Question 1: An angle-modulated signal is described by

\[ x_c(t) = 10 \cos \left[ 2\pi (10^6) t + 0.1 \sin (10^3) \pi t \right]. \]

(a) Considering \( x_c(t) \) as a PM signal with \( k_p = 10 \), find \( m(t) \).

(b) Considering \( x_c(t) \) as an FM signal with \( k_p = 10 \pi \), find \( m(t) \).

Solution: a) From the lecture notes we know that a PM modulated signal can be written in the form of:

\[ x_{PM}(t) = A \cos \left[ \omega t + k_p m(t) \right] \]

Now, \( m(t) \) can be obtained as:

\[ x_{PM}(t) = A \cos \left[ \omega t + k_p m(t) \right] = 10 \cos \left[ 2\pi (10^6) t + 10 m(t) \right] \]

\[ = 10 \cos \left[ 2\pi (10^6) t + 0.1 \sin (10^3) \pi t \right] \]

\[ \Rightarrow m(t) = 0.01 \sin (10^3) \pi t \]
b) we also know that an FM modulated signal can be written as
\[ x_{FM}(t) = A \cos \left[ \omega t + K_f \int_0^t m(\lambda) \, d\lambda \right] \]

so
\[ 10 \cos \left[ 2\pi (10^6) t + 0.1 \sin (10^3 \pi t) \right] = A \cos \left[ \omega t + K_f \int_0^t m(\lambda) \, d\lambda \right] \]

Assuming \( m(t) = a_n \cos (10^3 \pi t) \), we get
\[ 10 \pi \int_0^t m(\lambda) \, d\lambda = 10 \pi \frac{a_n}{100} \int_0^t \cos ((10^3 \pi \lambda) \, d\lambda \]
\[ = \frac{a_n}{100} \sin ((10^3 \pi) t) = 0.1 \sin ((10^3 \pi) t) \]

\[ \Rightarrow m(t) = 10 \cos ((10^3 \pi) t) \]
Question 2: Find the maximum frequency deviation $\Delta f$ of the output of the Armstrong FM transmitter (Lecture 8, 17(28)) and the carrier frequency $f_c$.
The first blocks create a NBFM modulated signal with a carrier frequency $f_c = 200$ kHz generated by the Crystal oscillator. This frequency is chosen because it's easy to construct stable crystal oscillator as well as balanced modulators at this frequency. The deviation of is chosen to be $\Delta 25$ Hz in order to maintain $\beta < 1$, as required in NBFM.

The first multiplication by 64 results as follows:

$$f_{c_2} = 200 \text{ kHz} \times 64 = 12.8 \text{ MHz}$$
$$\Delta f_{c_2} = 25 \times 64 = 1.6 \text{ kHz}$$

Then frequency converter (or mixer) with carrier frequency of 10.9 MHz shifts the entire spectrum as follows.

$$f_{c_3} = 12.8 \text{ MHz} - 10.9 \text{ MHz} = 1.9 \text{ MHz}$$

Consider the fact that using the frequency converter doesn't change the maximum frequency deviation

$$\Delta f_{c_3} = \Delta f_{c_2} = 1.6 \text{ kHz}$$
Further multiplication, by 48, yields:

\[ f_{c_4} = 1.9 \text{ MHz} \times 48 = 91.2 \text{ MHz} \]

\[ \Delta f_q = 1.6 \text{ kHz} \times 48 = 76.8 \text{ kHz}. \]

This scheme has an advantage of frequency stability, but it suffers from inherent noise caused by excessive multiplication and distortion at lower modulating frequencies, where \[ \beta = \frac{\Delta f}{f_m} \] is not small enough, and \[ \beta \ll 1 \] doesn't hold.