Adaptive Beamformers

So far, we assumed that the correlation matrices $\mathbf{S}_x$ or $\mathbf{S}_\xi$ and the desired signal direction were known. In practice, we have to estimate them from the incoming signal (wave). Hence , the beamformer will form a beam based on data extracted from the incoming signal – this is an adaptive beamformer.

There are 3 types of adaptive beamformers:

1. Estimate $\mathbf{S}_x$ or $\mathbf{S}_\xi$ from incoming signal data and invert them — the sample matrix inversion (SMI) or the direct matrix inversion (DMI).
2. Implement the inversion recursively – the recursive least squares (RLS) algorithm.
3. Using the classical steepest descent algorithm — the least mean squares (LMS) algorithm — less computation but slow convergence.

Estimating Correlation Matrices

How to find covariance matrices $\mathbf{S}_\xi$, $\mathbf{S}_s$?

This can be done using measured signals (samples or snapshots).

Measure the Rx signal at time moments $1, 2, \ldots, K$, i.e. $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_K$, and estimate $\mathbf{S}_x$ from these samples.

A good estimate of $\mathbf{S}_x$ is

$$\hat{\mathbf{S}}_x = \mathbf{C}_x = \frac{1}{K}\sum_{i=1}^{K} \mathbf{x}_i \mathbf{x}_i^+ \qquad (10.1)$$

where $\mathbf{C}_x$ is a sample (empirical) correlation matrix.

If $\mathbf{x}_i$ are not i.i.d Gaussian, the estimate in (10.1) may not be optimal in the ML sense, but it is still a good estimate, especially when K is large.

Note that $\mathbf{C}_x$ (and $\mathbf{S}_x$) is Hermitian and, if K > N, it is positive definite.

Q1: What happens if K < N ? Explain.

Q2: minimum K ?

Estimating Correlation Matrices

How to estimate $\mathbf{S}_\xi$?

The noise + interference correlation matrix, $\mathbf{S}_\xi$, can be estimated in a similar way provided that we are able to measure the incoming signals without the desired signal,

$$\hat{\mathbf{S}}_\xi = \frac{1}{K}\sum_{i=1}^{K} \xi_i \xi_i^+ \qquad (10.2)$$

Q. How to estimate $\mathbf{S}_s$?

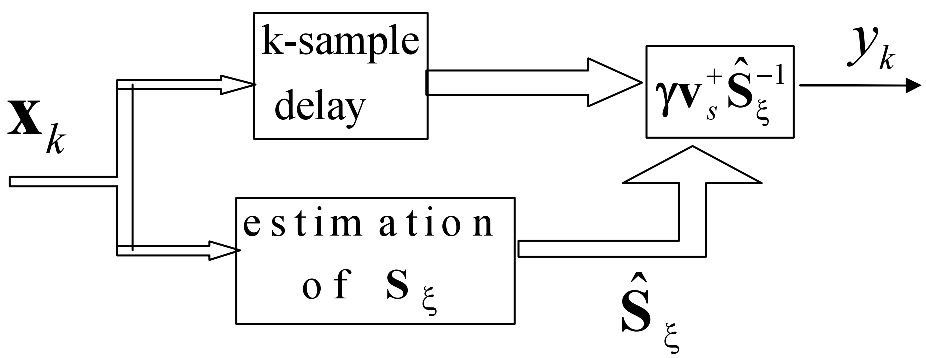
Sample Matrix Inversion (SMI)

Using the estimates above, we can find the optimum weights using one of the algorithms discussed earlier.

The MVDR weight vector is

$$\mathbf{w}_0^+ = \gamma \mathbf{v}_s^+ \hat{\mathbf{S}}_\xi^{-1}, \quad \gamma = \left(\mathbf{v}_s^+ \hat{\mathbf{S}}_\xi^{-1} \mathbf{v}_s \right)^{-1} \tag{10.3}$$

<u>Block Diagram of the SMI beamformer</u>



Performance Measures: SNIR

The signal to noise + interference ratio (SNIR) at the beamformer output is

$$\rho_{out} = SNIR_{out} = \frac{\sigma_s^2 \left| \hat{\mathbf{w}}^+ \mathbf{v}_s \right|^2}{\hat{\mathbf{w}}^+ \mathbf{S}_\xi \hat{\mathbf{w}}} \quad (10.4)$$

when there is one desired signal (plane-wave) of power σ_s^2.

Introduce the normalized output SNIR α:

$$\alpha = \frac{\rho_{out}}{\rho_{MVDR}} \quad (10.5)$$

where ρ_{MVDR} is the output SNIR of MVDR with known (exactly) $\mathbf{S}_\xi$.

Design Rule

If we require $\alpha = \alpha_0$, then the number of samples is

$$K = \frac{N - 2 + \alpha_0}{1 - \alpha_0} \approx \frac{N}{1 - \alpha_0} \tag{10.6}$$

Hence if $K = 2N - 3 \approx 2N \rightarrow \alpha = 0.5 \rightarrow 3dB$, i.e. the output SNIR is worse than that of the MVDR one with known $\mathbf{S}_\xi$ by 3 dB.

If we wish $\alpha = 0.95$ (5% loss in the SNR gain), then $K \approx 20N$.

Convergence: $\hat{\mathbf{S}}_K \rightarrow \mathbf{S}$ as $K \rightarrow \infty$ in several senses (MSE, Prob.).

Example 7.3.1 [1]

$u_s = 0,\ u_I = 0.15,\ INR = 10dB,$ 200 trials, ULA with $N = 10$ and $d = \lambda / 2$

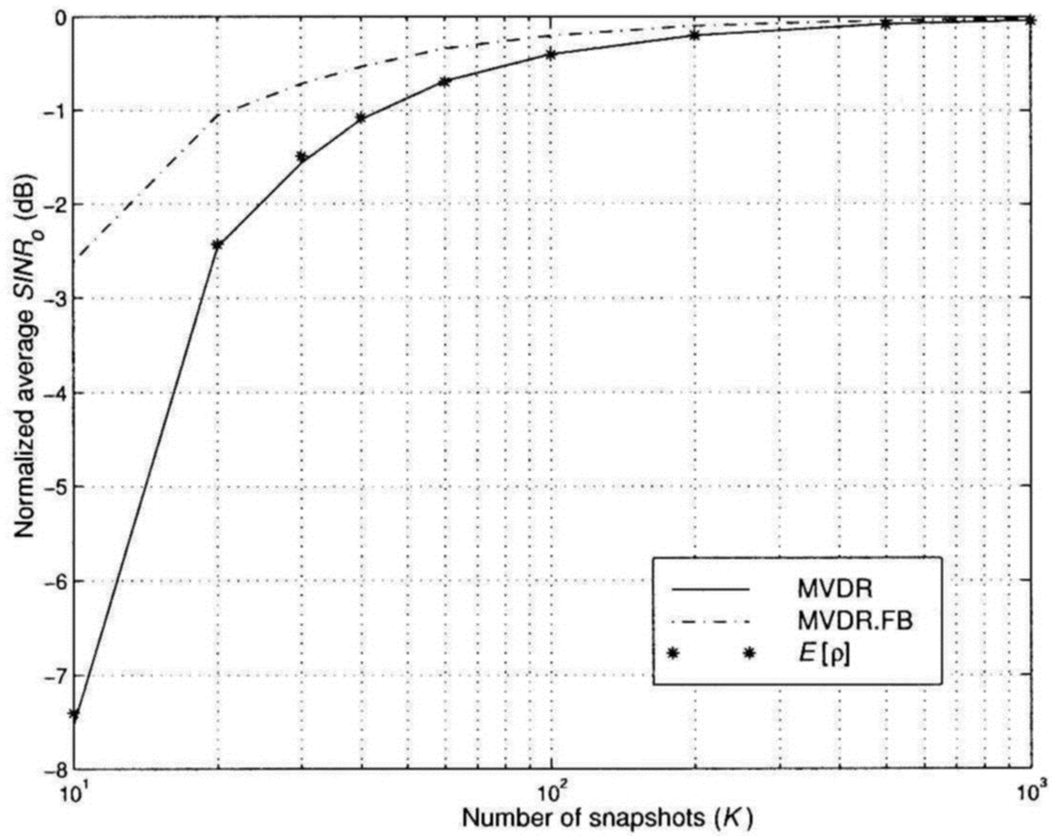


Figure 7.4 MVDR SMI beamformer: $\bar{\rho}$ and $E[\rho]$ versus K.

H.L. Van Trees, Optimum Array Processing, Wiley, 2002

Diagonal Loading (DL)

As earlier, we add a scaled identity matrix to $\hat{\mathbf{S}}_x$ to improve the robustness of the beamformer:

$$\hat{\mathbf{S}}_{xL} = \frac{1}{K}\sum_{i=1}^{K}\mathbf{x}_i\mathbf{x}_i^{+} + \sigma_L^2\mathbf{I} \qquad (10.7)$$

There are 3 reasons to use DL:

1. To improve the SNIR performance of the MPDR beamformer.
2. To implement beamformers when K<N.
3. To achieve better sidelobe control and main-beam shaping.

Q.: Why one cannot implement a beamformer for K<N without DL?

To demonstrate performance improvement, consider the example 7.3.4 in [1].

How Good Is The Beamforming ???

130 IEEE COMMUNICATIONS LETTERS, VOL. 1, NO. 5, SEPTEMBER 1997

Approaching Shannon's Capacity Limit by 0.27 dB Using Simple Hamming Codes

Helmut Nickl, Joachim Hagenauer, and Frank Burkert

***Abstract*—In this letter, we will show that the Shannon capacity limit for the additive white Gaussian noise (AWGN) channel can be approached within 0.27 dB at a bit error rate (BER) of 10^{-5} by applying long but simple Hamming codes as component codes to an iterative *turbo*-decoding scheme. In general, the complexity of soft-in/soft-out decoding of binary block codes is rather high. However, the application of a neurocomputer in combination with a parallelization of the decoding rule facilitates an implementation of the decoding algorithm in the logarithmic domain which requires only matrix additions and multiplications. But the storage requirement might still be quite high depending on the interleavers used.**

$$+\ln \underbrace{\frac{1+\sum_{i=2}^{2^{N-K}} \prod_{j=1,j\neq k}^{N} \lambda_j^{(1-x'_{ij})/2}}{1-\sum_{i=2}^{2^{N-K}} (-x'_{ik}) \prod_{j=1,j\neq k}^{N} \lambda_j^{(1-x'_{ij})/2}}}_{L_e(\hat{u}_k)} \qquad (1)$$

where $x'_{ij} \in \{+1, -1\}$ denotes the jth bit of the ith codeword of the dual code (the index $i = 1$ is used for the all-$+1$ codeword) and

How Good Is That?

Rate one-half code for approaching the Shannon limit by 0.1 dB

S. ten Brink

A serially concatenated code is presented which exhibits a turbo cliff at 0.28dB. The concatenation consists of an outer rate one-half repetition code and an inner rate one recursive convolutional code. The iterative decoding scheme was designed using the extrinsic information transfer chart (EXIT chart).

Introduction: The discovery of parallel concatenated codes (PCC) [1] has spurred the search for other code concatenations and corresponding iterative decoders which can operate close to the theoretical capacity limit. For a binary input/continuous output additive white Gaussian noise channel, the Shannon capacity limit [2] is $E_b/N_0 = 0.19$dB (code rate one-half). In this Letter we present a serially concatenated code (SCC) [3] which achieves a bit error rate (BER) of less than 10^{-5} at $E_b/N_0 = 0.28$dB. The code was designed using the EXIT chart [4, 5]. For large interleavers,

respective systematic counterparts
rate; the number of systematic bit
the number of coded bits, i.e. n_s/n
Letter, a doping ratio of $n_s/n_c = 1$:

Iterative decoder: The inputs to th
(BCJR algorithm [6]) are channel
bits and *a priori* log-likelihood rati
information (i.e. systematic) bits
extrinsic and channel information
a deinterleaver to become the *a pr*
in/soft out repetition decoder. Th
extrinsic information E_2 which is
priori knowledge A_1 to the inner
further iterative decoding steps. N
bility decoding rule for the repeti
swapping operation: For two ou
stemming from the same outer inf
values are easily calculated to D_2
the corresponding extrinsic L-valu

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How Good Is That?

58 IEEE COMMUNICATIONS LETTERS, VOL. 5, NO. 2, FEBRUARY 2001

On the Design of Low-Density Parity-Check Codes within 0.0045 dB of the Shannon Limit

Sae-Young Chung, *Member, IEEE*, G. David Forney, Jr., *Fellow, IEEE*, Thomas J. Richardson, and Rüdiger Urbanke

***Abstract*—We develop improved algorithms to construct good low-density parity-check codes that approach the Shannon limit very closely. For rate 1/2, the best code found has a threshold within 0.0045 dB of the Shannon limit of the binary-input additive white Gaussian noise channel. Simulation results with a somewhat simpler code show that we can achieve within 0.04 dB of the Shannon limit at a bit error rate of 10^{-6} using a block length of 10^7.**

***Index Terms*—Density evolution, low-density parity-check codes, Shannon limit, sum-product algorithm.**

Let v be a log-likelihood ratio (LLR) message from a degree-d_v variable node to a check node. Under sum-product decoding, v is equal to the sum of all incoming LLRs; i.e.,

$$v = \sum_{i=0}^{d_v-1} u_i \qquad (1)$$

Progress toward the Shannon limit

The original turbo codes: about **0.7 dB** from capacity

C. Berrou, A. Glavieux, and P. Thitimajshima, Near Shannon limit error-correcting coding and decoding: Turbo codes, *IEEE Int. Communications Conference*, 1993.

Irregular LDPC codes: about **0.1 dB** from capacity

T.J. Richardson and R. Urbanke, The capacity of low-density parity-check codes, *IEEE Transactions on Information Theory*, February 2001.

How about 0.01 dB from capacity? And 0.001 dB?

J. Boutros, G. Caire, E. Viterbo, H. Sawaya, and S. Vialle, Turbo code at 0.03 dB from capacity limit, *IEEE Symp. Inform. Theory*, July 2002.

S-Y. Chung, G.D. Forney, Jr., T.J. Richardson, and R. Urbanke, On the design of low-density parity-check codes within 0.0045 dB of the Shannon limit, *IEEE Communications Letters*, February 2001.

Conclusion: For all practical purposes, Shannon's puzzle has been now solved and Shannon's promise has been achieved!

A.Vardy, What's New and Exciting in Algebraic and Combinatorial Coding Theory? Plenary Talk at ISIT-06.

Summary

- Adaptive beamformers. Estimating the signal and interference correlation matrices.
- Sample matrix inversion. Required number of snapshots. Performance measures. Comparison with MVDR and known correlation matrices.
- Diagonal loading. Performance improvement. Choice of LNR.

References

1. H.L. Van Trees, Optimum Array Processing, Wiley, New York, 2002.
2. S. Loyka, Review of Matrix Theory, 2005.

Homework

Fill in the details in the derivations above. Answer the questions. Do the examples yourself.