Lab # 2: Rayleigh and Ricean Fading

Preparation

1. Read Ch. 5 of Rappaport's textbook.
2. Let x and y be i.i.d. Gaussian random variables of zero mean and unit variance. What is the mean and variance of $|z|^2$, where $z = x + \mathrm{j}\cdot y$ and j is imaginary unit? How to obtain a random variable having the same distribution as $|z|^2$ but unit variance?
3. Find the PDF and CDF of $|z|$ and $|z|^2$.

Laboratory

In this experiment, we will simulate Rayleigh fading by assuming that the channel gain is distributed as $|z|$ in the previous part.

1. Generate the empirical PDF (hystogram) of $|z|$ by generating n i.i.d. realizations of x and y and obtain n realizations of $|z|$ via $z = x + \mathrm{j}\cdot y$. Plot the histogram of these samples for n=100 and compare it with the theoretical PDF of Rayleigh distribution. Do they agree? Hint: you need to scale the histogram in a such a way that it becomes an empirical PDF.

2. Repeat #1 for n=10 and 1000. Comment on differences if any and explain them.

3. Now generate the empirical CDF (i.e. the outage probability). Hint: you can obtain it using the empirical PDF. Do it for n=10, 100, 1000 and comment on differences. To see clearly the low outage region (e.g. 10^{-2} … 10^{-3}) use the exponential scale for the y-axis and dB for the x-axis, as in Lecture 4 (slide 18). How many samples n do you need to predict accurately the outage probability around 10^{-3}? Determine this quantity experimentally. *Hint*: fix n and run simulations several times; observe variations of the empirical CDF from one run to another around the point 10^{-3}; increase n until these variations become negligible.

4. In this experiment, we test the theoretical derivation of Rayleigh distribution in Lecture 4. Let N be the number of multipath components. Assume that all of them have unit amplitude and i.i.d. phase uniformly distributed in $[0, 2\pi]$. Normalize the multipath signal so that the total power is 1 regardless of N. Use n=1000 realizations of such a channel and plot its empirical PDF for N=1, 2, 3, 5 and 10, and compare it to the theoretical Rayleigh PDF on the same graph. How many multipath components are required for the resulting distribution be reasonably close to the Rayleigh one? Repeat this experiment for the CDF. How many multipath components do you need for Rayleigh distribution to be accurate around $P_{out} \approx 10^{-2}$?

5. Repeat #5 when each multipath component has i.i.d. amplitude uniformly distributed in the interval [0,1]. How do your results/conclusions change? Now assume the amplitudes of individual multipath components are Rayleigh distributed. Comment on differences/similarities to the previous cases.

6. Repeat #1 and #2 for Ricean distribution, i.e. when there is a steady LOS component with K = 1, 2, 10. Find both empirical PDF and CDF. Comment on the impact of K on the CDF of the distribution. Do your results, i.e. empirical PDF and CDF, agree with those in Lecture 4 (p. 23, 24)?

References:

1. T.S. Rappaport, Wireless Communications: Principles and Practice, Prentice Hall, New Jersey, 2002. (2nd Edition)
2. J.G. Proakis et al, Contemporary Communication Systems Using MATLAB and Simulink, Thomson & Books/Cole, 2004.
3. W.H. Tranter et al., Principles of Communication Systems Simulation, Prentice Hall, 2004.
4. M.C. Jeruchim et al, Simulation of Communication Systems: Modeling, Methodology, and Techniques, Kluwer, New York, 2000.