Angle Modulation

- Angle modulation: frequency modulation (FM) or phase modulation (PM).
- Basic idea: vary frequency (FM) or phase (PM) according to the message signal.
- While AM is (almost) linear, FM or PM is highly nonlinear.
- FM/PM provide many advantages (main – noise immunity) over AM, at a cost of larger bandwidth.
- Demodulation may be complex, but modern ICs allow cost-effective implementation.
- Example: FM radio (high quality, not expensive receivers).

Angle Modulation: Basic Definitions

- Angle-modulated signal (PM or FM) can be expressed as:

$$x(t) = A_c \cos(\psi(t))$$

- Phase modulation:

$$\psi(t) = \omega_c t + \varphi(t), \quad \varphi(t) = \Delta\varphi \cdot m(t)$$

- Frequency modulation:

$$\psi(t) = \omega_c t + \int_0^t \Omega(\tau) d\tau, \quad \Omega(t) = \Delta\Omega \cdot m(t)$$

for a short period of time (small t): $\psi(t) \approx [\omega_c + \Omega(0)]t + \varphi_0$

- Max phase deviation: $\Delta\varphi = \mathrm{Max}\{|\varphi(t)|\} = \mathrm{Max}\{|\psi(t) - \omega_c t|\}$
- Max frequency deviation: $\Delta\Omega = \mathrm{Max}\{|\Omega(t)|\} = \mathrm{Max}\{|\omega(t) - \omega_c|\}$
- Normalized message signal: $|m(t)| \leq 1$

Note: deviation is w.r.t. unmodulated value.

Angle Modulation: Parameters

- Instantaneous frequency:

$$\omega(t) = \frac{d\psi(t)}{dt} = \begin{cases} \omega_c + \dfrac{d\varphi(t)}{dt} = \omega_c + \Delta\varphi \dfrac{dm(t)}{dt}, & PM \\ \omega_c + \Omega(t) = \omega_c + \Delta\Omega \cdot m(t), & FM \end{cases}$$

- Instantaneous phase:

$$\psi(t) = \int_0^t \omega(\tau) d\tau = \begin{cases} \omega_c t + \varphi(t) = \omega_c t + \Delta\varphi \cdot m(t), & PM \\ \omega_c t + \int_0^t \Omega(\tau) d\tau = \omega_c t + \Delta\Omega \int_0^t m(\tau) d\tau, & FM \end{cases}$$

- Effect of mod. signal amplitude: $M(t) = A \cdot m(t), \ \max\left[|m(t)|\right] = 1$

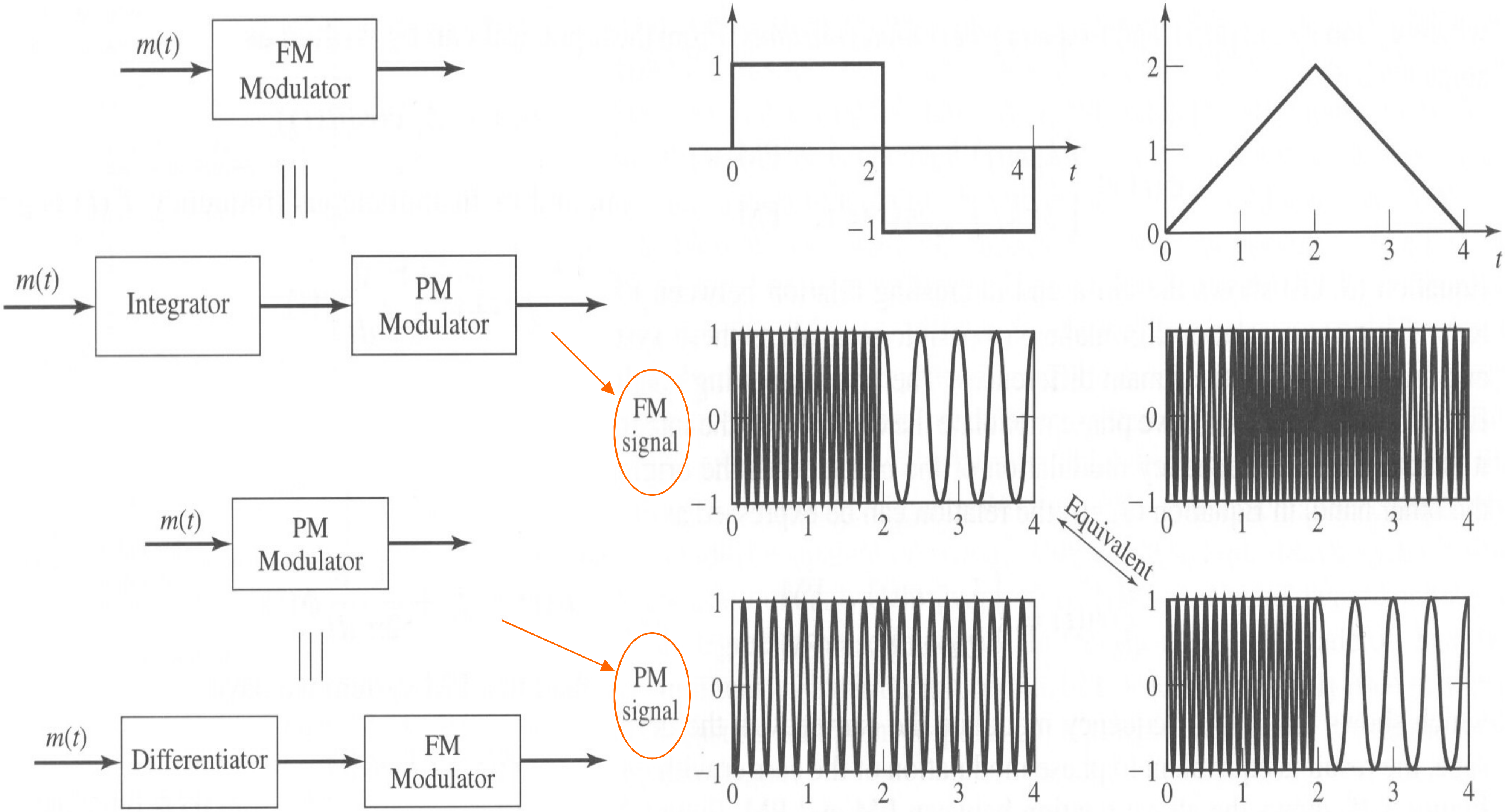
$$\begin{cases} \Delta\varphi = k_p A, & PM \\ \Delta\Omega = k_f A, & FM \end{cases}$$

k_f, k_p - modulation constants, Hz/V & rad./V

↑ measured in lab 3.

Angle Modulation: Examples

rectangular pulse sawtooth pulse



Example: Sinusoidal Modulating Signal

- Assume that $m(t) = \cos(\omega_m t)$
- Instantaneous phase:

$$\psi(t) = \begin{cases} \omega_c t + \Delta\varphi \cdot \cos(\omega_m t), & PM \\ \omega_c t + \dfrac{\Delta\Omega}{\omega_m} \sin(\omega_m t), & FM \end{cases}$$

- Modulated signal:

$$x(t) = \begin{cases} A_c \cos\left[\omega_c t + \Delta\varphi \cdot \cos(\omega_m t)\right], & PM \\ A_c \cos\left[\omega_c t + \dfrac{\Delta\Omega}{\omega_m} \sin(\omega_m t)\right], & FM \end{cases}$$

- Modulation indices:

$$\begin{cases} \beta_p = \Delta\varphi, & PM \\ \beta_f = \dfrac{\Delta\Omega}{\omega_m}, & FM \end{cases}$$

⇦ Valid in general case as well, with ω_m -> max. modulating frequency

Spectrum of Angle-Modulated Signal

- Consider sinusoidal modulating signal:

$$x(t) = A_c \cos\left[\omega_c t + \beta \cdot \sin(\omega_m t)\right] = \mathrm{Re}\left[A_c e^{j\beta\cdot\sin(\omega_m t)} e^{j\omega_c t}\right]$$

- Complex envelope is expanded in Fourier series:

$$C(t) = A_c e^{j\beta\cdot\sin(\omega_m t)} = A_c \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_m t}$$

- Expansion coefficients are

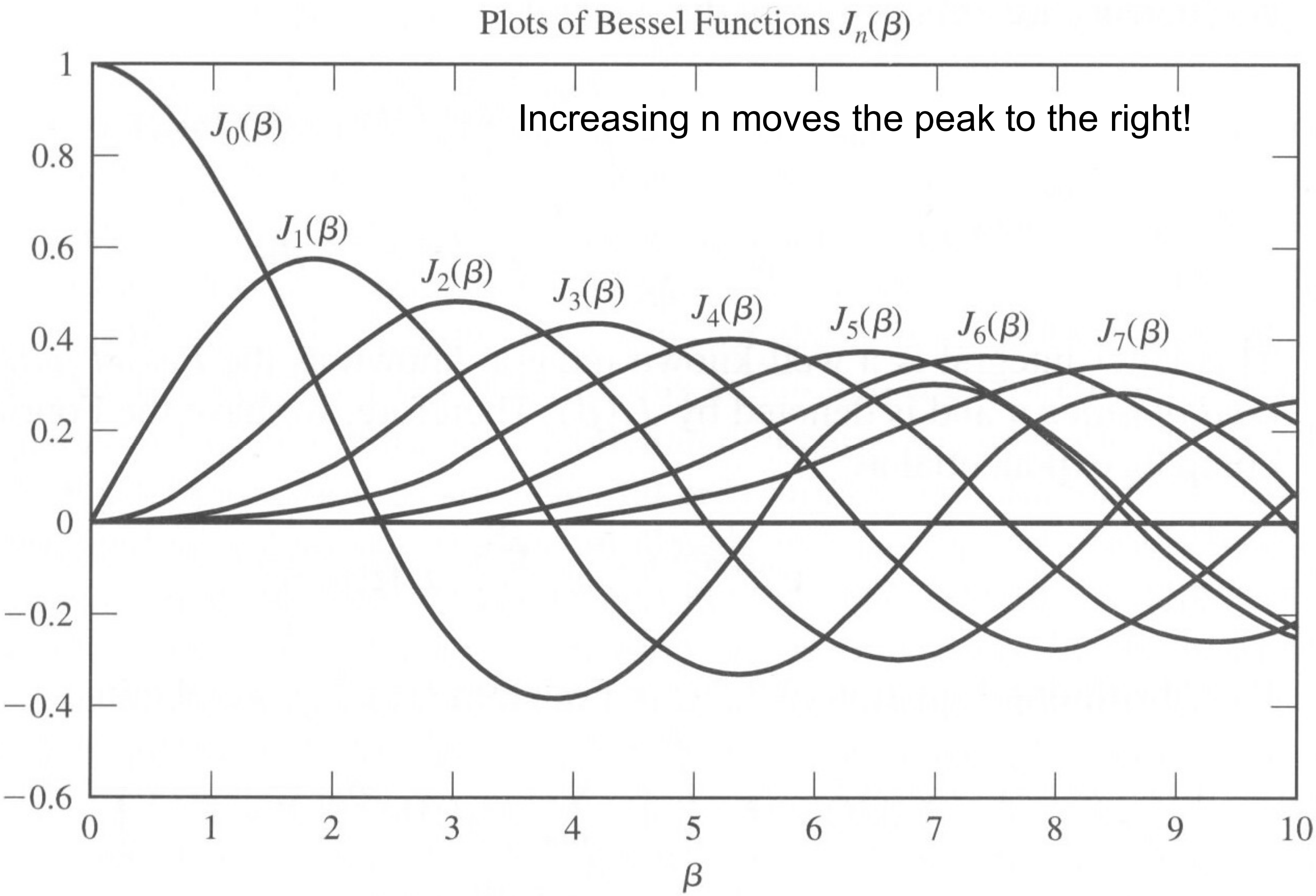
$$c_n = \frac{1}{T_m}\int_0^{T_m} e^{j\beta\sin\omega_m t} e^{-jn\omega_m t} dt \overset{u=\omega_m t}{=} \frac{1}{2\pi}\int_0^{2\pi} e^{j(\beta\sin u - nu)} du = J_n(\beta)$$

- Finally,

$$\boxed{x(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos\left[(\omega_c + n\omega_m)t\right]}$$

$J_n(\beta)$ - Bessel function of 1st kind & n-th order, $J_{-n}(\beta) = (-1)^n J_n(\beta)$

Spectrum of Angle Modulation: $J_n(\beta)$



Spectrum of Angle Modulation: $J_n(\beta)$

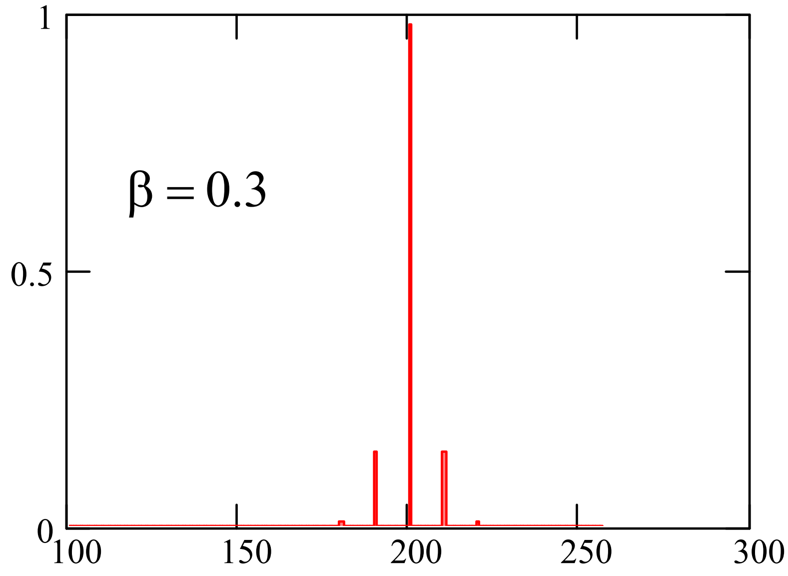
n	$\beta=0.1$	$\beta=0.2$	$\beta=0.5$	$\beta=1$	$\beta=2$	$\beta=5$	$\beta=8$	$\beta=10$	n
0	0.998	0.990	0.938	0.765	0.224	-0.178	0.172	-0.246	0
1	0.050	0.100	0.242	0.440	0.577	-0.328	0.235	0.043	1
2	0.001	0.005	0.031	0.115	0.353	0.047	-0.113	0.255	2
3				0.020	0.129	0.365	-0.291	0.058	3
4				0.002	0.034	0.391	-0.105	-0.220	4
5					0.007	0.261	0.186	-0.234	5
6					0.001	0.131	0.338	-0.014	6
7						0.053	0.321	0.217	7
8						0.018	0.223	0.318	8
9						0.006	0.126	0.292	9
10						0.001	0.061	0.207	10
11							0.026	0.123	11
12							0.010	0.063	12
13							0.003	0.029	13
14							0.001	0.012	14
15								0.004	15
16								0.001	16

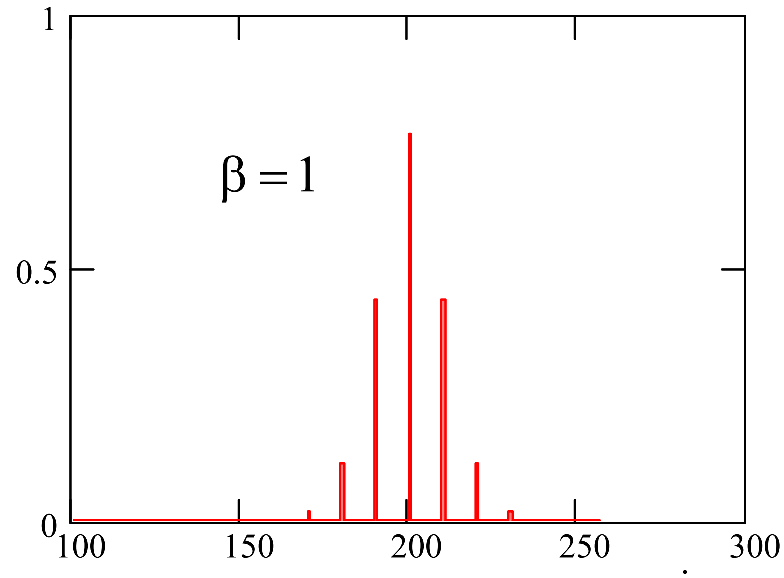
the last significant spectral component:

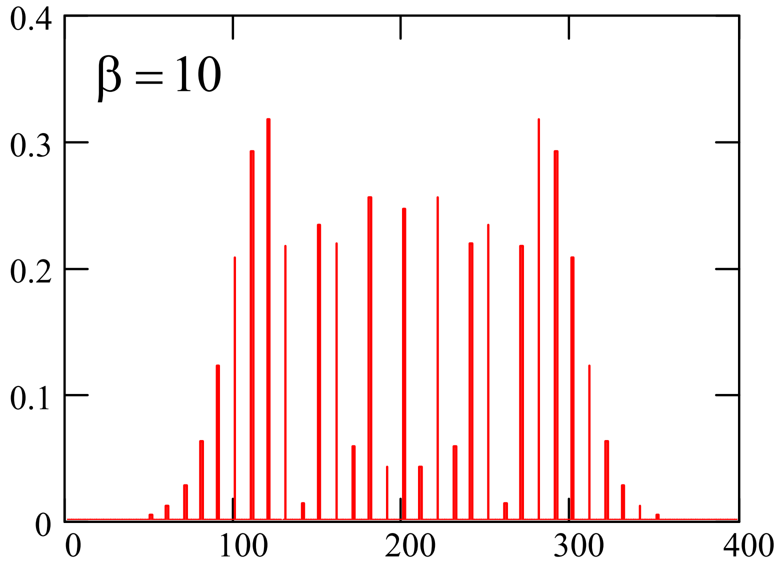
$$n = [\beta + 1]$$

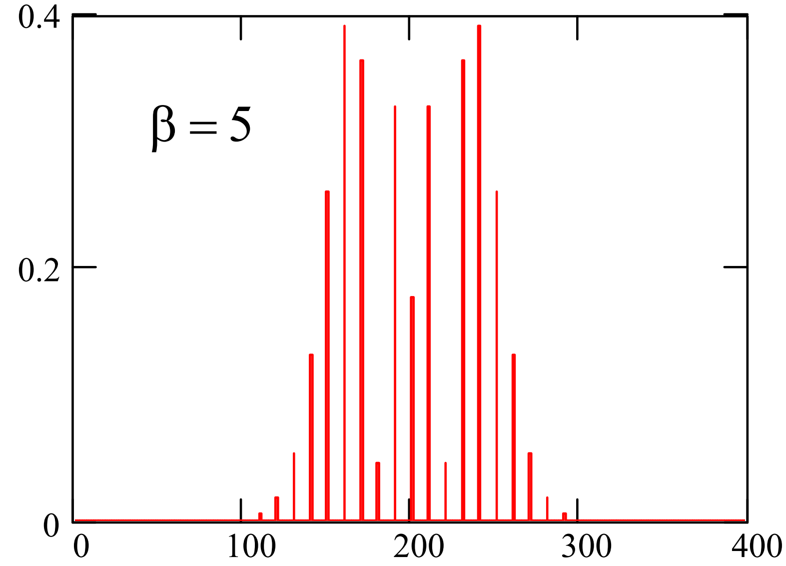
$$x(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left[(\omega_c + n\omega_m)t\right]$$

FM/PM Spectrum: Examples









Bandwidth of Angle-Modulated Signal

- Power bandwidth (98% of the power) of angle-modulated signal (Carson's rule): $\boxed{\Delta\omega \approx 2(\beta+1)\omega_m}$

- Power bandwidth of PM and FM signals:

$$\Delta\omega \approx 2(\beta+1)\omega_m = \begin{cases} 2(\Delta\varphi+1)\omega_m, & PM \\ 2(\Delta\Omega+\omega_m), & FM \end{cases}$$

- These expressions hold for a general modulating signal as well, ω_m - the max. modulating frequency.
- <u>Angle modulation with large index expands spectrum!</u>

Example: FM Radio

- FM signal with a sinusoidal message has the following parameters:

$$A_c = 10,\ f_c = 100MHz,\ \Delta f_p = 80kHz, F_{\max} = 10kHz$$

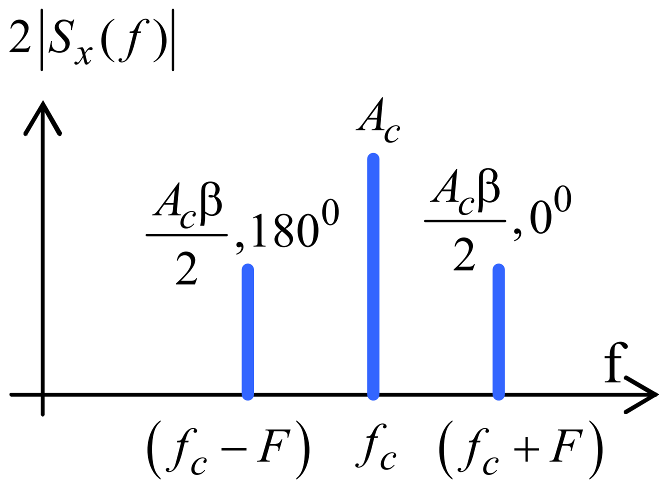
1. Find modulation index
2. Find signal bandwidth, compare to AM
3. Find signal power
4. Find time-domain expression x(t) of the signal; sketch it (as on OS)
5. Find its spectrum (as on SA) and plot it.

Narrowband Angle Modulation

- Modulation index is low, $\beta << 1$
- Modulated signal can be expressed as:

$$x(t) = A_c \cos\left[\omega_c t + \beta \cdot \sin(\omega_m t)\right]$$
$$\approx A_c \cos \omega_c t + \frac{A_c \beta}{2} \cos(\omega_c + \omega_m) t - \frac{A_c \beta}{2} \cos(\omega_c - \omega_m) t$$

- Similar to AM signal, the bandwidth is (both, PM & FM)



$$\Delta\omega \approx 2\omega_m$$

Wideband Angle Modulation

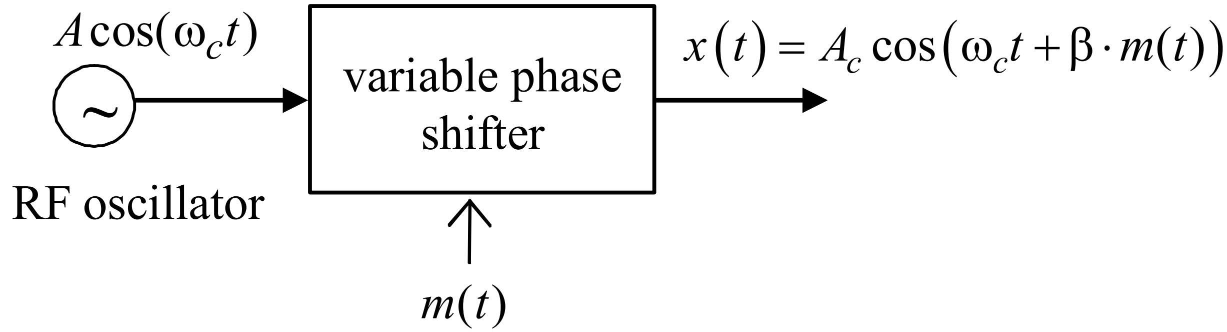
- Modulation index is high, $\beta >> 1$
- The signal bandwidth is:

$$\Delta\omega \approx 2\beta\omega_m = \begin{cases} 2\Delta\varphi\cdot\omega_m, & PM \\ 2\Delta\Omega, & FM \end{cases}$$

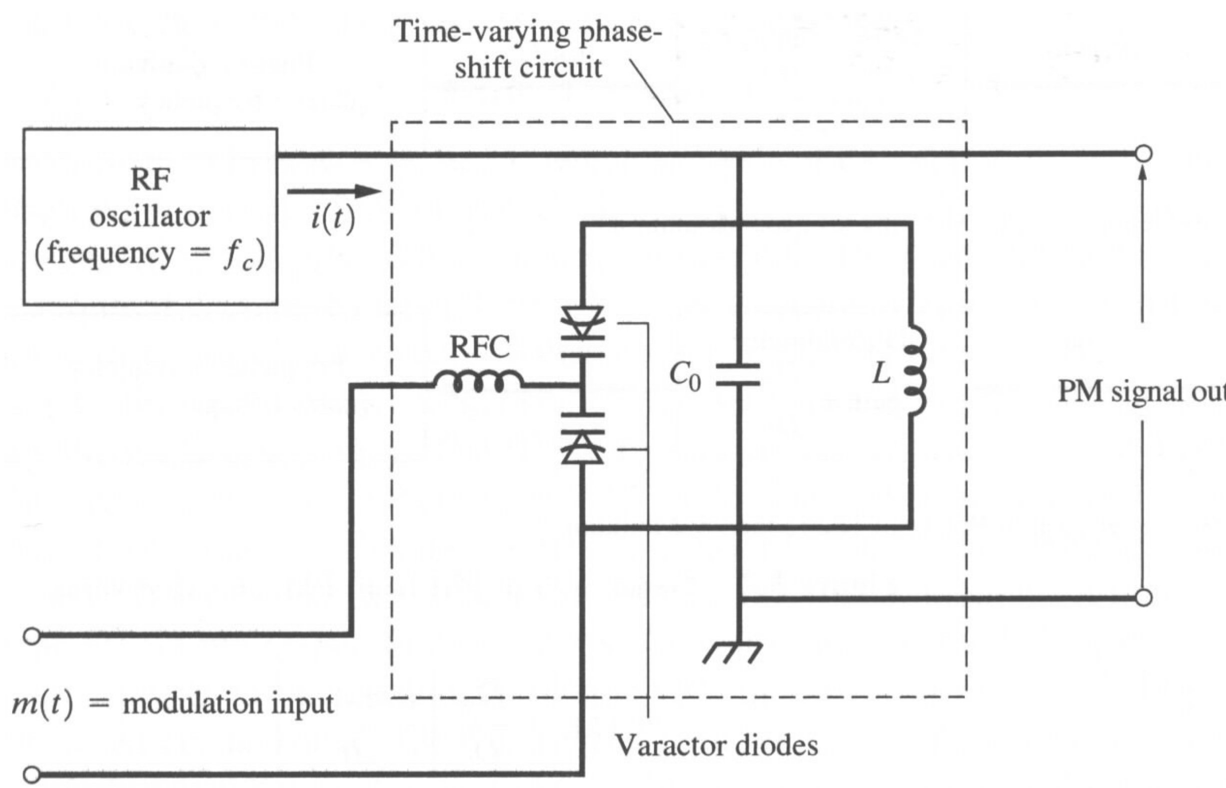
- Different for PM and FM!
- Wideband FM: the bandwidth is twice the frequency deviation. Does not depend on the modulating frequency.
- Wideband PM: the bandwidth depends on modulating frequency.
- Modulation index $\beta =$ bandwidth expansion factor.

PM Modulator

General principle:



Practical implementation:



L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

$$\Delta\varphi \approx k\Delta f,\ \Delta f = f_c - f_0$$

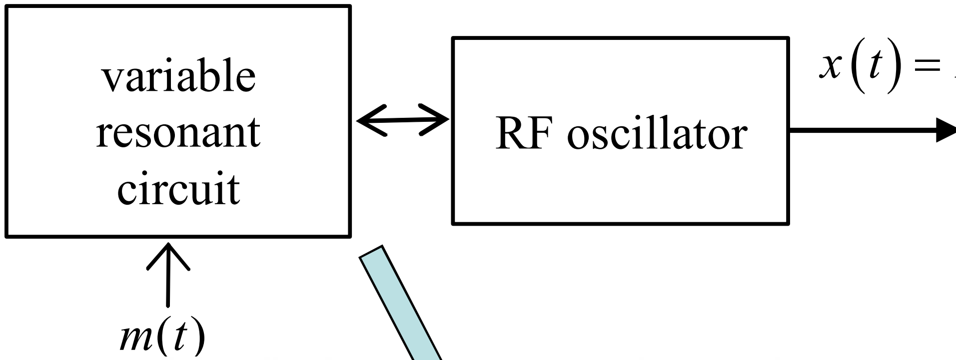
k = modulation constant

f_0 = resonant frequency

Q.: sketch $\Delta\varphi(\Delta f)$

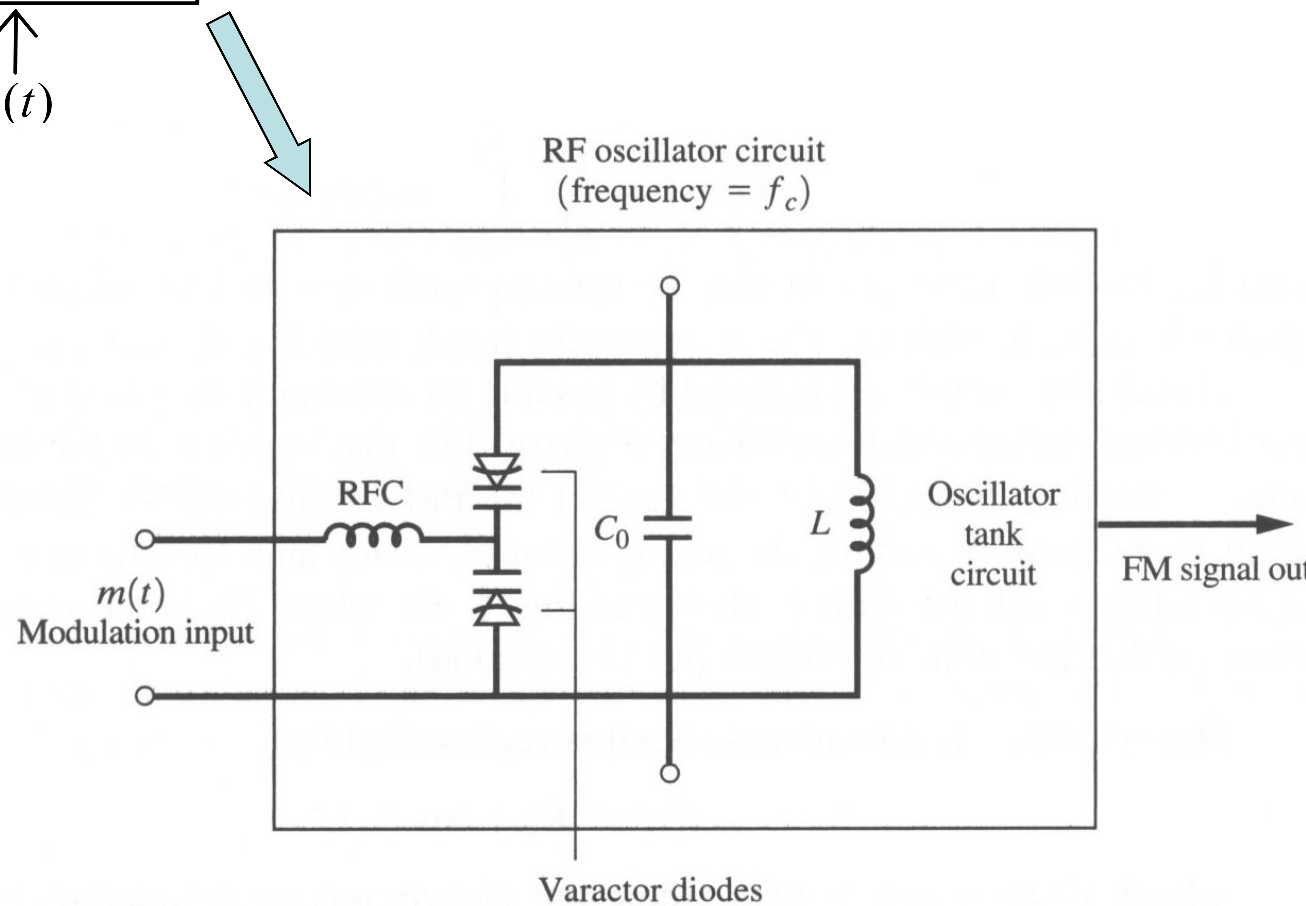
FM Modulator

General principle:



$$x(t) = A_c \cos\left[\omega_c t + \int_0^t \Omega(\tau)d\tau + \varphi_0\right]$$

Practical implementation:



L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Difficulty: frequency stability.

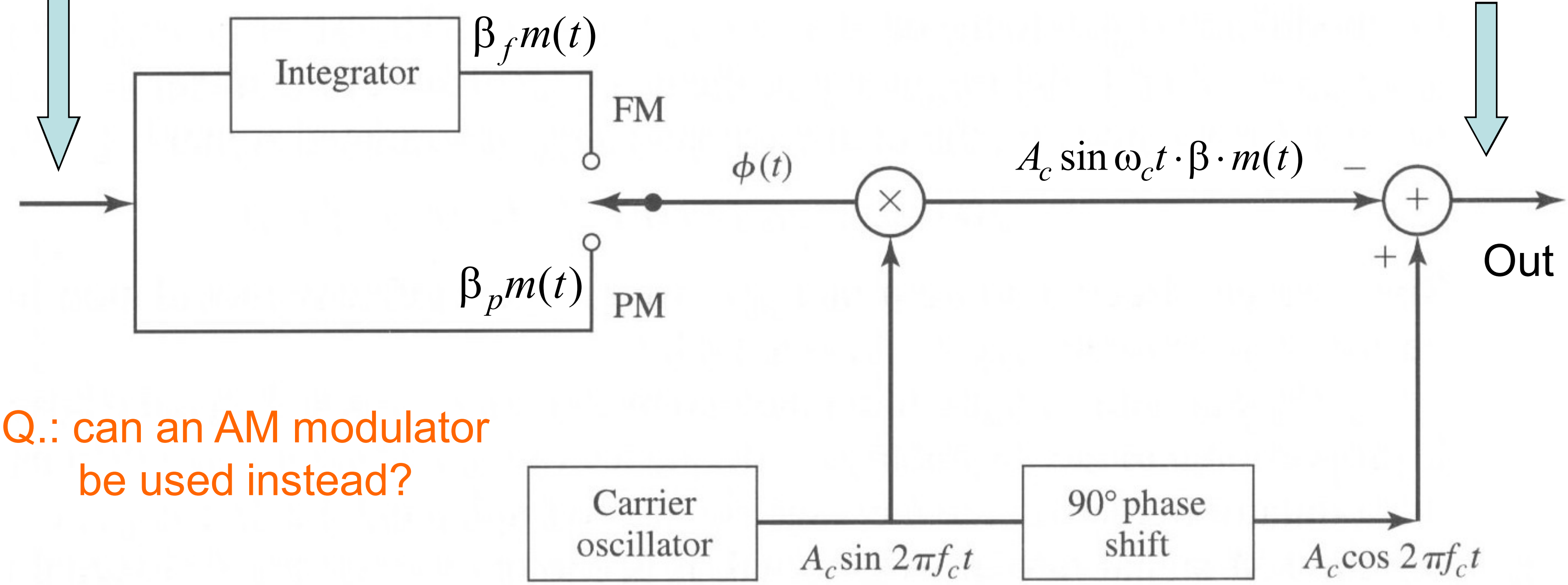
Suitable for narrowband FM only.

Narrowband Angle Modulator

Small modulation index: $\beta \ll 1$

$$x(t) = A_c \cos\left[\omega_c t + \beta \cdot m(t)\right] \approx$$
$$\approx A_c \cos \omega_c t - A_c \sin \omega_c t \cdot \beta \cdot m(t)$$

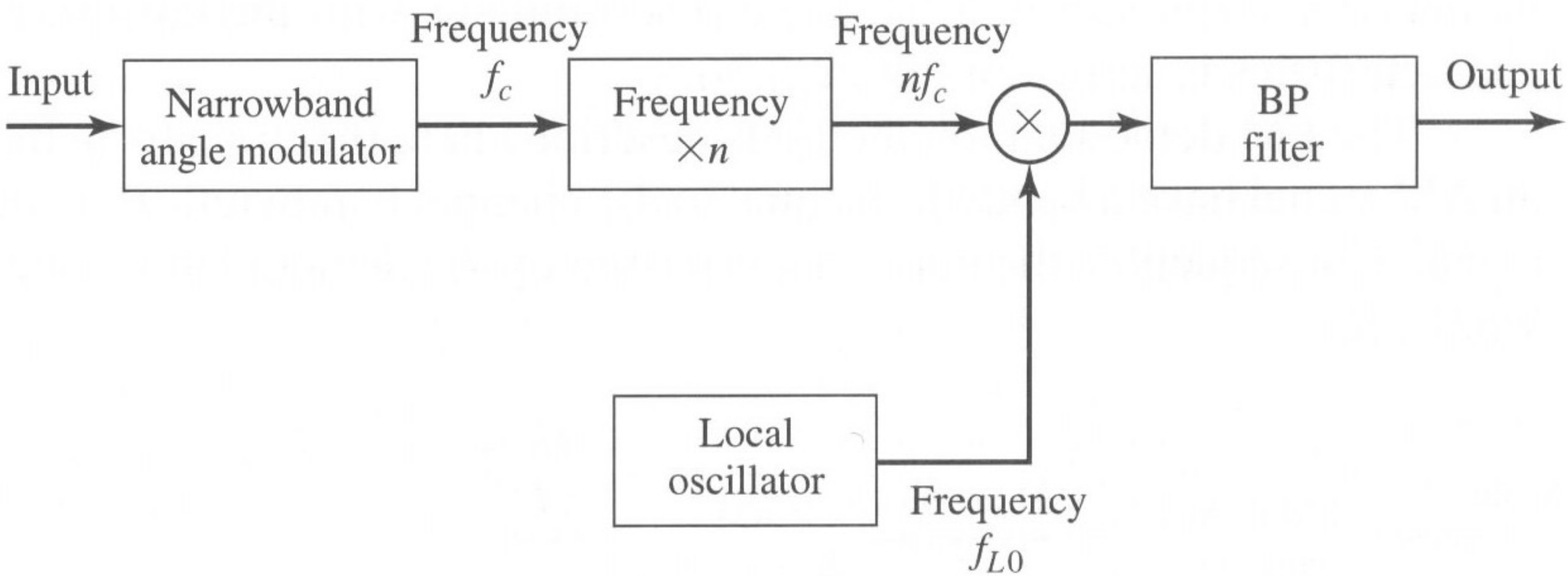
Message signal



Q.: can an AM modulator be used instead?

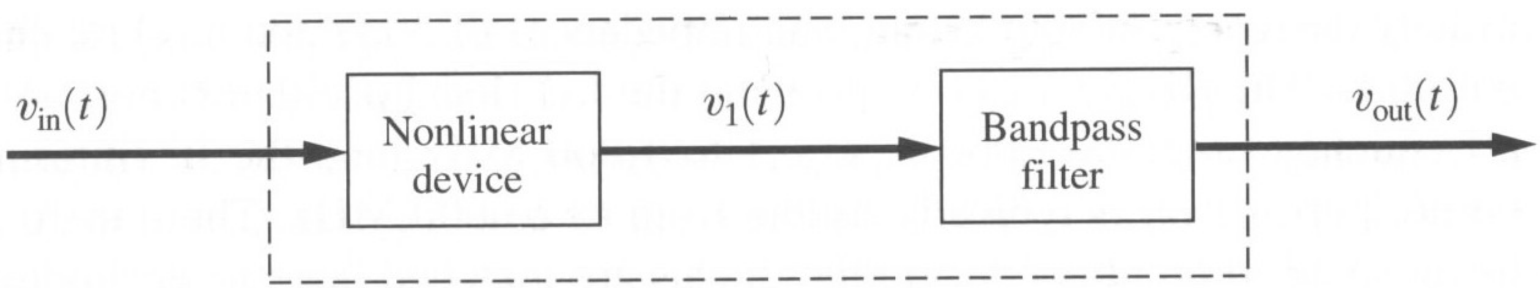
J.Proakis, M.Salehi, Communications Systems Engineering, Prentice Hall, 2002

Indirect Wideband Angle Modulator



J.Proakis, M.Salehi, Communications Systems Engineering, Prentice Hall, 2002

Frequency multiplier:



$$\cos^2\psi(t) = \frac{1}{2}\left[1+\cos\left(2\psi(t)\right)\right] \xrightarrow{\text{BPF}} \frac{1}{2}\cos\left(2\psi(t)\right)$$

Indirect Wideband FM Transmitter (Amstrong)

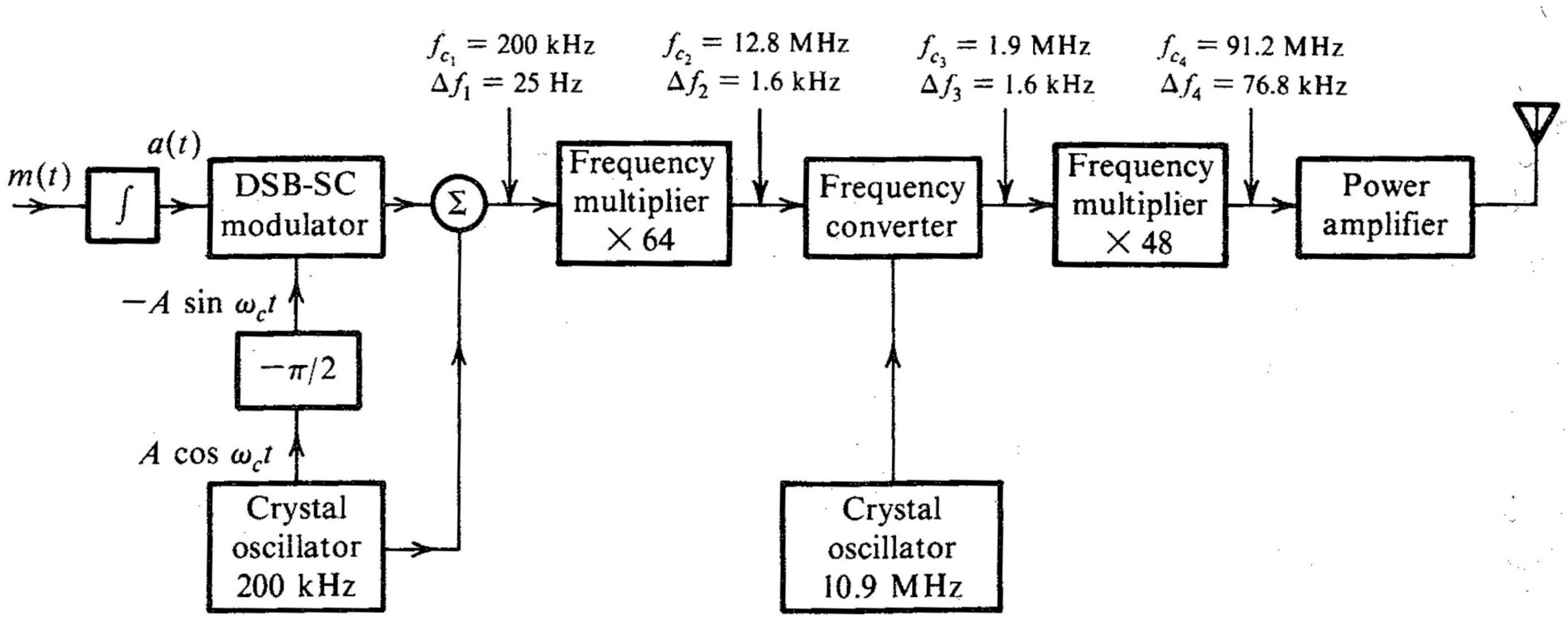
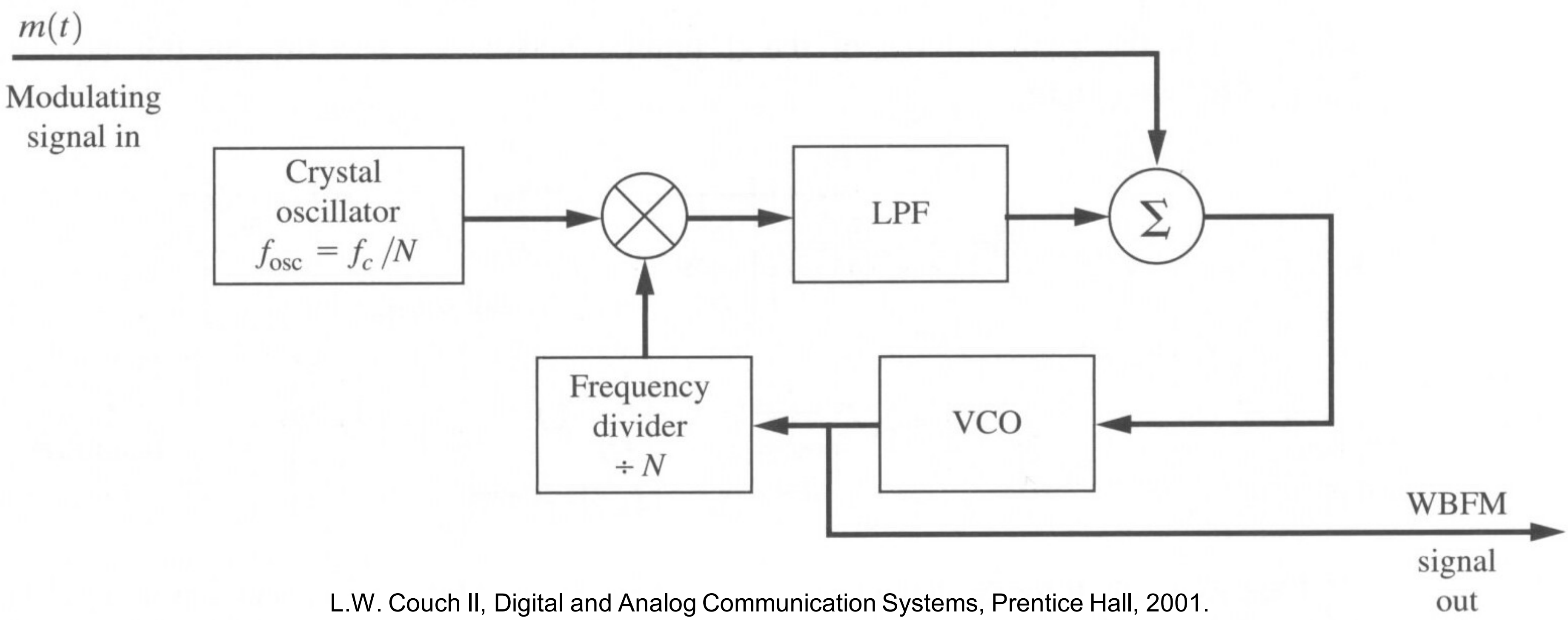


Figure 5.10 Armstrong indirect FM transmitter.

B.P. Lathi, Modern Digital and Analog Communications Systems, Oxford University Press

Direct Wideband Angle Modulator



L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Explain how it operates

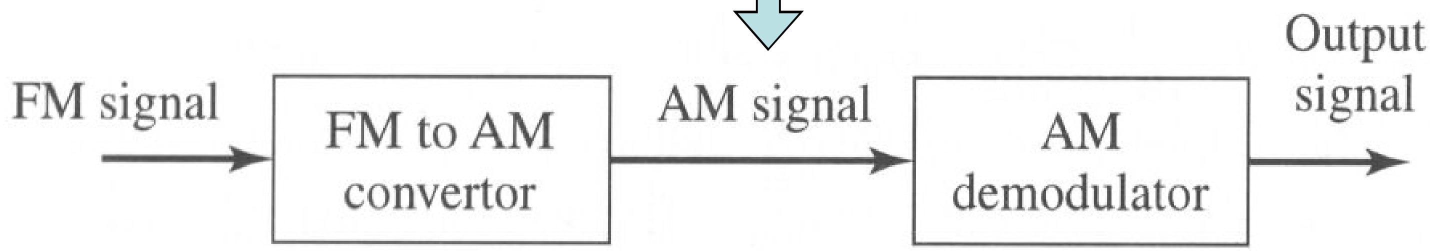
- Hint: consider it without feedback first
- Explain why feedback is required
- Explain why frequency divider is required

FM Demodulators

- FM-to-AM conversion:

$$x(t) = A_c \cos\left[\omega_c t + \int_0^t \Omega(\tau) d\tau + \varphi_0\right]$$

$$y(t) = A_c \left(H_0 + \alpha\Omega(t)\right) \cos[*]$$



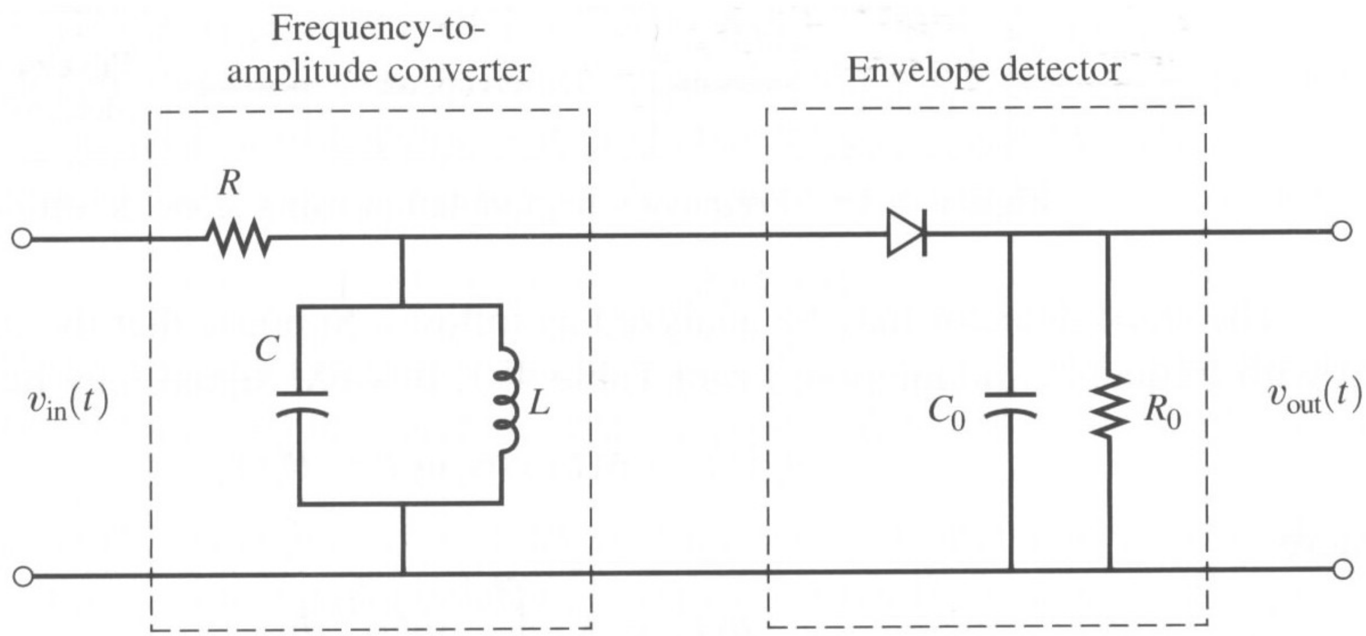
$\sim \Omega(t)$

$$\left|H(f)\right| = H_0 + \alpha(f - f_c) \quad \text{for} \left|f - f_c\right| < \frac{B_c}{2}$$

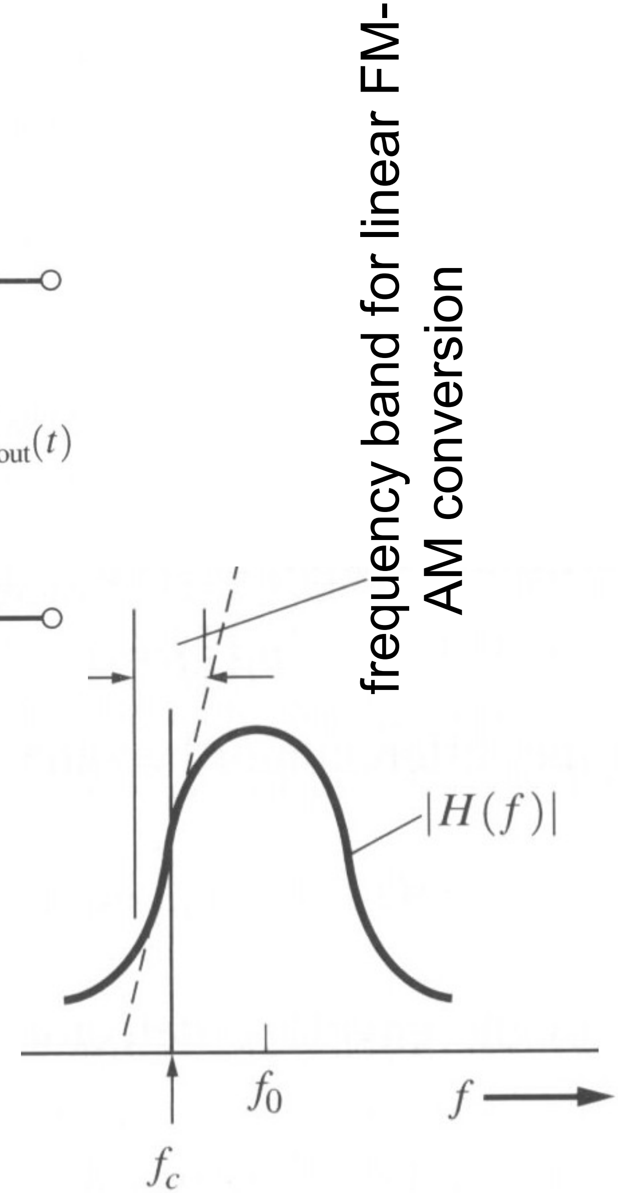
- Possible candidate: $\left|H(f)\right| = 2\pi f$ (differentiator)

FM Slope Detector

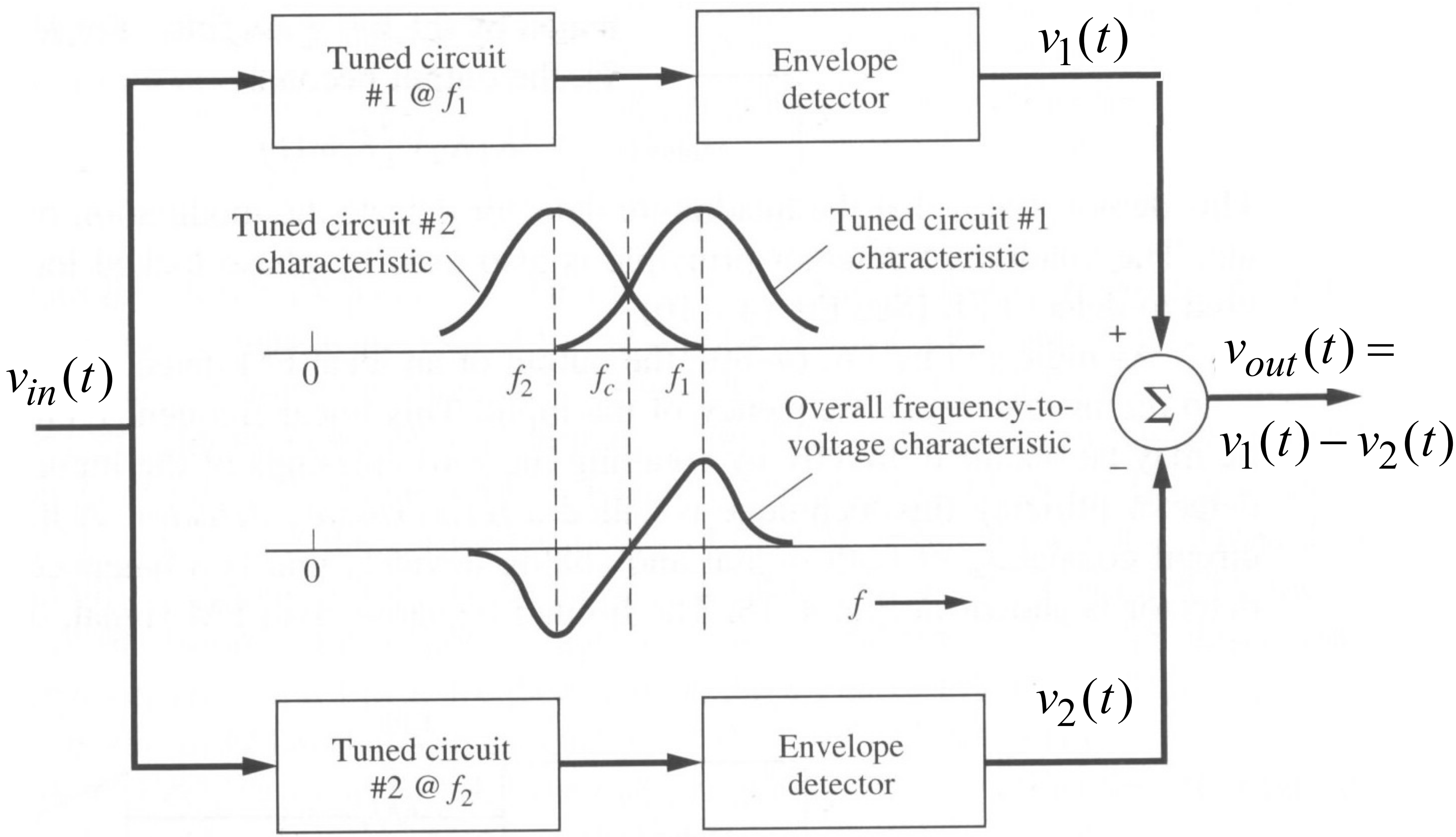
- Circuit diagram:



- Magnitude frequency response:

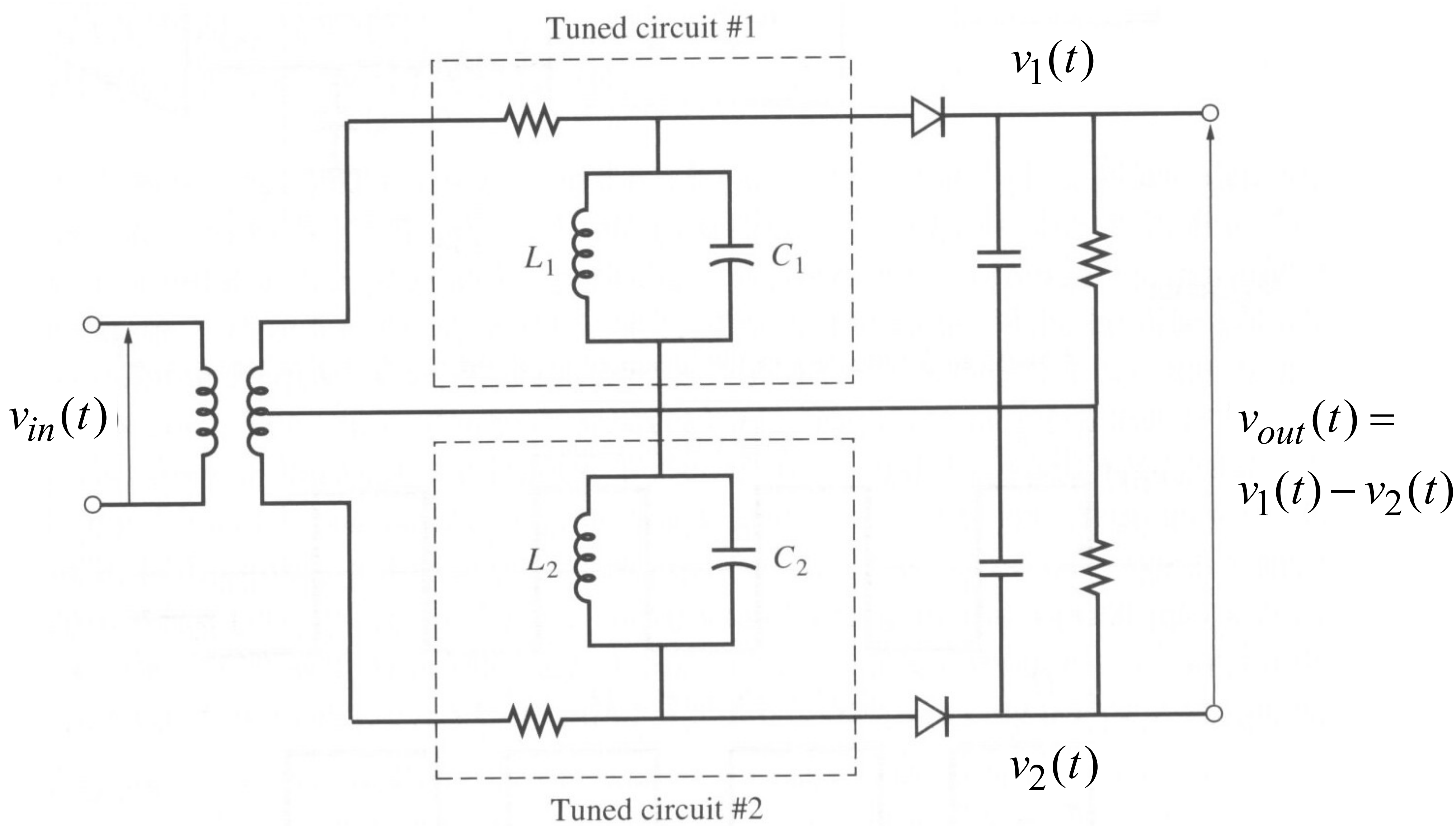


Balanced Discriminator: Block Diagram



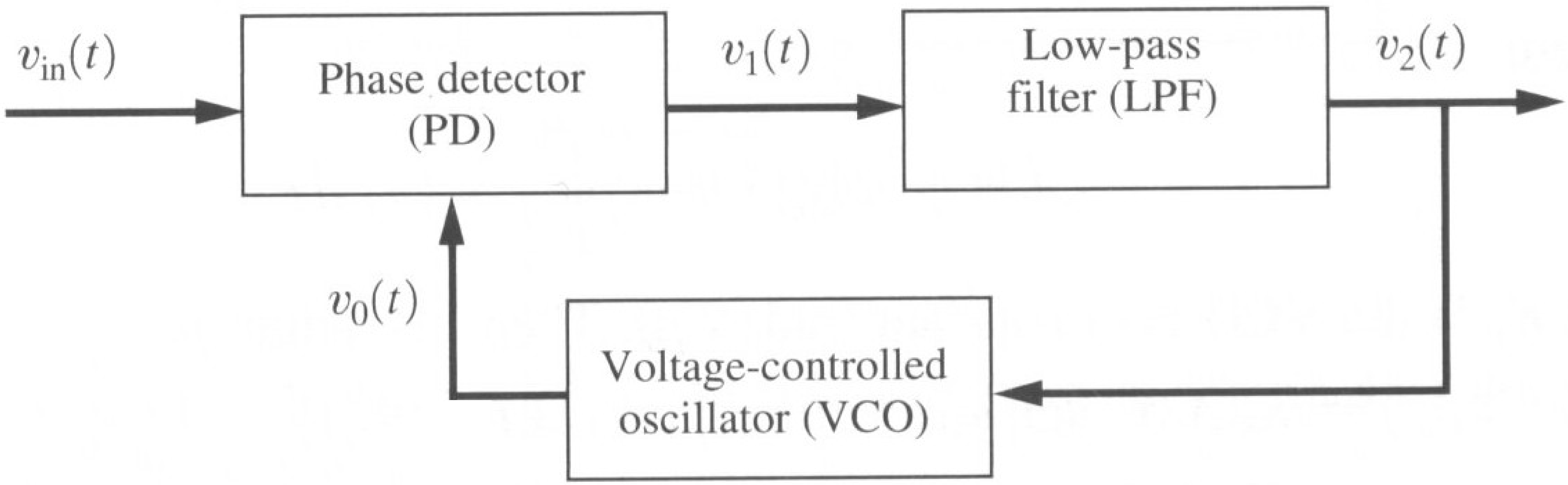
L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Balanced Discriminator: Circuit Diagram



L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Phased Locked Loop (PLL) Detector



$$v_{in}(t) = A_{in} \sin\left[\omega_c t + \varphi_{in}(t)\right]$$

$$v_1(t) = \frac{A_{in} A_0}{2} \sin\left[\varphi_{in}(t) - \varphi_0(t)\right] + (2\omega_c)\text{term}$$

$$v_0(t) = A_0 \cos\left[\omega_c t + \varphi_0(t)\right]$$

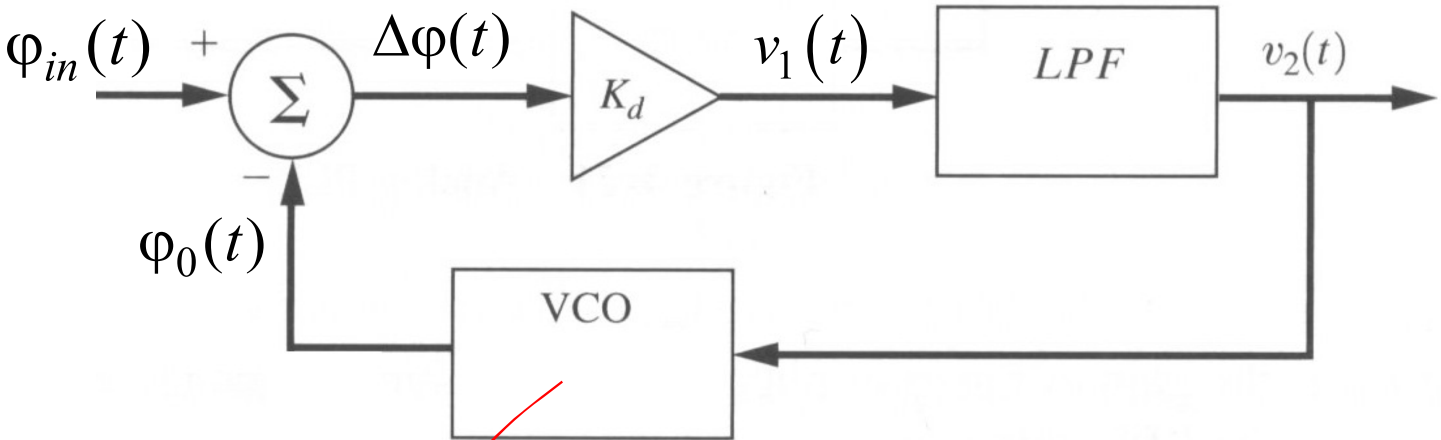
$$\omega_{VCO}(t) = \frac{d}{dt}\left(\omega_c t + \varphi_0(t)\right) = \omega_c + \alpha v_2(t)$$

Informally,

$$\varphi_{in}(t) \approx \varphi_0(t)$$

$$v_2(t) \approx \frac{1}{\alpha} \frac{d}{dt} \varphi_{in}(t)$$

PLL Detector: Linear Model



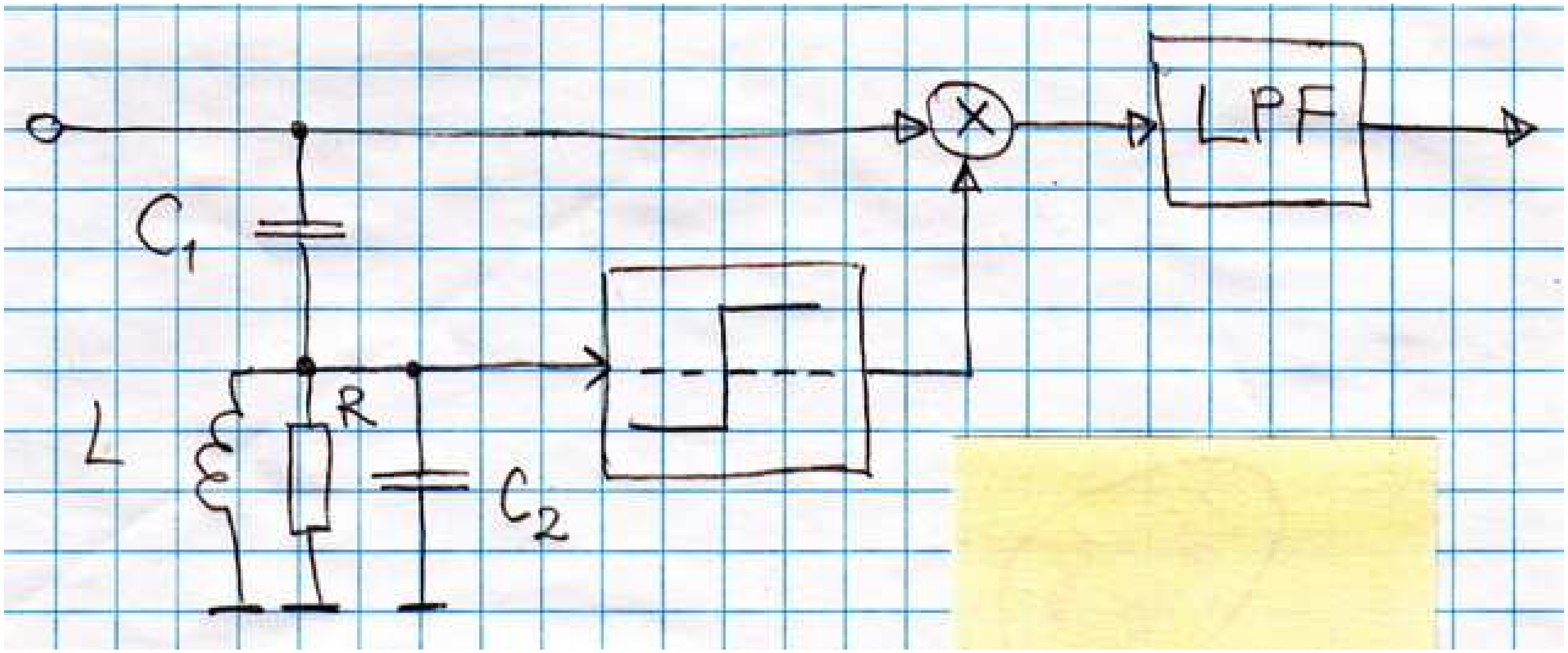
$$v_1(t) = K_d \frac{A_{in}A_0}{2} \sin\left[\varphi_{in}(t) - \varphi_0(t)\right] + +(2\omega_c)\text{term}$$

$$v_2(t) = K_d \frac{A_{in}A_0}{2} \sin\left[\varphi_{in}(t) - \varphi_0(t)\right] \approx \approx K_d \frac{A_{in}A_0}{2}\left(\varphi_{in}(t) - \varphi_0(t)\right) = a_2 \Delta\varphi(t)$$

$$\frac{d}{dt}\varphi_0(t) = \alpha v_2(t)$$

$$v_2(t) = \frac{1}{\alpha}\frac{d}{dt}\left[\varphi_{in}(t) - \Delta\varphi(t)\right] \approx \frac{1}{\alpha}\frac{d}{dt}\varphi_{in}(t)$$

FM Demodulator: Lab 3

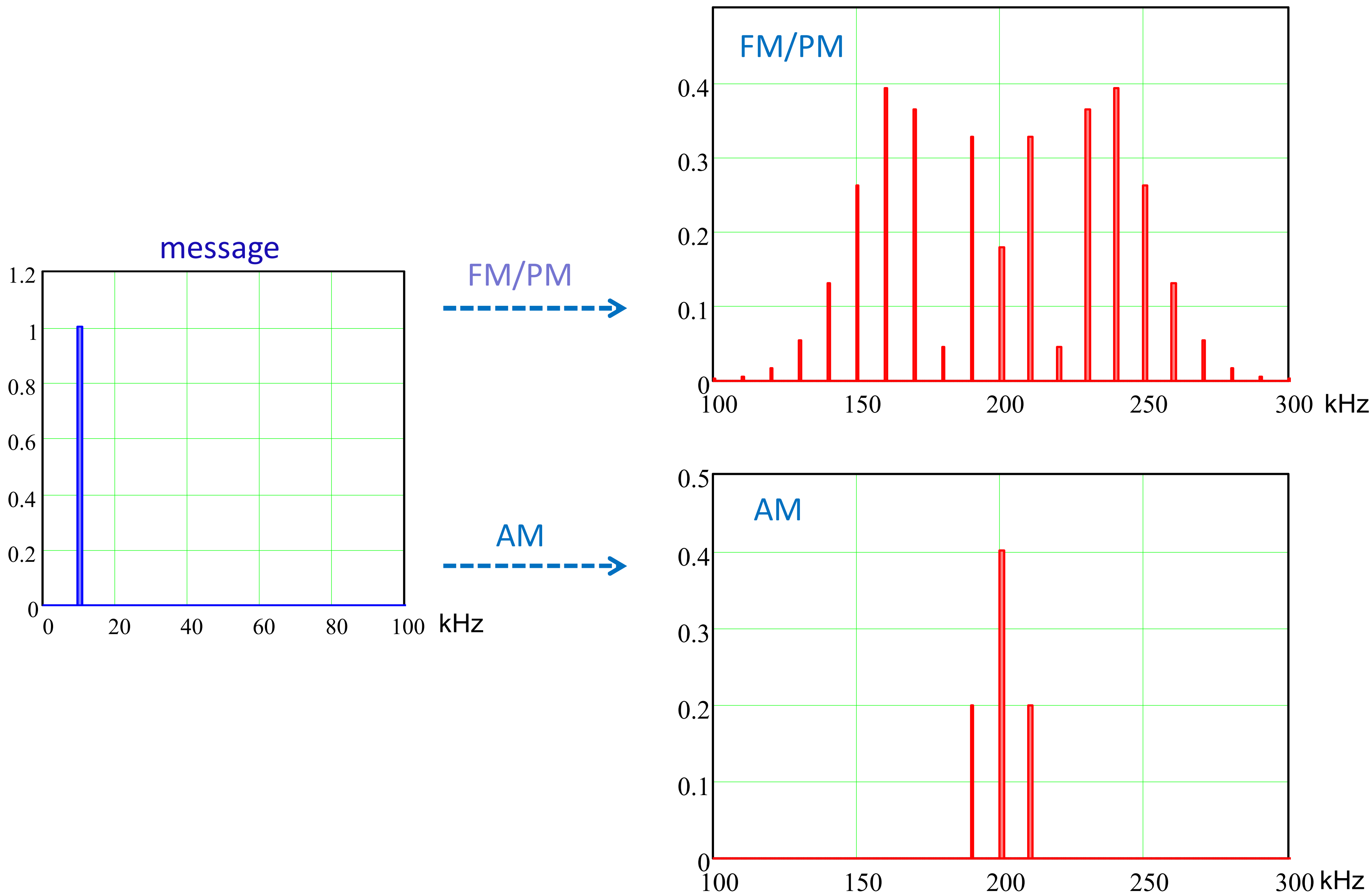


- explain its operation !
- Hint: C1=5 pF, C2=120 pF, L=1 mH, R=100k Ohm;
- observe that C1 << C2 and use it.

Comparison of AM and FM/PM

- AM is simple (envelope detector) but no noise/interference immunity (low quality).
- AM bandwidth is twice or the same as the modulating signal (no bandwidth expansion).
- Power efficiency is low for conventional AM.
- DSB-SC & SSB – good power efficiency, but complex circuitry.
- FM/PM – spectrum expansion & noise immunity. Good quality.
- More complex circuitry. However, ICs allow for cost-effective implementation.

Spectrum: FM/PM vs. AM



Important Properties of Angle-Modulated Signals: Summary

- FM/PM signal is a nonlinear function of the message.
- The signal's bandwidth increases with the modulation index.
- The carrier spectral level varies with the modulation index, being 0 in some cases.
- Narrowband FM/PM: the signal's bandwidth is twice that of the message (the same as for AM).
- The amplitude of the FM/PM signal is constant (hence, the power does not depend on the message).

Summary

- Angle modulation: PM & FM
- Spectra of angle-modulated signals. Modulation index.
- Narrowband (low-index) & wideband (large-index) modulation. Signal bandwidth.
- Relation between PM and FM.
- Generation of angle-modulated signals. Narrowband & wideband modulators.
- Demodulation of PM and FM signals. Slope detector & balanced discriminator. PLL detector.
- Comparison of AM and FM/PM.

- **Homework**: Reading: Couch, 5.6, 4.13, 4.14. Study carefully all the examples, make sure you understand them and can solve with the book closed.