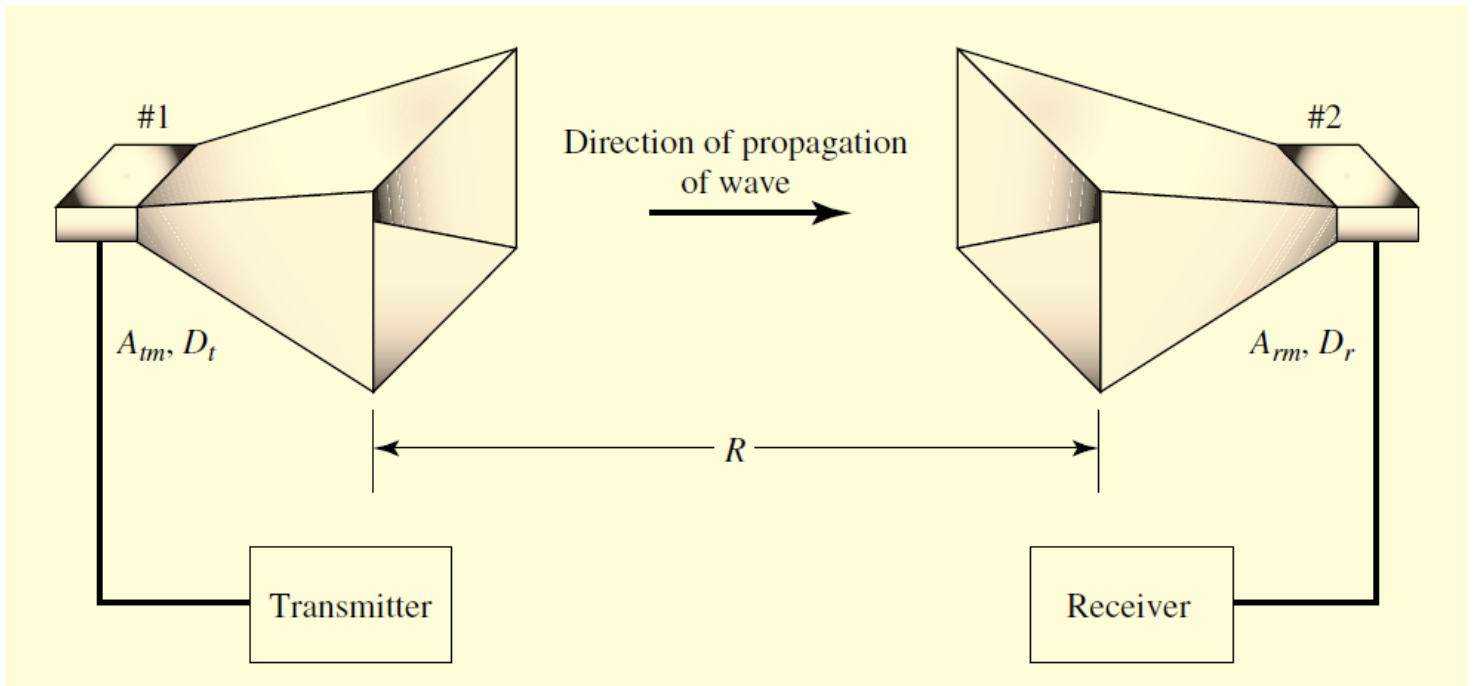


J.D. Kraus, Antennas, McGraw Hill, 1988

On reception, antenna gathers energy from incident free-space wave and sends it into TL.

Reciprocity theorem: almost all antenna properties are the same on transmission and reception. Hence, no need to study them twice.

Antenna = transition region between guided waves and free space waves (horn antennas are shown here)



C.A. Balanis, Antenna Theory: Analysis and Design, John Wiley & Sons, 2005.

Full link budget computation (from Lecture 2):

$$P_r = P_t \frac{G_t G_r}{L_p}, \quad L_p = \left(\frac{4\pi R}{\lambda} \right)^2, \quad R \geq \frac{2D^2}{\lambda} \quad (2.5)$$

$$G = \frac{4\pi}{\lambda^2} S_e, \quad S_e = \eta_e ab, \quad \eta_e \approx 1 \quad (0.5 \dots 1) \quad (2.4)$$

Smart (Adaptive) Antennas

Antenna array (of simple antennas or sensors, e.g. dipole or monopole) + signal processing via optimum combining.

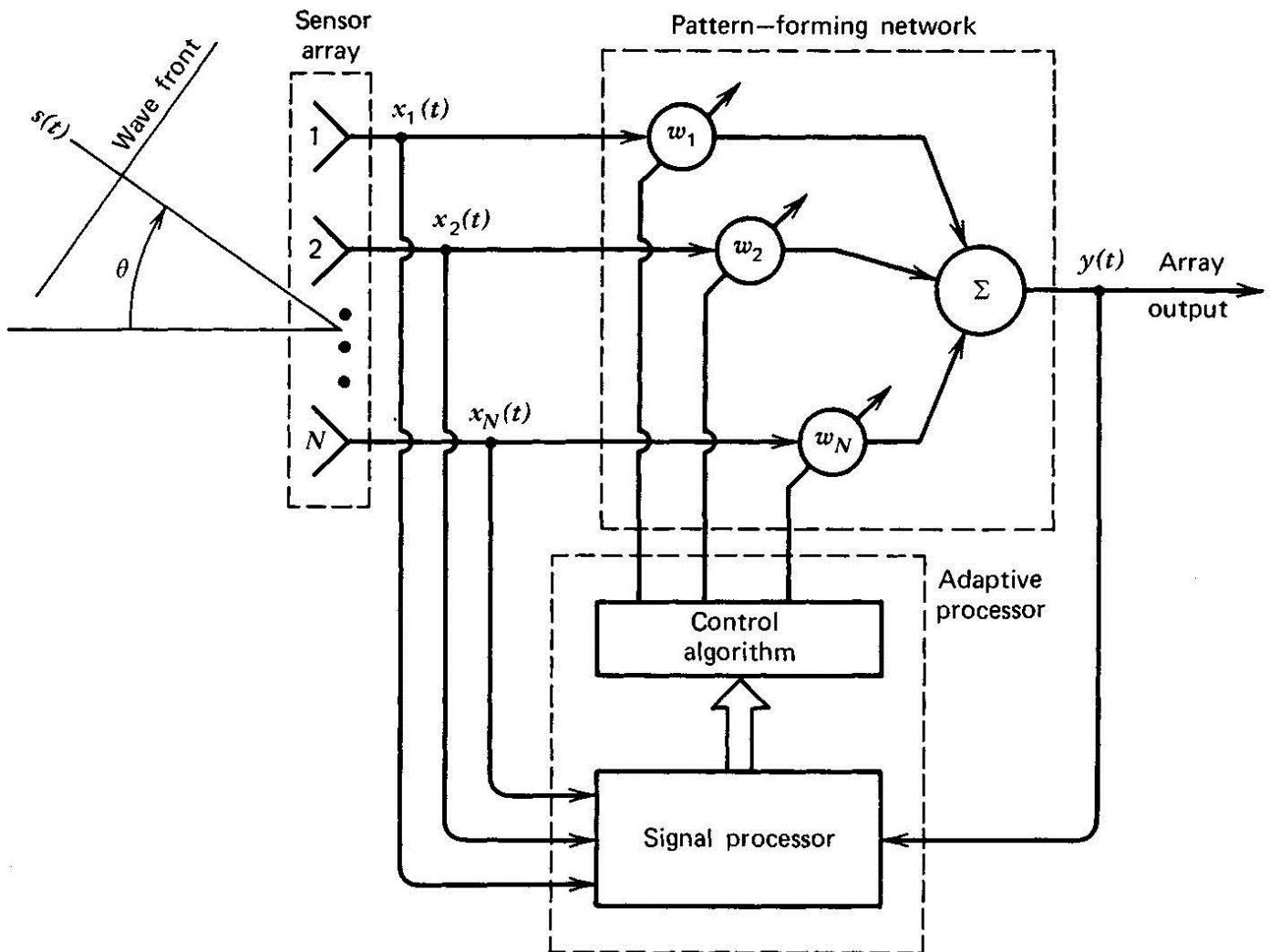


Figure 1.1 Functional diagram of an N -element adaptive array.

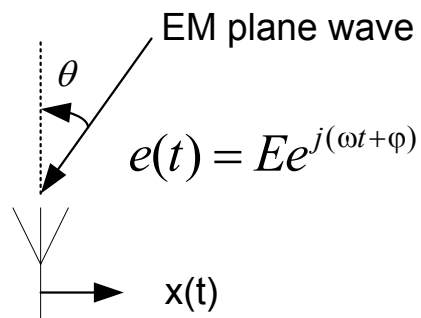
R.A. Monzingo, T.W. Miller, Introduction to Adaptive Arrays, Wiley, New York, 1980.

Antenna pattern

Antenna pattern: describes directional properties of an antenna.

Assume a plane wave incident upon the antenna from a particular direction in **free space**. The output signal is

$$x(t) = A(\theta)e^{j(\omega t + \varphi(\theta))} \quad (3.1)$$



The amplitude pattern is

$$F(\theta) = A(\theta) / A_{\max} \quad (3.2)$$

Similarly, power pattern

$$F^2(\theta) \quad (3.3)$$

Radiation intensity:

$$\Phi(\theta) = c \cdot F^2(\theta) \quad (3.4)$$

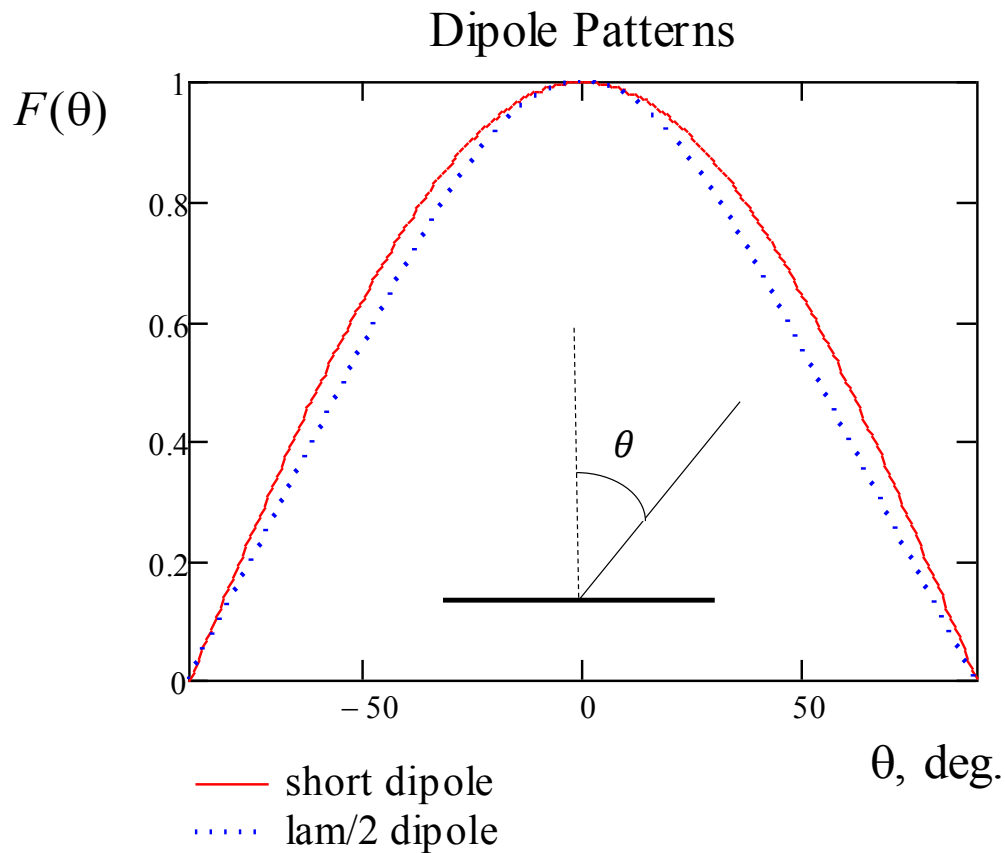
c – constant.

Reciprocity: the Tx and Rx patterns are the same.

θ - incidence angle or angle-of-arrival (AoA).

Antenna Pattern: Examples

Short and half-wavelength dipoles (the angle is from the broadside, in deg.)



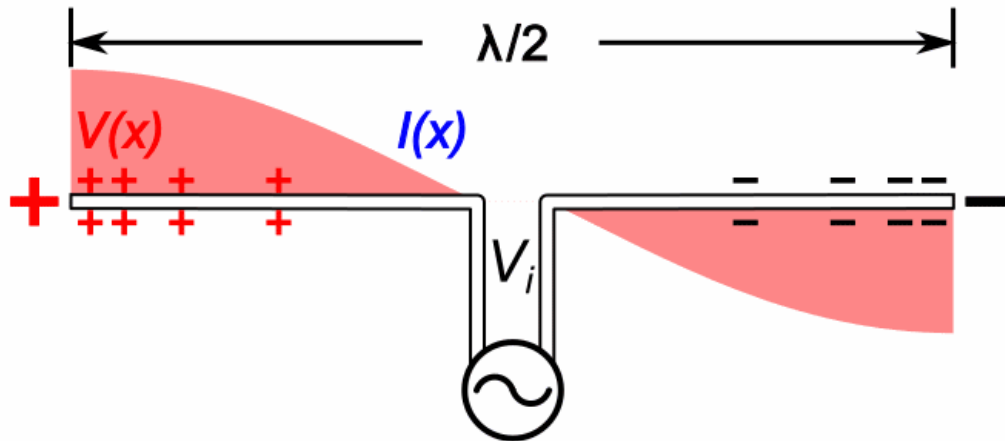
Short dipole ($L \ll \lambda$):

$$F(\theta) = |\cos \theta|$$

Half-wave dipole ($L = \lambda / 2$):

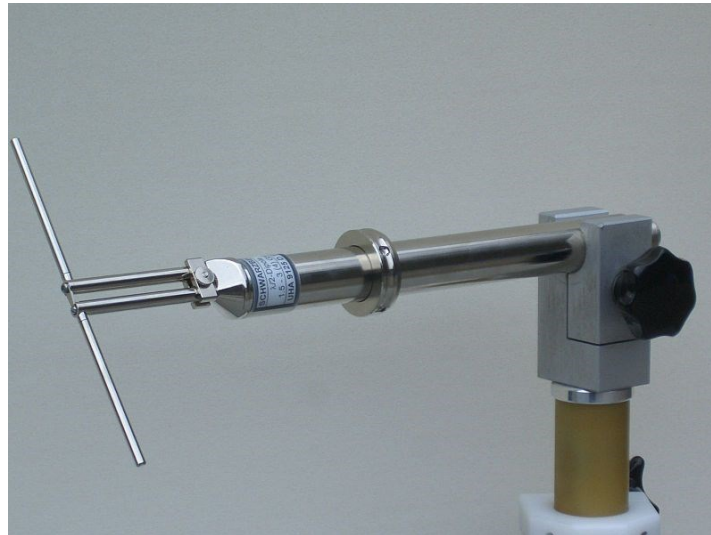
$$F(\theta) = \left| \frac{\cos\left(\frac{\pi}{2} \sin \theta\right)}{\cos \theta} \right|$$

Half-wave dipole with voltage source attached



https://en.wikipedia.org/wiki/Dipole_antenna

...and practical..



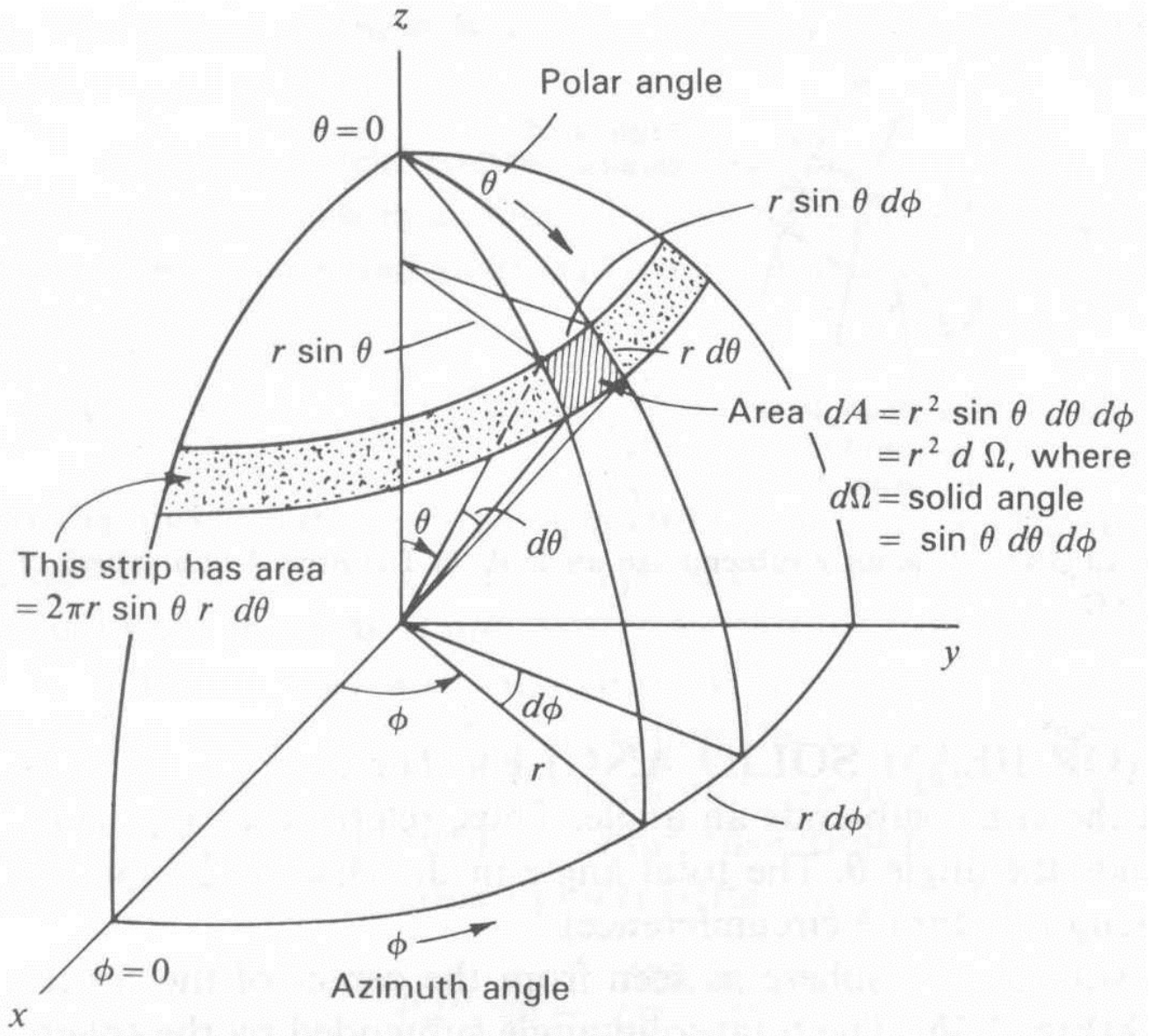
Antenna array of dipoles, SCR-270 radar (1943)



A reflective dipole array antenna in front of a large reflector (horizontal wires) to make it uni-directional; from https://en.wikipedia.org/wiki/Dipole_antenna

(SCR-270 radar antenna was part of a mobile radar system, operating at 104 MHz, that could detect and track aircraft at 125 miles. This antenna is a 1943 production version of the radar that tracked the incoming Japanese raid on Oahu, Hawaii on 7 December 1941)

Solid Angle and Spherical Coordinate System



J.D. Kraus, Antennas, McGraw Hill, 1988.

Antenna Gain

Antenna directivity: the ratio of maximum radiation intensity (power per unit solid angle) to the average radiation intensity.

Gain = directivity if the antenna is lossless (assumed below).

$$\Phi_{\max} = c; \quad \bar{\Phi} = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} \Phi(\theta, \varphi) \sin \theta d\theta d\varphi \quad (3.5)$$

$$\text{Gain : } G = \frac{\Phi_{\max}}{\bar{\Phi}} = \frac{4\pi}{\int_0^{2\pi} \int_0^{\pi} F^2(\theta, \varphi) \sin \theta d\theta d\varphi} \quad (3.6)$$

θ - elevation angle, φ - azimuth.

Antenna gain compares max. radiation intensity of a given antenna with that of an isotropic antenna for the fixed total power radiated.

In receive mode, antenna gain tells us how much more power is received by given antenna as compared to the isotropic one.

Examples: short dipole $G=1.5$ (1.8 dB), half-wave $G=1.6$ (2.2 dB). **Q.: prove this!**

Antenna theory: the gain/directivity characterize directional properties of an antenna in free space, no interference, no multiple users, no multipath, etc.

Wireless system applications: need to account for interference, multiple users, multipath, etc.

Isotropic & Omnidirectional Antennas

Omnidirectional antenna: equal radiation in all directions of a given plane (horizontal),

$$F(\theta, \varphi) = F(\theta) \quad (3.7)$$

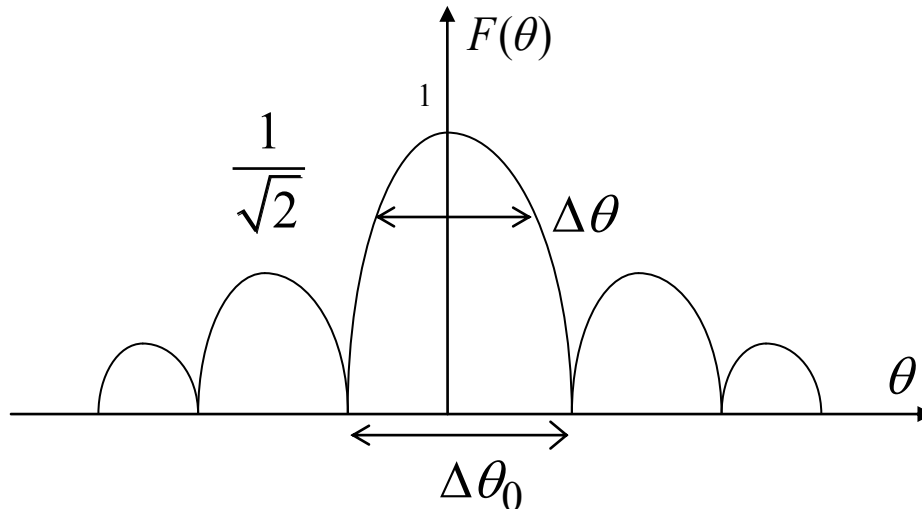
$$G = \frac{2}{\int_0^{\pi} F^2(\theta) \sin \theta d\theta} \quad (3.8)$$

Isotropic antenna: equal radiation in all directions,

$$F(\theta, \varphi) = 1 \quad (3.9)$$

Q.: $G = ?$

Antenna Beamwidth



Antenna beamwidth:

3dB beam width $\Delta\theta$: distance between the half-power (HP) points, where

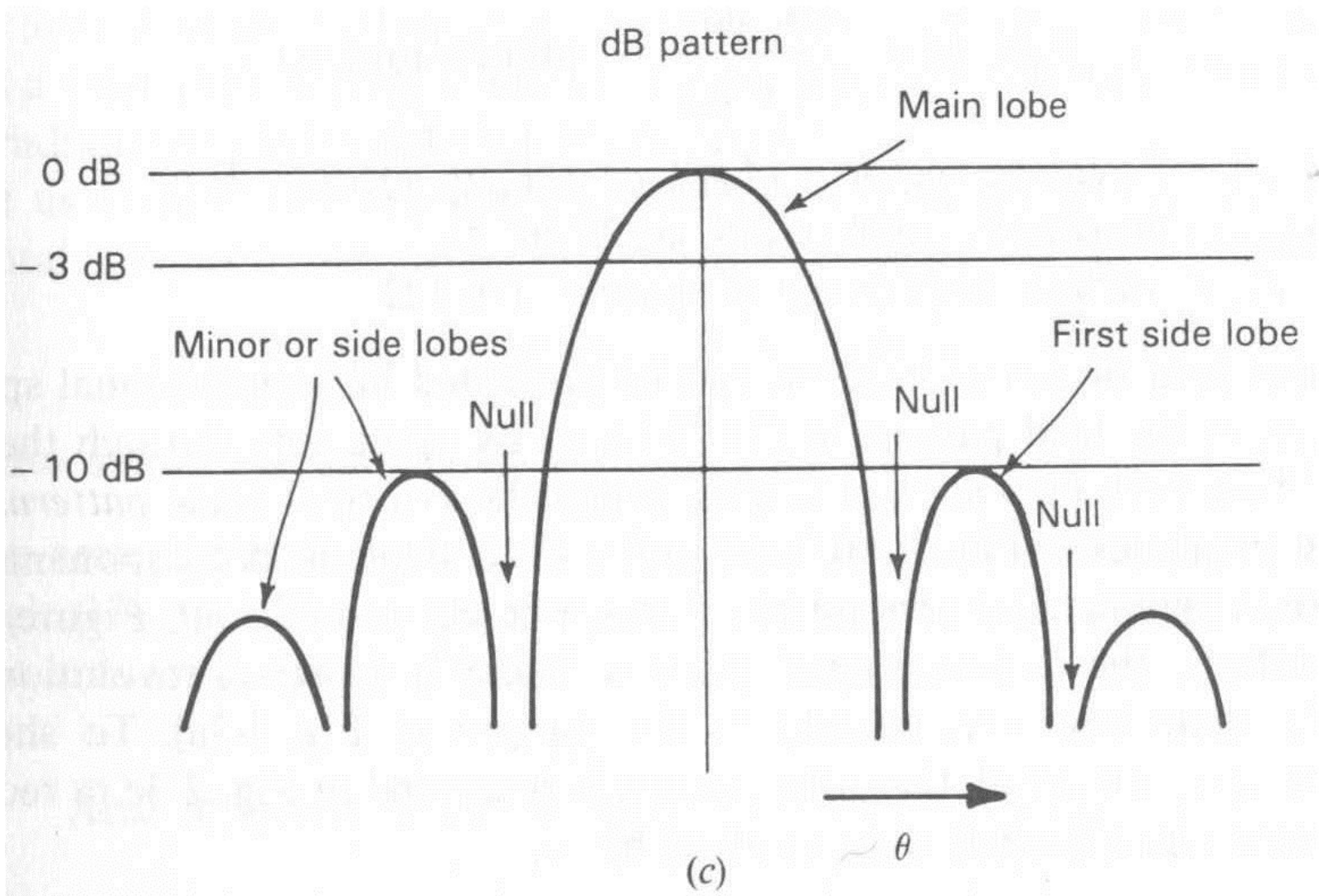
$$F(\theta) = 1/\sqrt{2} \quad (-3dB) \quad (3.10)$$

Zero-to-zero beamwidth, $\Delta\theta_0$, is distance between the points of $F(\theta) = 0$.

Examples: the HP beamwidth of short dipole = 90 deg., of half-wave dipole = 78 deg.

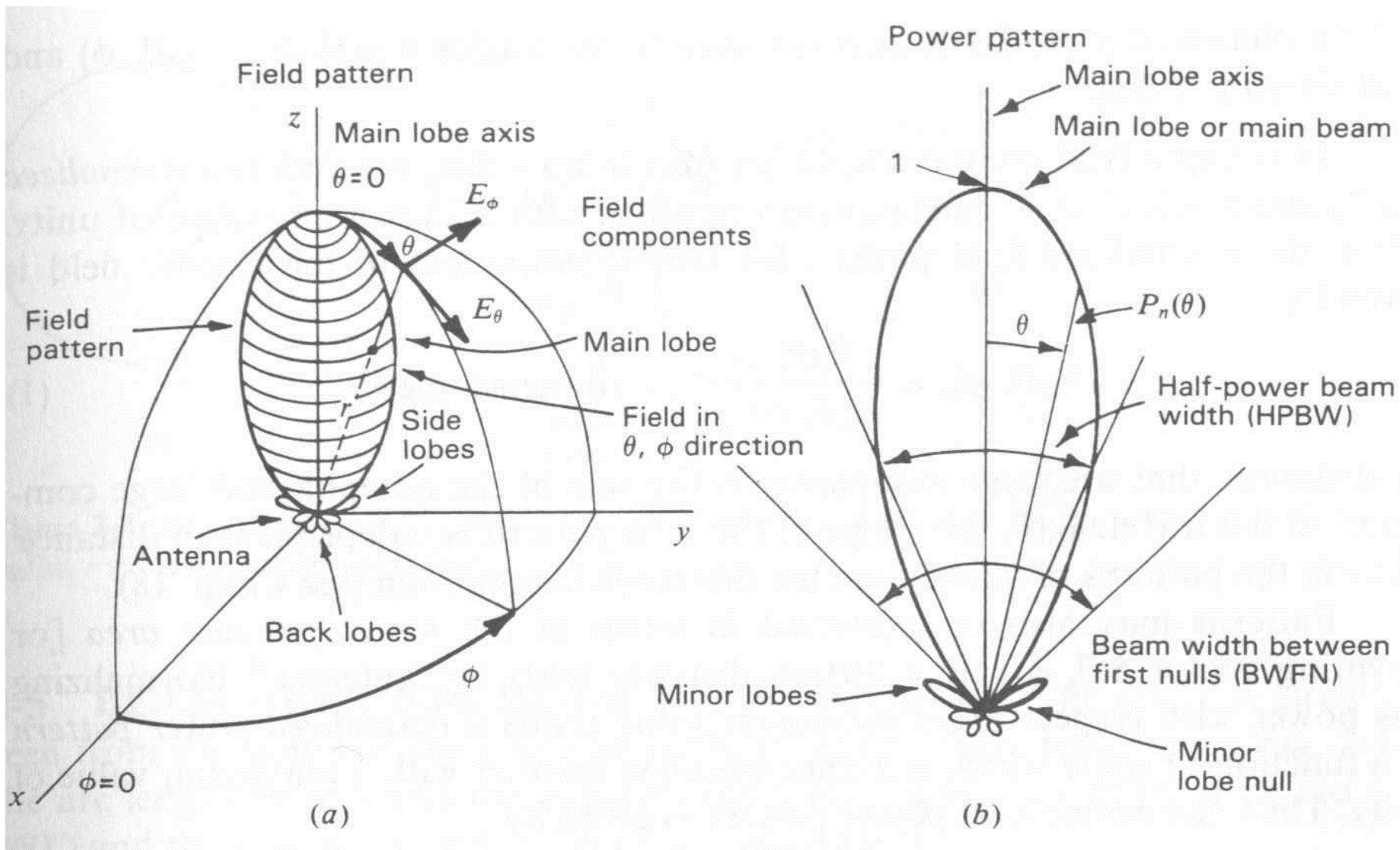
Q.: prove this !

Antenna Pattern Components



J.D. Kraus, Antennas, McGraw Hill, 1988.

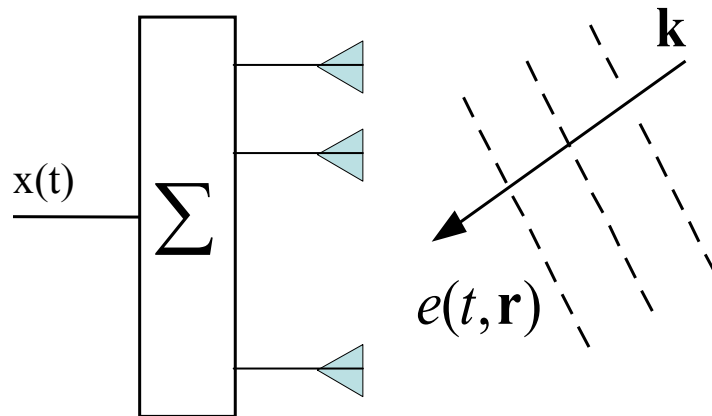
Antenna Pattern in 3-D



Uniform Linear Array (of isotropic elements)

Plane wave: input signal (field) is a plane wave or a combination of plane waves (function of space and time) and output signal is a conventional signal (function of time only).

Plane wave signal



$$e(t, \mathbf{r}) = E \cdot e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} \quad (3.11)$$

E – wave amplitude

ω – frequency (radial)

\mathbf{k} - wave vector (sometimes called "number")¹

\mathbf{r} - position vector (identifies a point in space)

$$|\mathbf{k}| = \frac{2\pi}{\lambda} = \frac{\omega}{c}; \quad \lambda - \text{wavelength}$$

Direction of \mathbf{k} is the same as the direction of the wave front, the surface of constant amplitude and phase is a plane (for a given moment of time) $\rightarrow \mathbf{k} \cdot \mathbf{r} = \text{const}$.

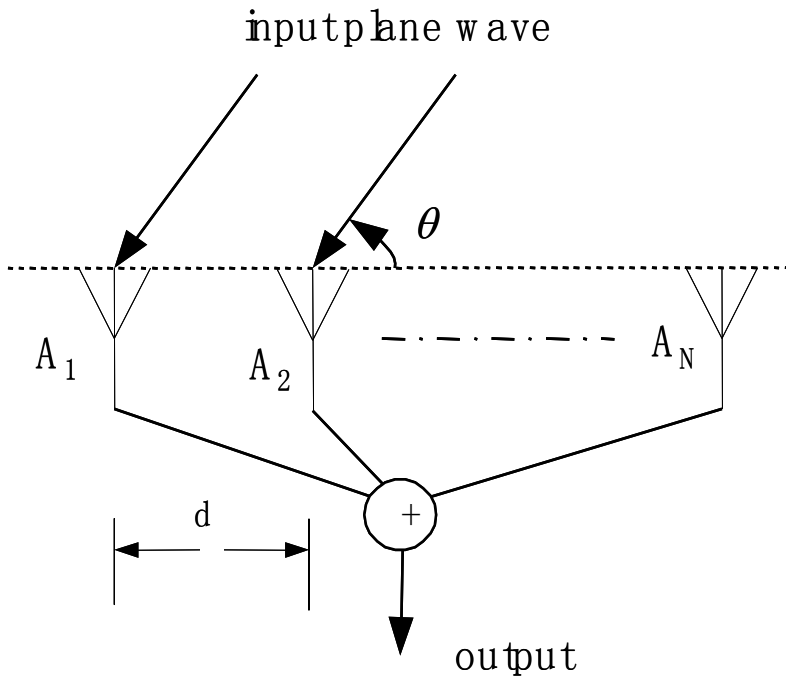
¹ Notations: bold capital (\mathbf{K}) – matrices; bold lower case (\mathbf{k}) – vectors; lower case regular (k) – scalars; \mathbf{k}_i - i -th column of \mathbf{K} .

Why **plane waves** are important?

- Almost any reasonable (physical) field can be decomposed into plane waves (Fourier transform in 3-D).
- EM field from a distant source (far zone) looks locally like a plane wave.

Note: if $\mathbf{k} \parallel \mathbf{r}$, then $\Delta\varphi = -kr = -\frac{2\pi}{\lambda}r \rightarrow$ phase shift due to propagation along distance r .

ULA response to a plane wave



$$x_i(t) = a \cdot e^{i(\omega t + \varphi_i)} \quad (3.12)$$

$x_i(t)$ - output signal of i -th antenna element

(assumed to be isotropic)

a - signal amplitude (the same for all elements)

φ_i - signal phase

Drop $e^{j\omega t}$ -> it is everywhere, further work with complex amplitude,

$$A_i = ae^{j\varphi_i}, \text{ assume } \varphi_1 = 0: \quad \varphi_2 = \frac{2\pi}{\lambda} d \cos \theta \quad (3.13)$$

$$\varphi_i = \frac{2\pi}{\lambda} d (i-1) \cos \theta \quad (3.14)$$

ULA Pattern (factor)

Total signal amplitude at the output,

$$y = \sum_{i=1}^N A_i = a \sum_{i=1}^N e^{j(i-1)\psi} = a \frac{\sin \frac{N\psi}{2}}{\sin \frac{\psi}{2}} e^{j \frac{(N-1)\psi}{2}} \quad (3.15)$$

$$\psi = \frac{2\pi}{\lambda} d \cos \theta \quad (3.16)$$

Array pattern = normalized response to a plane wave versus AoA,

$$F(\theta) = \left| \frac{\sin \frac{N\psi}{2}}{N \sin \frac{\psi}{2}} \right| \quad (3.17)$$

Assuming array is located along oz,

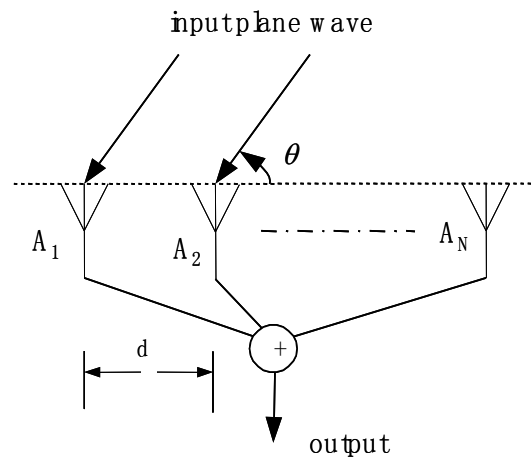
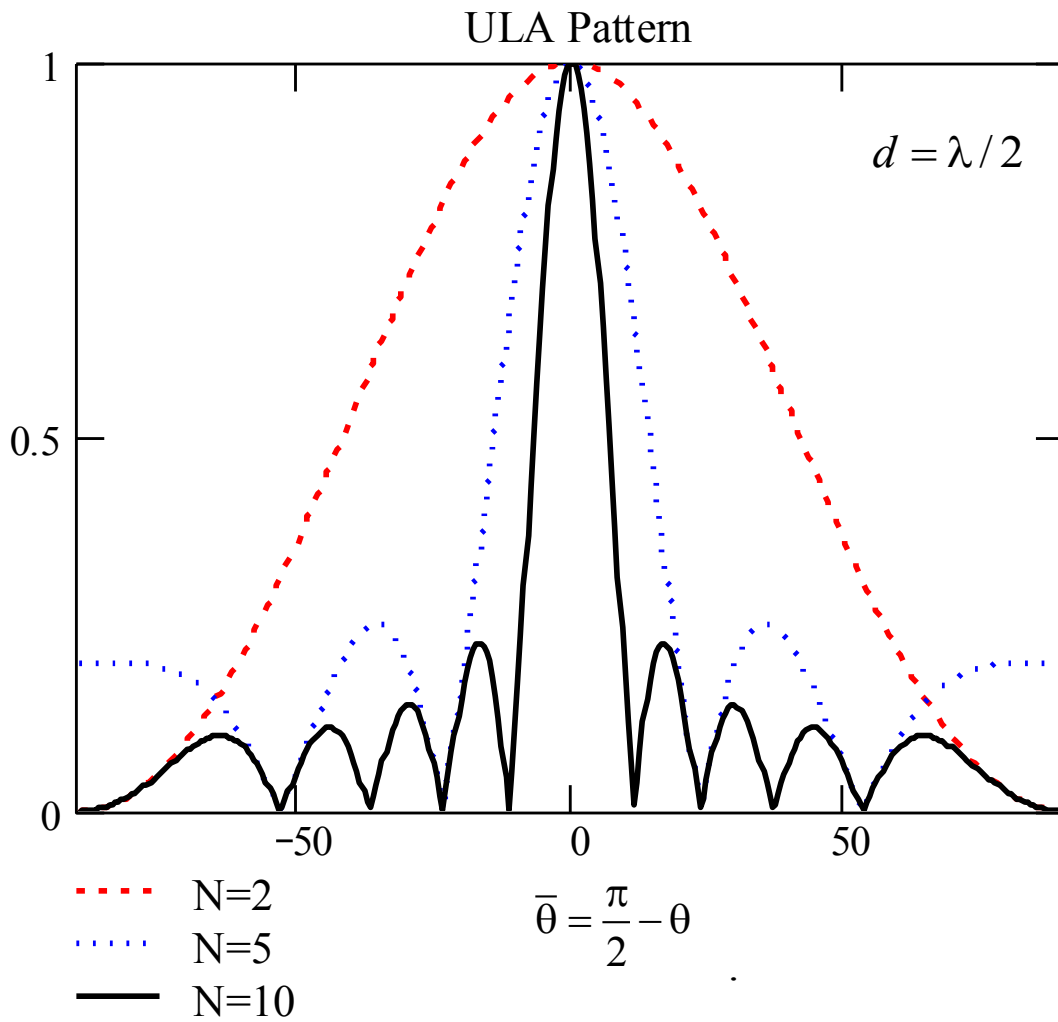
$$F(\mathbf{k}) = \left| \frac{\sin(d \cdot k_z \frac{N}{2})}{N \sin(\frac{d \cdot k_z}{2})} \right| \quad (3.18)$$

Note:

$$\psi_{\max} = \frac{2\pi}{\lambda} d; \quad \psi_{\min} = -\frac{2\pi}{\lambda} d \quad (3.19)$$

We started with isotropic elements and managed to obtain a highly-selective pattern!

ULA Pattern



ULA Pattern Parameters

3 dB beamwidth $\rightarrow |F(\theta)|^2 = 0.5 \rightarrow$ no closed-form solution

For large N ($N \gg 1$),

$$\Delta\theta_0 \approx 51 \frac{\lambda}{L}; \quad L = Nd = \text{array aperture (length)} \quad (3.20)$$

Null-to-null beamwidth

$$\Delta\theta_0 = 2 \frac{\lambda}{L} [\text{rad}] = 114 \frac{\lambda}{L} [\text{deg}]; \quad (3.21)$$

First sidelobe = -13.2dB

Gain (directivity): if $d = n \frac{\lambda}{2}$, $G = N$

$n =$ integer number.

Generic Representation of Array Response

Array elements spatially sample incoming wave

$$\mathbf{x}(t) = [e(t, \mathbf{p}_0) \quad e(t, \mathbf{p}_1) \quad \dots \quad e(t, \mathbf{p}_{N-1})]^T \quad (3.22)$$

where $\mathbf{p}_0 \dots \mathbf{p}_{N-1}$ — position vectors of array elements.

Using plane-wave model of $e(t, \mathbf{p}) = E e^{i(\omega t - \mathbf{k} \cdot \mathbf{p})}$,

$$\mathbf{x}(t) = E \cdot e^{i\omega t} \left[e^{-j\mathbf{k} \cdot \mathbf{p}_0} \quad e^{-j\mathbf{k} \cdot \mathbf{p}_1} \quad \dots \quad e^{-j\mathbf{k} \cdot \mathbf{p}_{N-1}} \right]^T \quad (3.23)$$

Array manifold vector is

$$\mathbf{v}(\mathbf{k}) = \left[e^{-j\mathbf{k} \cdot \mathbf{p}_0} \quad e^{-j\mathbf{k} \cdot \mathbf{p}_1} \quad \dots \quad e^{-j\mathbf{k} \cdot \mathbf{p}_{N-1}} \right]^T \quad (3.24)$$

It gives us a phase at each array element.

Array response to a plane wave can be expressed as

$$y(t) = \sum_{i=1}^{N-1} x_i(t) \quad (3.25)$$

$$= E \cdot e^{i\omega t} \sum_{k=0}^{N-1} v_i(\mathbf{k}) \quad (3.26)$$

Array pattern (factor) is

$$F(\mathbf{k}) \sim \left| \sum_{k=0}^{N-1} v_i(\mathbf{k}) \right| \quad (3.27)$$

Generic case: assume that each elements has impulse response of the form $h_i(\tau)$, introduce vector impulse response of the array

$$\mathbf{h}(\tau) = [h_0(\tau) \quad h_1(\tau) \quad \dots \quad h_{N-1}(\tau)]^T \quad (3.28)$$

$e(t, \mathbf{p})$ may be a generic wave, the array output can be expressed as

$$y(t) = \int_{-\infty}^t \mathbf{h}(t - \tau)^T \mathbf{e}(\tau) d\tau \quad (3.29)$$

where

$$\mathbf{e}(\tau) = [e(\tau, \mathbf{p}_0) \quad e(\tau, \mathbf{p}_1) \quad \dots \quad e(\tau, \mathbf{p}_{N-1})]^T \quad (3.30)$$

Very similar to conventional response (convolution integral) of an LTI system.

Fourier transform to frequency domain,

$$y(\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt = \mathbf{h}(\omega)^T \cdot \mathbf{e}(\omega) \quad (3.31)$$

$$\mathbf{h}(\omega) = FT\{\mathbf{h}(t)\}, \mathbf{e}(\omega) = FT\{\mathbf{e}(t)\} \quad (3.32)$$

$\mathbf{h}(\omega)$ = vector frequency response of the array, $\mathbf{e}(\omega)$ = spectrum of incoming field at element locations,

$$\mathbf{e}(\omega) = e(\omega) \cdot \mathbf{v}(\mathbf{k}) \quad (3.33)$$

Array scalar frequency response is

$$h(\omega) = y(\omega) / e(\omega) = \mathbf{h}(\omega)^T \mathbf{v}(\mathbf{k}) \quad (3.34)$$

It depends on the frequency response of individual elements ($\mathbf{h}(\omega)$) and on the array geometry ($\mathbf{v}(\mathbf{k})$)

Varying $\mathbf{h}(\omega)$ may perform beamforming, e.g. to steer the beam in desired direction.

Example: delay-and-sum beamformer

$$h_i(\tau) = \delta(\tau + \tau_i) \quad (3.35)$$

$$\tau_i = \frac{\mathbf{k}_0 \mathbf{p}_i}{\omega} \quad (3.36)$$

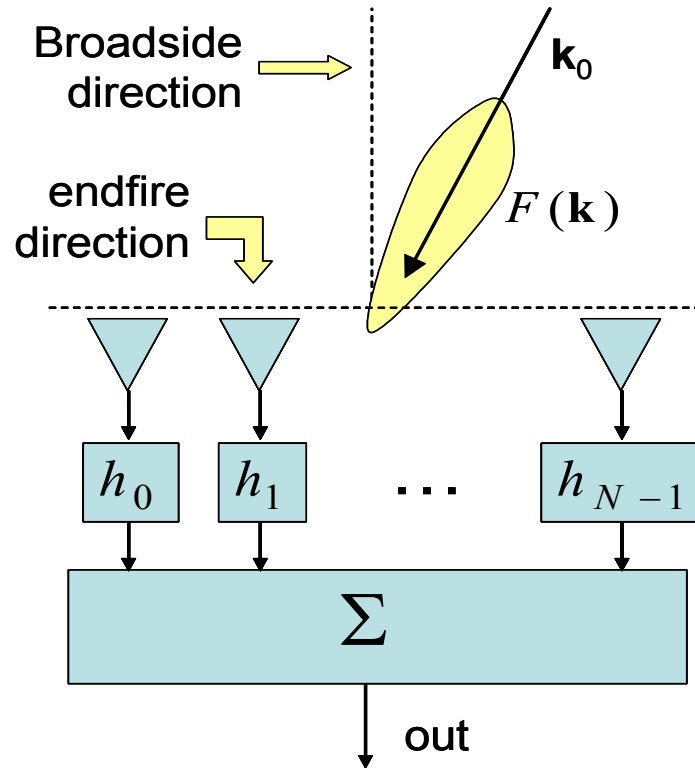
$$h_i(\omega) = e^{j\omega\tau_i} = e^{j\mathbf{k}_0 \mathbf{p}_i} \quad (3.37)$$

The array pattern (factor) is

$$F(\mathbf{k}) = \frac{1}{N} \mathbf{h}^T \mathbf{v}(\mathbf{k}) = \frac{1}{N} \sum_{i=0}^{N-1} e^{j\mathbf{k}_0 \mathbf{p}_i} e^{-j\mathbf{k} \mathbf{p}_i} \quad (3.38)$$

$$= \frac{1}{N} \sum_{i=0}^{N-1} v_i(\mathbf{k} - \mathbf{k}_0) \quad (3.39)$$

Block Diagram of Antenna Array



$$\begin{aligned}
 F(\mathbf{k}) &= \frac{1}{N} \left| \mathbf{h}^T \mathbf{v}(\mathbf{k}) \right| \\
 &= \frac{1}{N} \left| \sum_{i=0}^{N-1} e^{j\mathbf{k}_0 \mathbf{p}_i} e^{-j\mathbf{k} \mathbf{p}_i} \right| \\
 &= \frac{1}{N} \left| \sum_{i=0}^{N-1} v_i(\mathbf{k} - \mathbf{k}_0) \right|
 \end{aligned} \tag{3.40}$$

\mathbf{k}_0 = steering direction.

Beam steering: the beam is directed towards \mathbf{k}_0 !

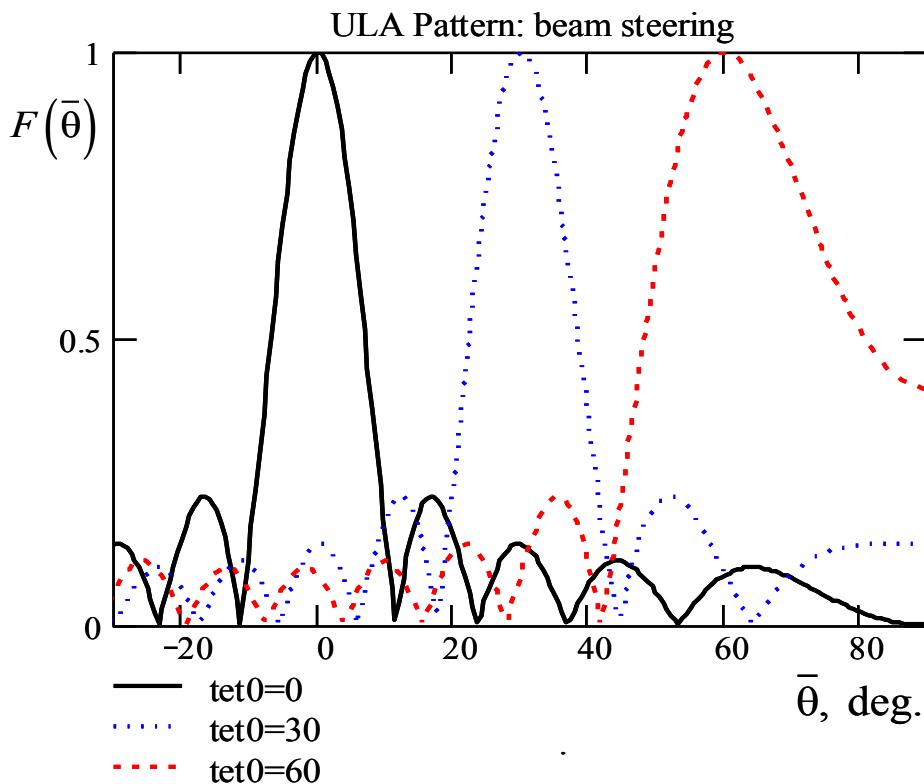
In terms of angles:

$$F(\theta) = \left| \frac{\sin \frac{N(\psi - \psi_0)}{2}}{N \sin \frac{\psi - \psi_0}{2}} \right| \quad (3.41)$$

where $\psi = \frac{2\pi}{\lambda} d \cos \theta$, $\psi_0 = \frac{2\pi}{\lambda} d \cos \theta_0$

Half-power beamwidth,

$$\bar{\theta}_0 = \frac{\pi}{2} - \theta_0, \quad \bar{\theta}_0 < \frac{\pi}{2}, \quad \Delta\theta \approx \frac{51^\circ \lambda}{Nd \cos \bar{\theta}_0} \quad (3.42)$$



Summary

- Introduction to antennas and arrays.
- Antenna pattern. Main beam and sidelobes. Nulls.
- Half-power beamwidth and zero-to-zero beamwidth. Sidelobe level. Antenna gain/directivity.
- Uniform linear array. Array pattern, beamwidth, sidelobe level.
- Beam steering. Pattern parameters.

References

H.L. Van Trees, Optimum Array Processing, Wiley, New York, 2002.

R.A. Monzingo, T.W. Miller, Introduction to Adaptive Arrays, Wiley, New York, 1980. (also 2nd Edition, 2011, available in pdf)

C.A. Balanis, Antenna Theory: Analysis and Design, Wiley, 1997.

R.C. Hansen, Phased Array Antennas, Wiley, New York, 1998.

W.L. Stutzman, G.A. Thiele, Antenna Theory and Design, Wiley, 1998.

A.W. Rudge et al (Eds.), The Handbook of Antenna Design, IEE & Peter Peregrinus, Norflok, 1986.