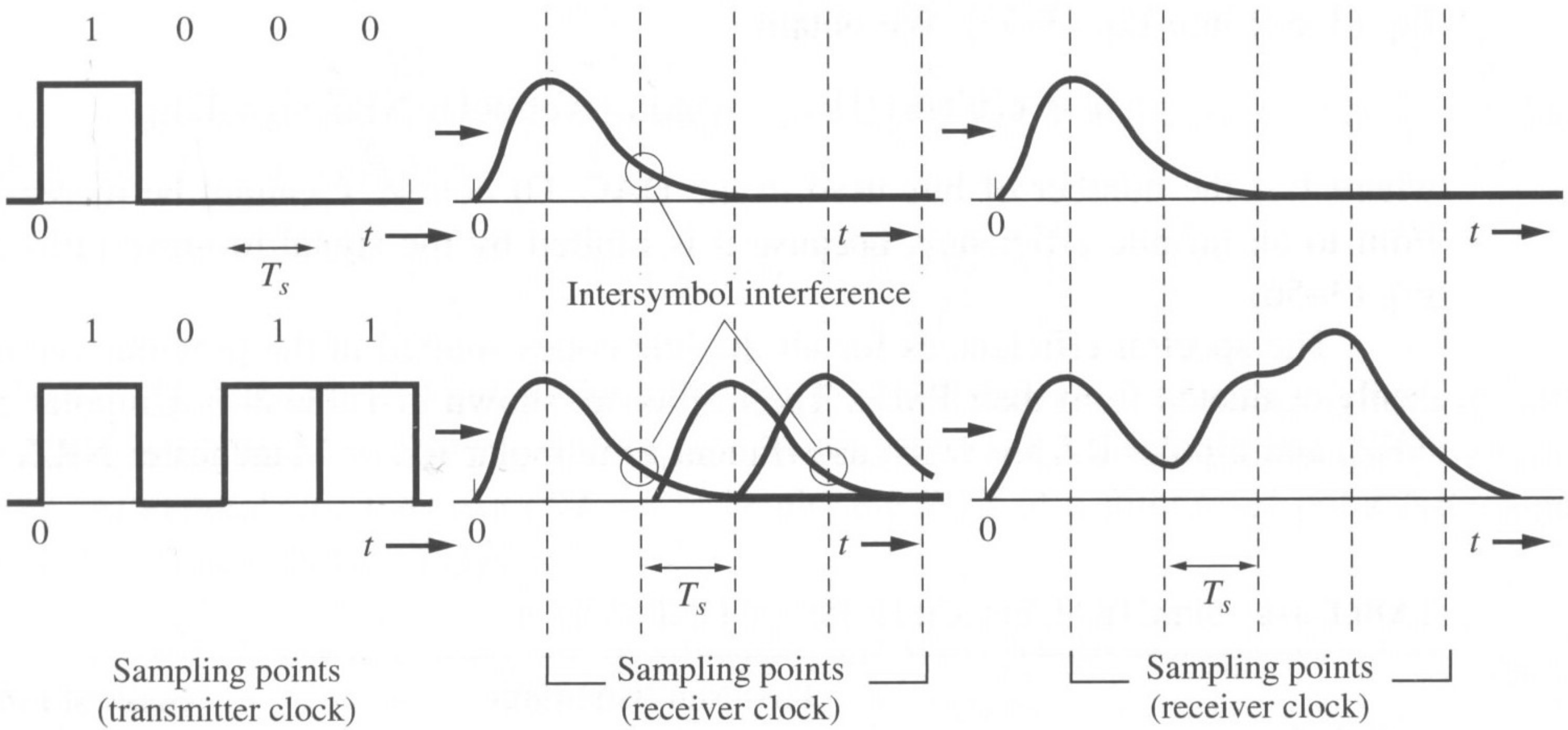
Band-limited Channels and Intersymbol Interference

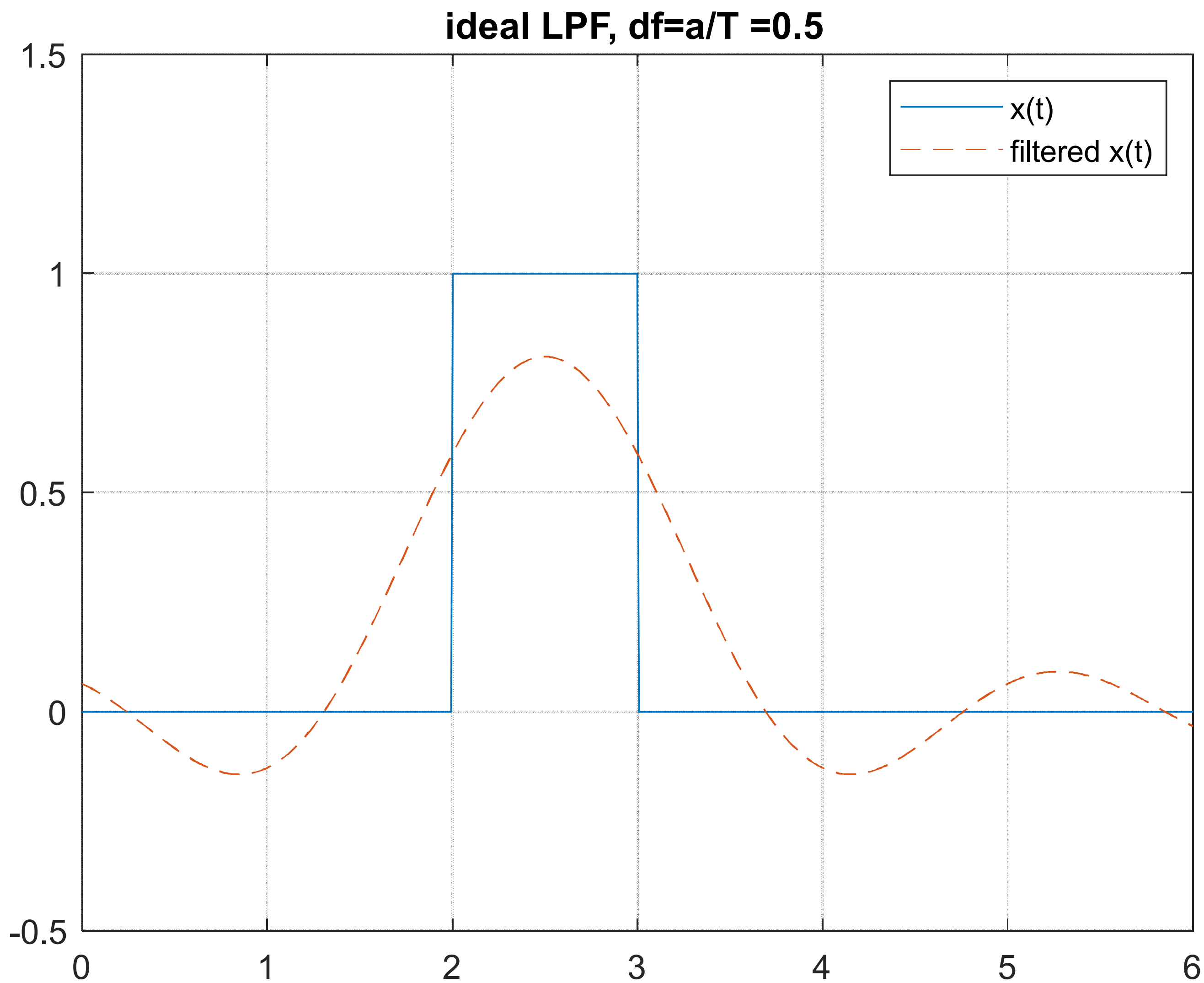
- Rectangular pulses are suitable for infinite-bandwidth channels (practically – wideband).
- Practical channels are band-limited -> pulses spread in time and are smeared into adjacent slots. This is intersymbol interference (ISI).

Input binary waveform   Individual pulse response   Received waveform

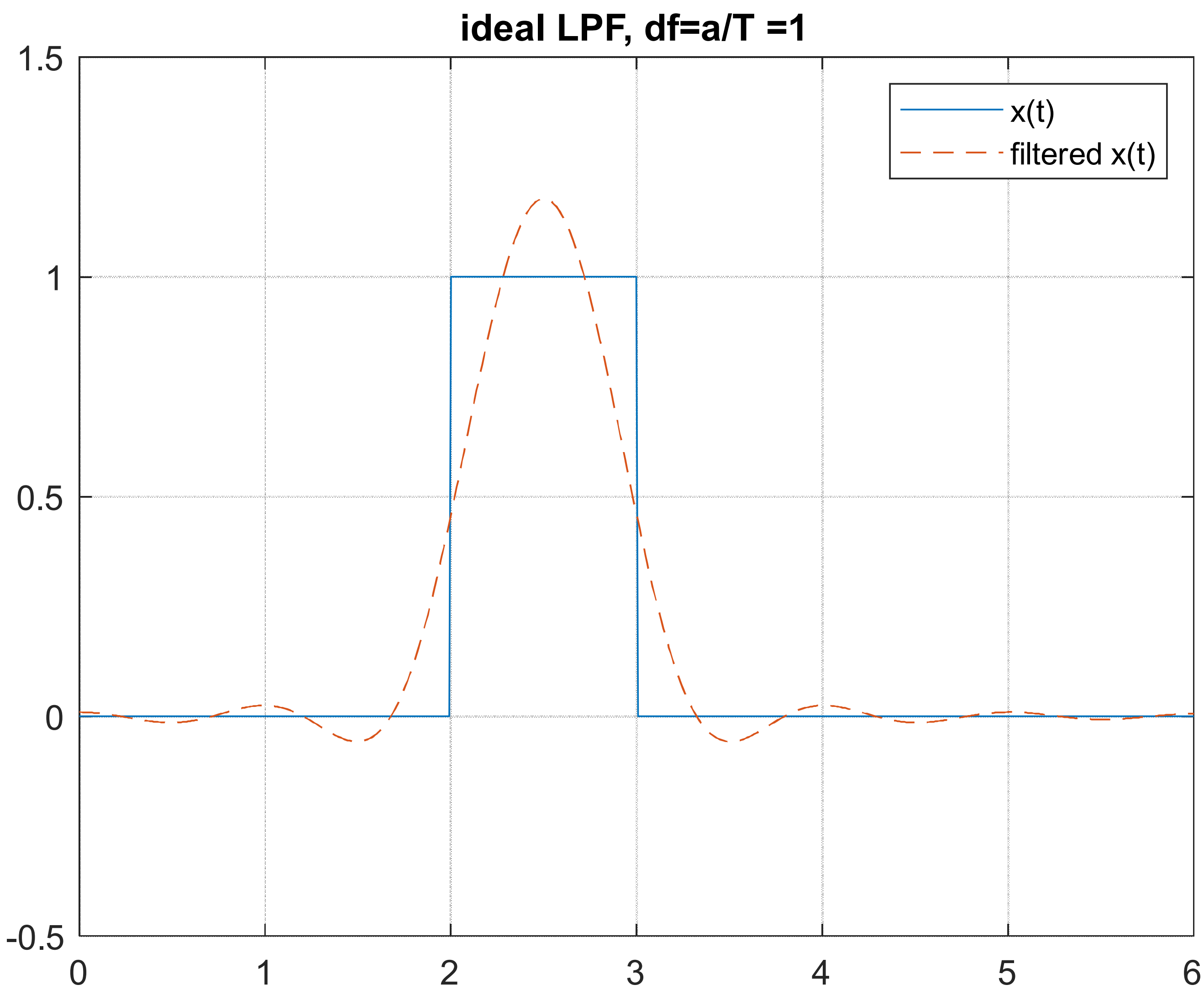


L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

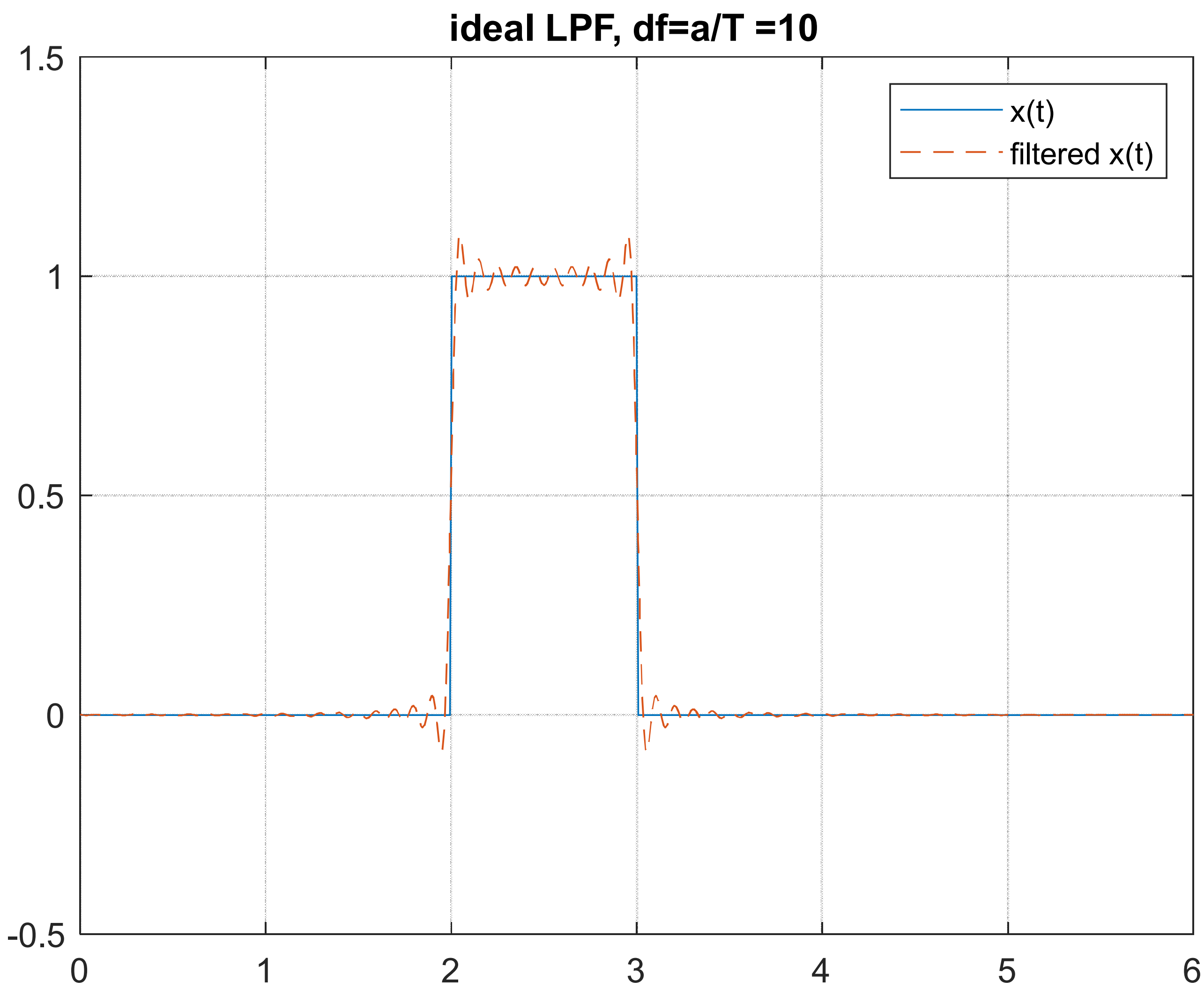
Band-limited Channels: Ideal LPF



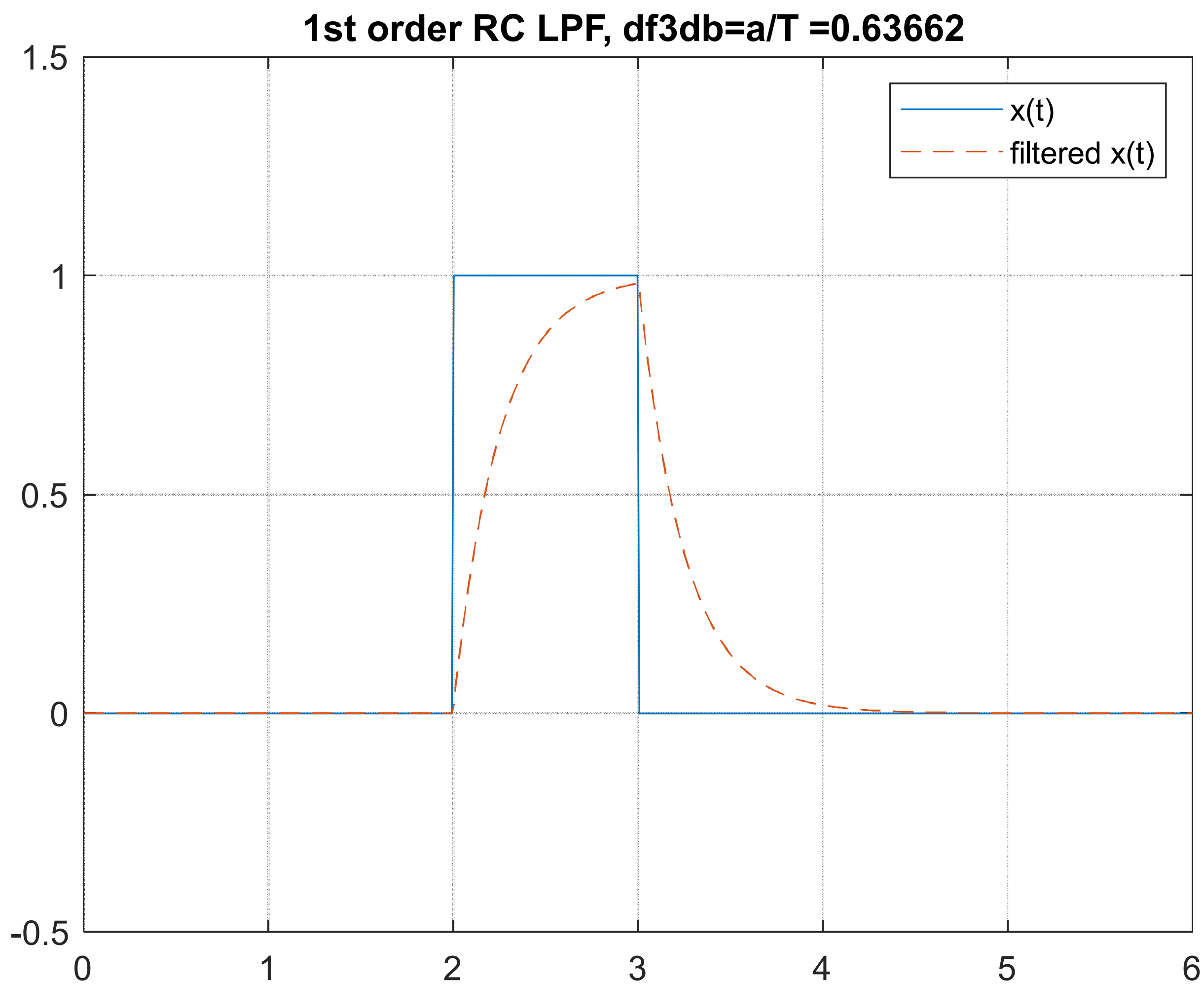
Band-limited Channels: Ideal LPF



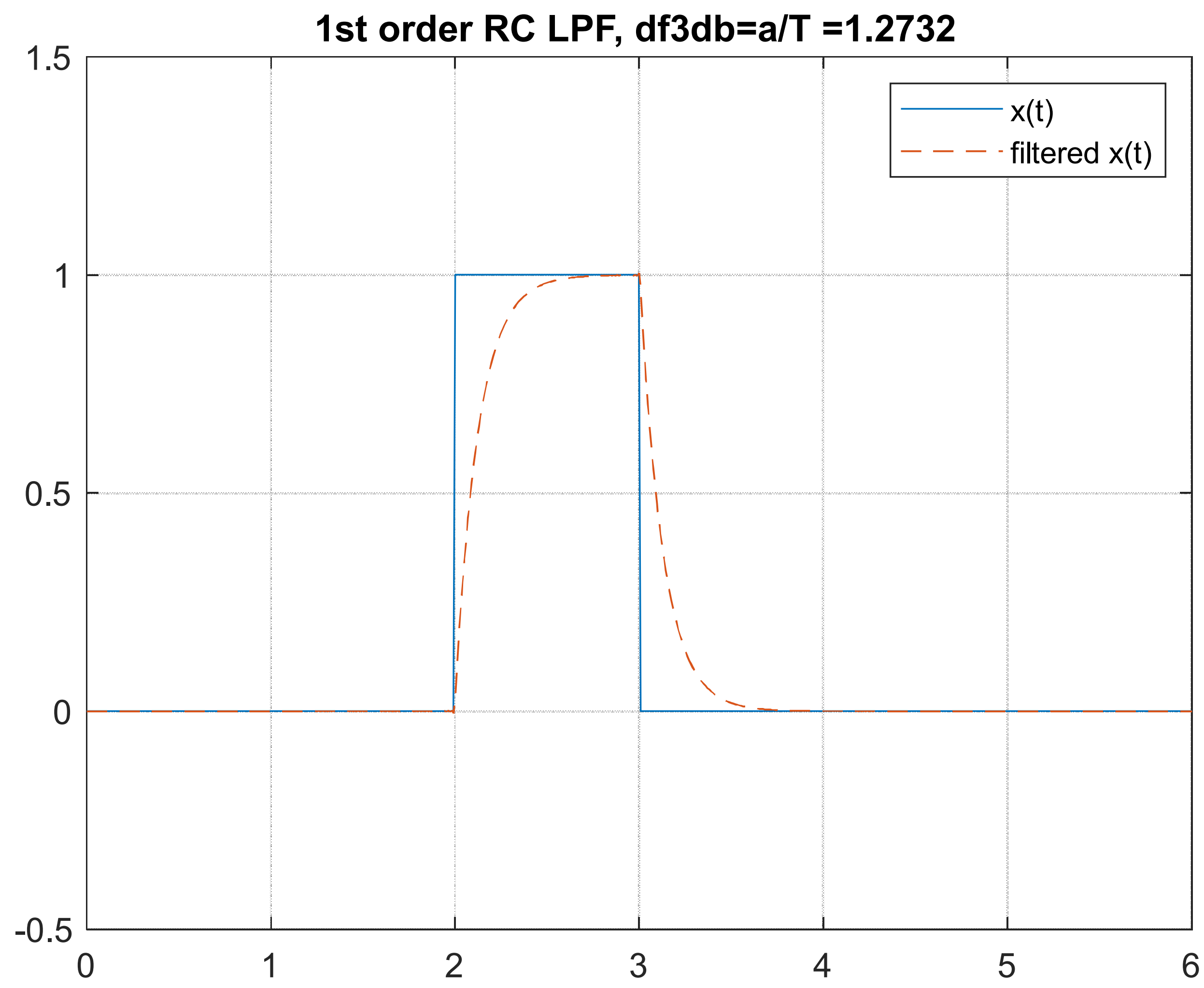
Band-limited Channels: Ideal LPF



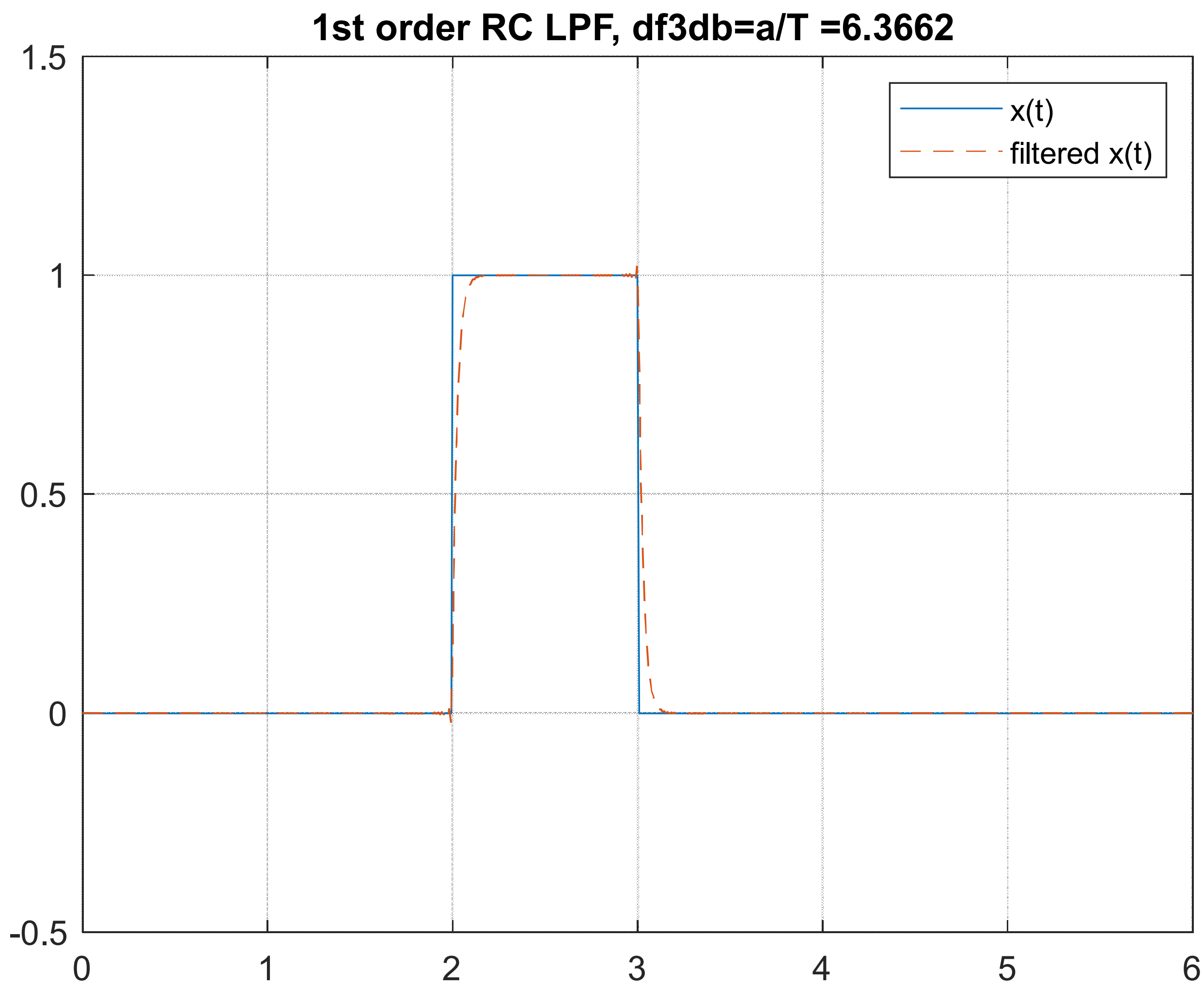
Band-limited Channels: RC LPF



Band-limited Channels: RC LPF

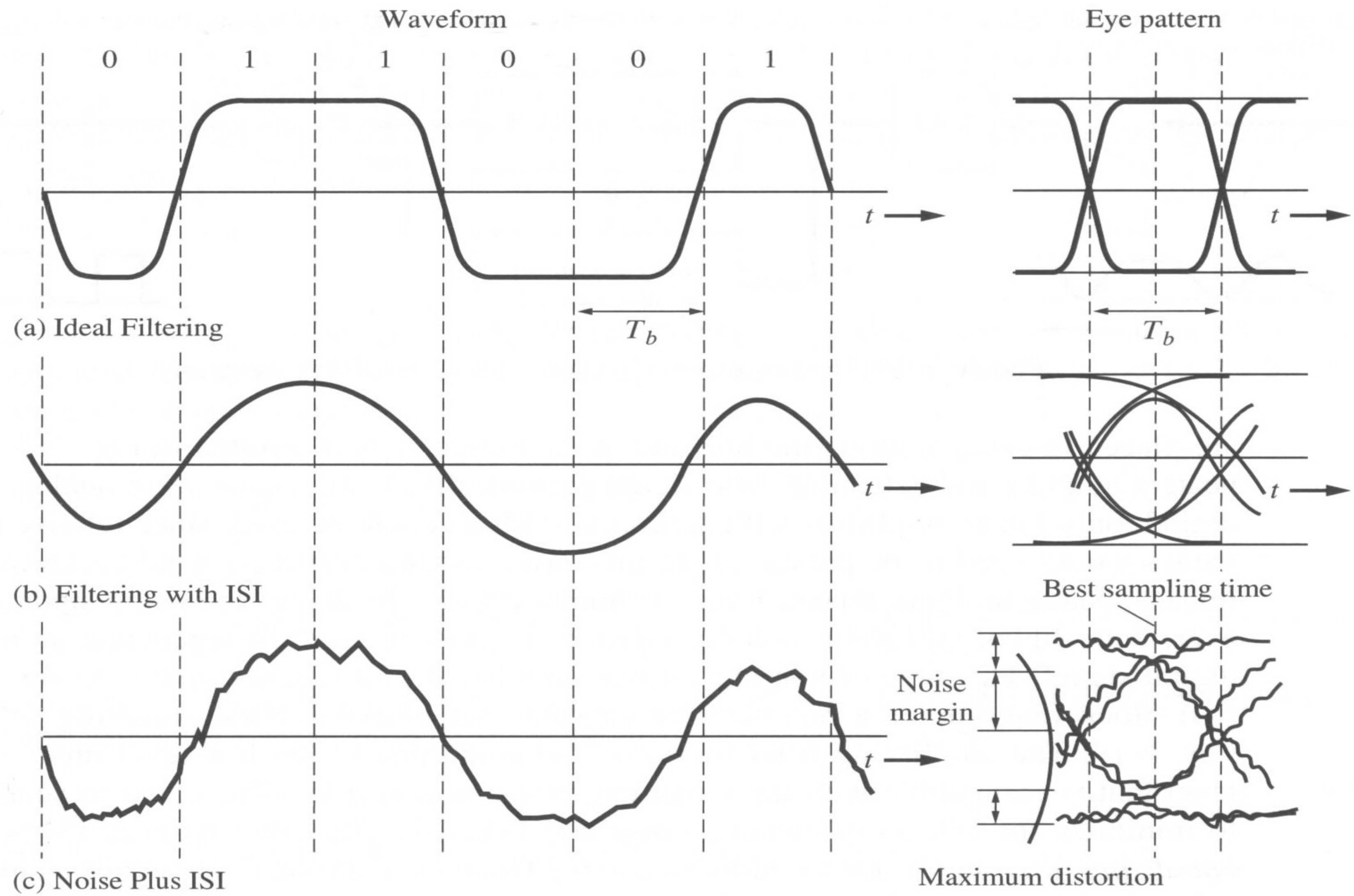


Band-limited Channels: RC LPF



Eye Diagram

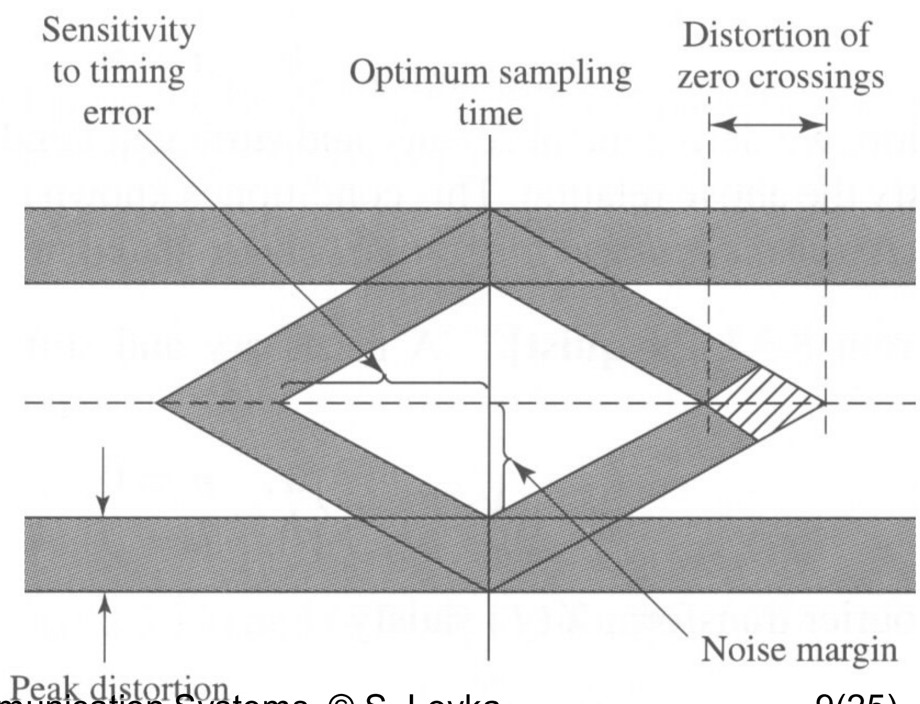
- Convenient way to observe the effect of ISI and channel noise on an oscilloscope.



Eye Diagram

- Oscilloscope presentations of a signal with multiple sweeps (triggered by a clock signal!), each is slightly larger than symbol interval.
- Quality of a received signal may be estimated.
- Normal operating conditions (no ISI, no noise) -> eye is open.
- Large ISI or noise -> eye is closed.

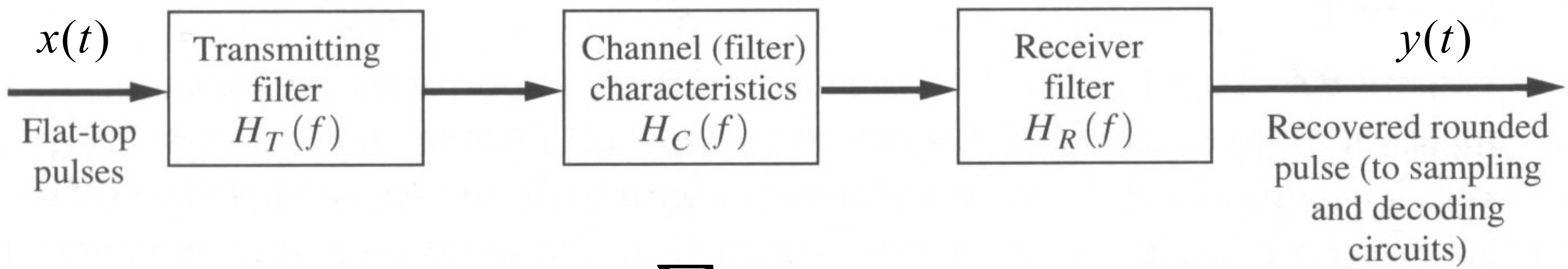
- Timing error allowed – width of the eye, called eye opening (preferred sampling time – at the largest vertical eye opening).
- Sensitivity to timing error -> slope of the open eye evaluated at the zero crossing point.
- Noise margin -> the height of the eye opening.



J.Proakis, M.Salehi, Communications Systems Engineering, Prentice Hall, 2002

Transmission over a Band-Limited Channel

baseband transmission system



- Input PAM signal: $x(t) = \sum_n a_n s_T(t - nT)$

T – symbol interval

R_s = f_0=1/T – symbol rate

- Output signal: $y(t) = \sum_n a_n s(t - nT),$

$$s(t) = s_T(t) * h_T(t) * h_C(t) * h_R(t) = s_T(t) * h(t)$$

- Sampled (at *t=mT*) output:

transmitted symbol

$$y_m = \sum_n a_n s_{m-n} = a_m s_0 + \sum_{n \neq m} a_n s_{m-n},$$

$$y_m = y(mT),\ s_{m-n} = s(mT - nT)$$

sampled output

ISI

Pulse Shaping to Eliminate ISI

- Nyquist (1928) discovered 3 methods to eliminate ISI
  - zero ISI pulse shaping
  - controlled ISI (eliminated later on by, say, equalizer)
  - zero average ISI (negative and positive areas under the pulse in an adjacent interval are equal)
- Zero ISI pulse shaping:

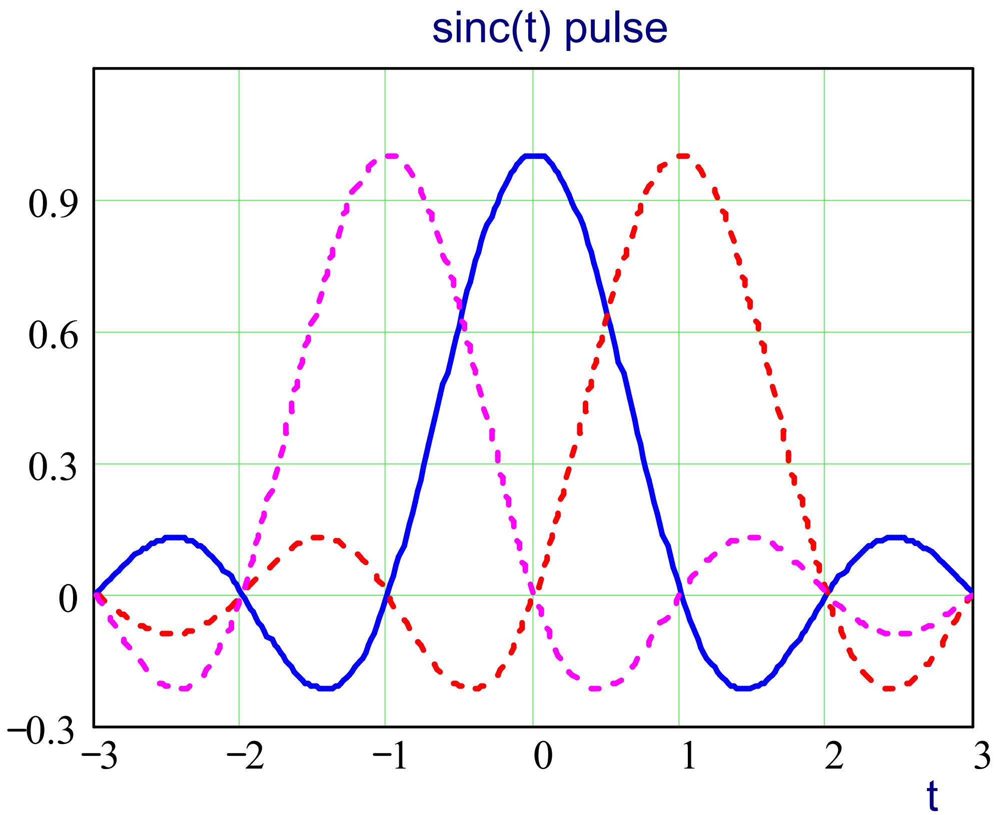
$$\boxed{s(nT) = \begin{cases} s_0, & n = 0 \\ 0, & n \neq 0 \end{cases}}$$

$$\boxed{y_m = a_m s_0 + \sum_{n \neq m} a_n s_{m-n} \Rightarrow y_m = a_m s_0}$$

- Example: $s(t) = \operatorname{sinc}(R_s t) \Rightarrow s(nT) = \operatorname{sinc}(n) = 0,\ n \neq 0$

Note: $R_s = f_0 = 1/T$ – transmission rate [symbols/s]

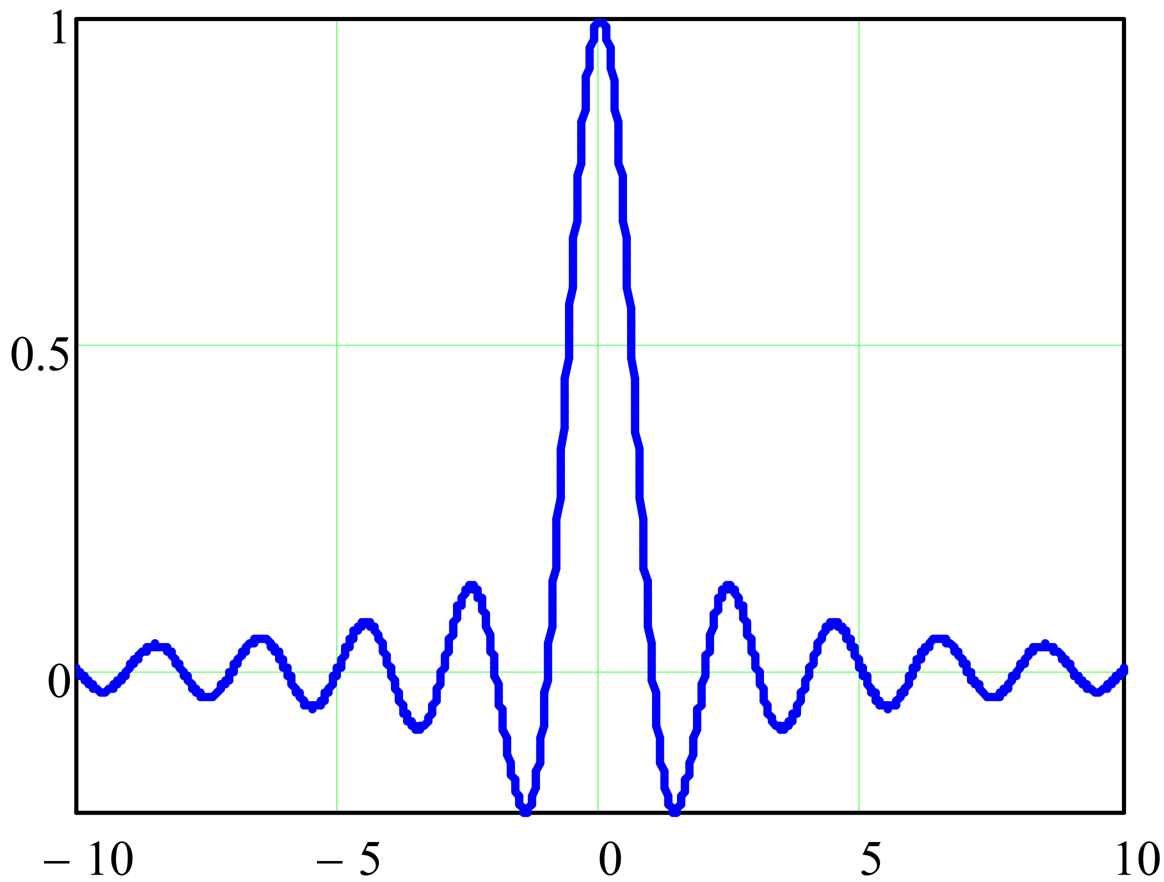
Zero ISI: sinc Pulse



$$S_s(f) = ?$$

- Example: $s(t) = \text{sinc}(R_s t) \Rightarrow s(nT) = \text{sinc}(n) = 0,\ n \neq 0$
- Hence, *sinc* pulse allows to eliminate ISI at sampling instants. However, it has some (2) serious drawbacks.

Zero ISI: sinc pulse, drawbacks



Nyquist Criterion for Zero ISI

- Provides a generic solution to the zero ISI problem.
- A necessary and sufficient condition for s(t) to satisfy

$$\boxed{s(nT)=\begin{cases}1, & n=0\\ 0, & n\neq 0\end{cases}} \quad \text{is} \quad \Longleftrightarrow \quad \boxed{\sum_{m=-\infty}^{\infty} S_s\left(f+\frac{m}{T}\right)=T}$$

- Proof: homework (see the text by Proakis and Salehi, Sec.8.3.1).
  - Hint: consider ideal sampling of s(t) and find its FT
- Consider a channel badndlimited to [0,F_{max}].Three cases:

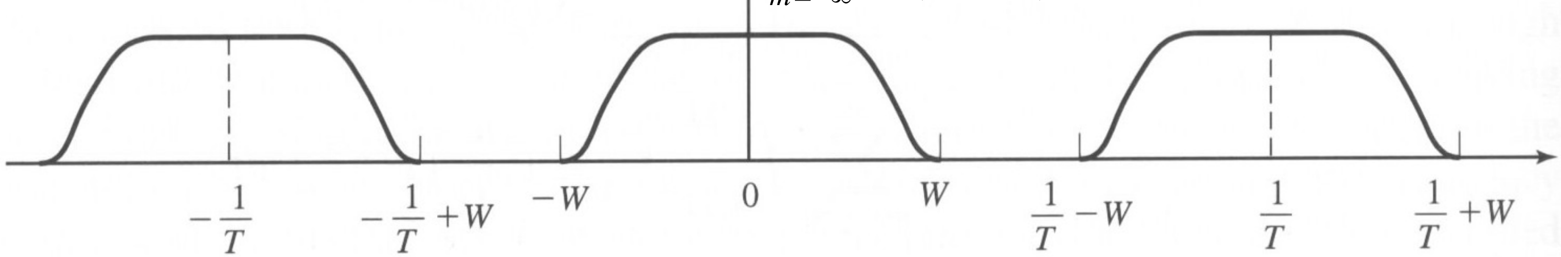
  1) $1/T=R_s>2F_{\max}$ -> no way to eliminate ISI.
  2) $R_s=2F_{\max}$ -> only $\mathrm{sinc}(R_s t)$ eliminates ISI. The highest possible transmission rate f_0 for transmission with zero ISI is $2F_{max}$, not F_{max} (what a surprise!)
  3) $R_s<2F_{\max}$ -> many signals may eliminate ISI in this case.

Note: R_s=1/T – transmission rate [symbols/s]

Nyquist Criterion for Zero ISI

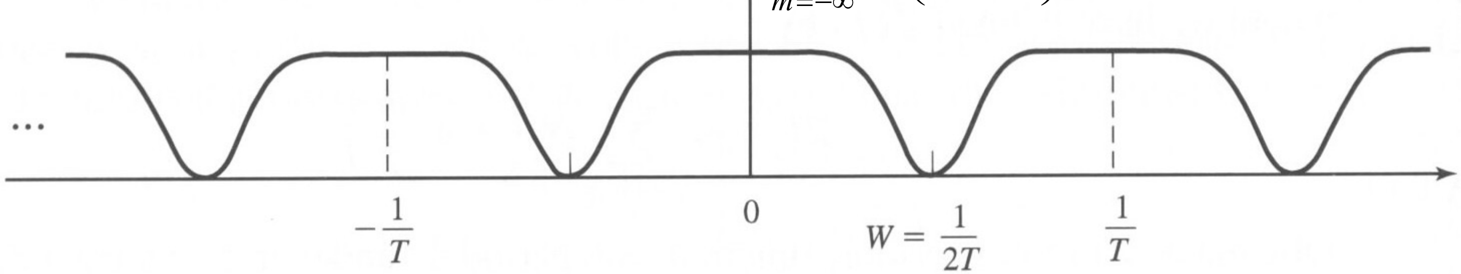
Case 1: $1/T = R_s > 2F_{\max}$

$$\sum_{m=-\infty}^{\infty} S_s\left(f + \frac{m}{T}\right) = Z(f) \qquad W = F_{\max}$$



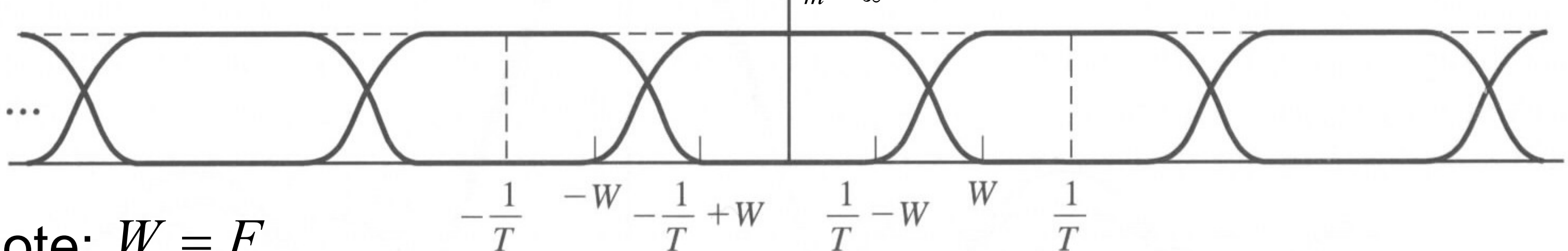
Case 2: $R_s = 2F_{\max}$

$$\sum_{m=-\infty}^{\infty} S_s\left(f + \frac{m}{T}\right) = Z(f)$$



Case 3: $R_s < 2F_{\max}$

$$\sum_{m=-\infty}^{\infty} S_s\left(f + \frac{m}{T}\right) = Z(f)$$



Note: $W = F_{\max}$

Harry Nyquist

Certain Factors Affecting Telegraph Speed[1]

By H. NYQUIST

SYNOPSIS: This paper considers two fundamental factors entering into the maximum speed of transmission of intelligence by telegraph. These factors are signal shaping and choice of codes. The first is concerned with the best wave shape to be impressed on the transmitting medium so as to permit of greater speed without undue interference either in the circuit under consideration or in those adjacent, while the latter deals with the choice of codes which will permit of transmitting a maximum amount of intelligence with a given number of signal elements.

It is shown that the wave shape depends somewhat on the type of circuit over which intelligence is to be transmitted and that for most cases the optimum wave is neither rectangular nor a half cycle sine wave as is frequently used but a wave of special form produced by sending a simple rectangular wave through a suitable network. The impedances usually associated with telegraph circuits are such as to produce a fair degree of signal shaping when a rectangular voltage wave is impressed.

Consideration of the choice of codes show that while it is desirable to use those involving more than two current values, there are limitations which prevent a large number of current values being used. A table of comparisons shows the relative speed efficiencies of various codes proposed. It is shown that no advantages result from the use of a sine wave for telegraph transmission as proposed by Squier and others[2] and that their arguments are based on erroneous assumptions.

SIGNAL SHAPING

SEVERAL different wave shapes will be assumed and comparison will be made between them as to:

1. Excellence of signals delivered at the distant end of the circuit, and
2. Interfering properties of the signals.

Consideration will first be given to the case where direct-current impulses are transmitted over a distortionless line, using a limited range of frequencies. Transmission over radio and carrier circuits will next be considered. It will be shown that these cases are closely related to the preceding one because of the fact that the transmitting medium in the case of either radio or carrier circuits closely approximates a distortionless line. Telegraphy over ordinary land lines

[1] Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa. February 4-8, 1924, and reprinted from the Journal of the A. I. E. E. Vol. 43, p.

Born: 7 Feb. 1889, Värmland, Sweden
Died: 4 Apr. 1976 (aged 87), Texas, US

https://en.wikipedia.org/wiki/Harry_Nyquist

Raised Cosine Pulse

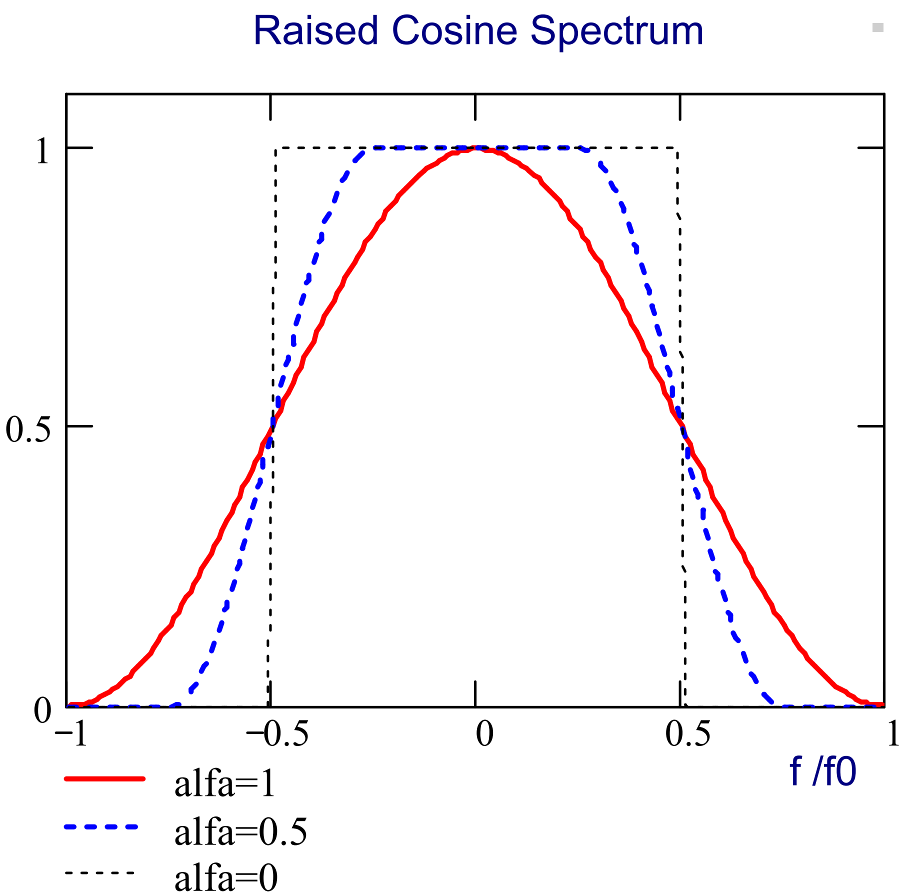
- When $R_s = f_0 < 2F_{\max}$ raised cosine pulse is widely used.
- Its spectrum is

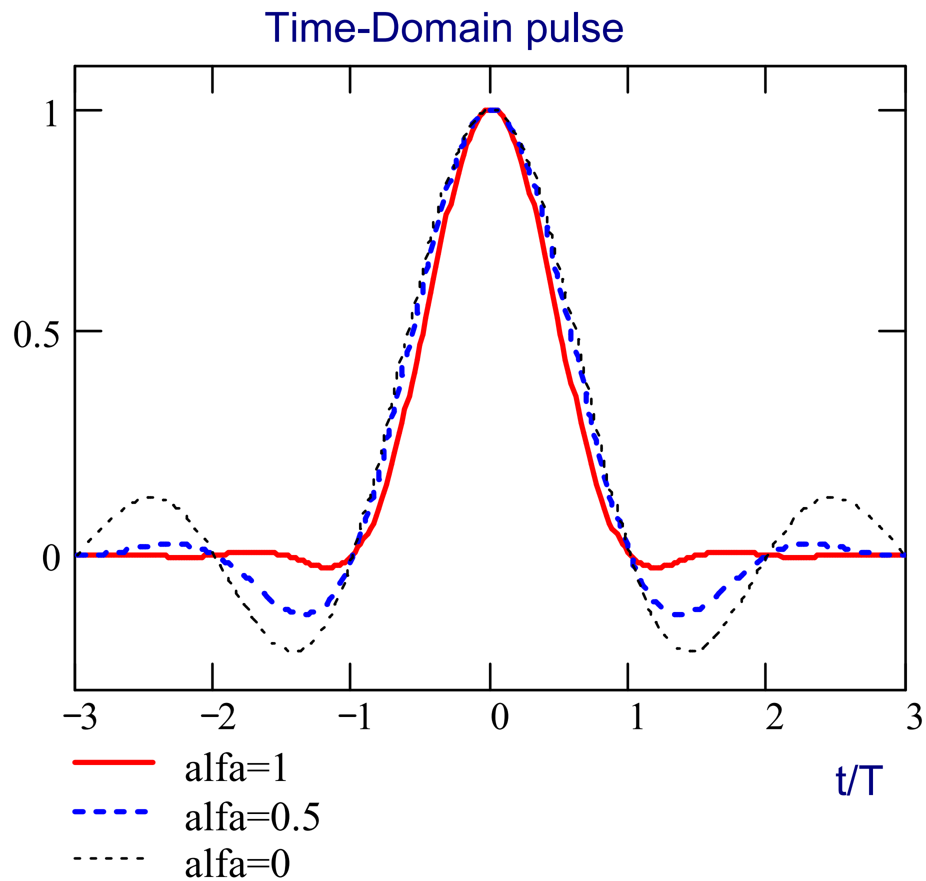
$$S_{rc}(f) = \begin{cases} T, & 0 \le |f| \le (1-\alpha) f_0 / 2 \\ \frac{T}{2}\left[1 + \cos\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2} f_0\right)\right], & \frac{1-\alpha}{2} f_0 \le |f| \le \frac{1+\alpha}{2} f_0 \\ 0, & |f| > \frac{1+\alpha}{2} f_0 \end{cases}$$

- where α is roll-off factor, $0 \le \alpha \le 1$
- The bandwidth above $f_0/2$ is called excess bandwidth.
- Time-domain waveform of the pulse:

$$s(t) = \operatorname{sinc}(t/T)\frac{\cos(\pi\alpha t/T)}{1 - 4\alpha^2 t^2 / T^2}$$

Raised Cosine Pulse (RCP)





Note that: (1) $\alpha = 0$, pulse reduces to $\mathrm{sinc}\left(tf_0\right)$ and $f_0 = 2F_{\max}$

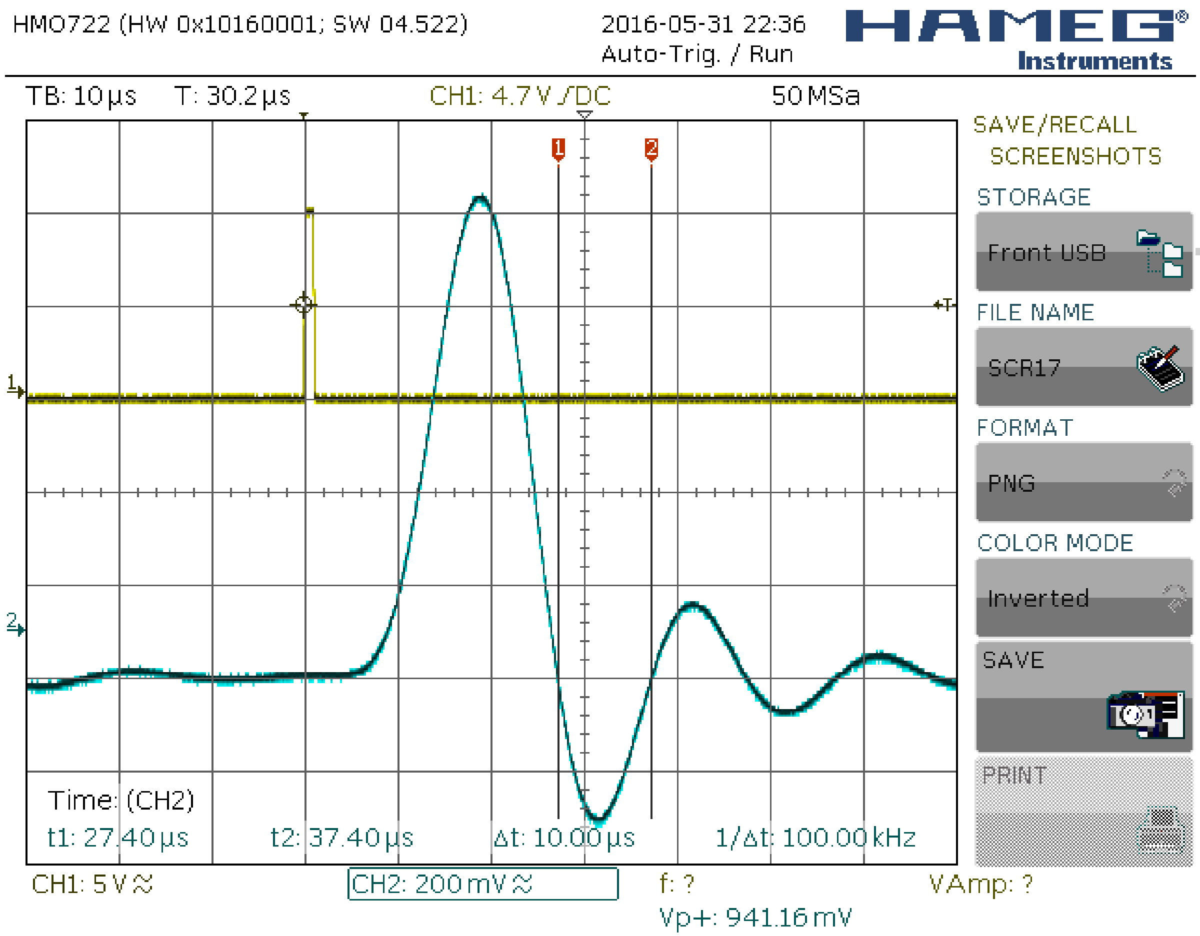
(2) $\alpha = 1$, symbol rate $f_0 = F_{\max}$

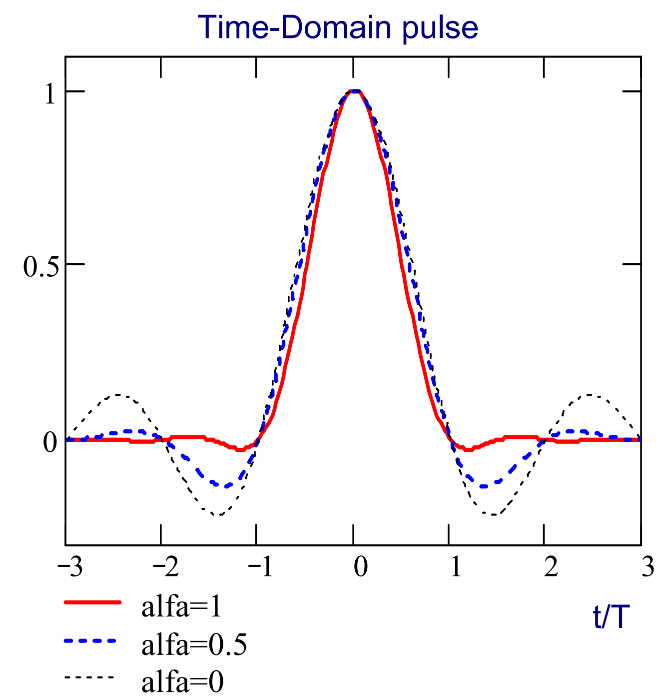
(3) tails decay as t^3 -> mistiming is not a big problem

(4) smooth shape of the spectrum -> easier to design filters

Practical sinc(t):

(as measured in Lab 1)





RCP: Example

- R_b=1Mb/s, binary PAM (BPAM) or BPSK, RCP with $\alpha = 1$

- Bandwidth = ?

- Solution: BPAM

$$R_b = R_s = f_0 = 1 \text{ Msymb./s}$$

$$\Delta f_{BPAM} = \frac{1+\alpha}{2} f_0 = 1 \text{ MHz}$$

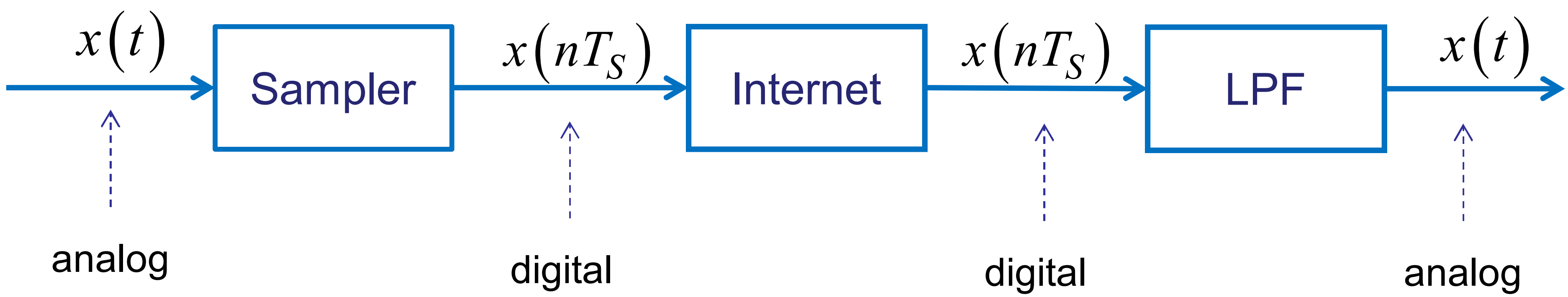
- Solution: BPSK

$$\Delta f_{BPSK} = 2\Delta f_{BPAM} = 2 \text{ MHz}$$

- 4-PAM, 4-PSK (QPSK): bandwidth = ?

Zero ISI via Sampling Theorem

- Standard view: analog – digital - analog

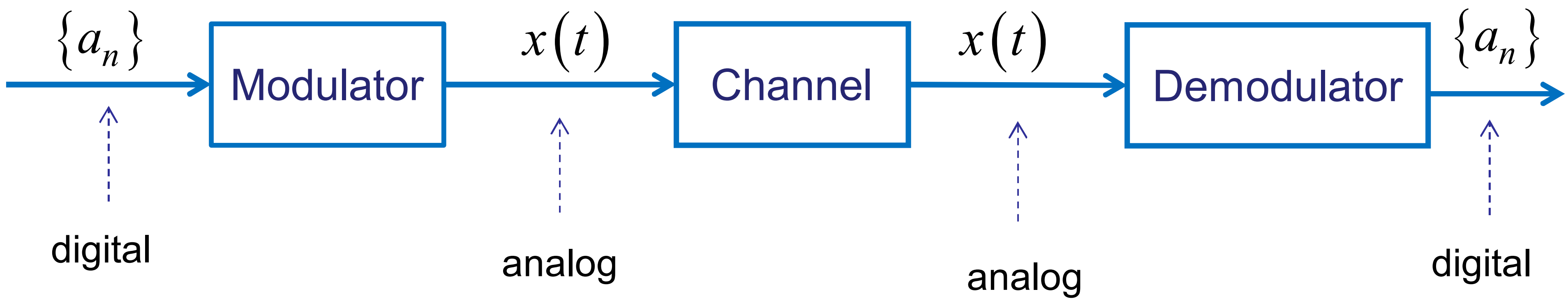


$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} x(nT_S)\,\mathrm{sinc}\left(\frac{t}{T_S} - n\right)}$$

$$\Delta f_x \leq \frac{1}{2} f_s$$

Zero ISI via Sampling Theorem

- PAM: digital - analog - digital

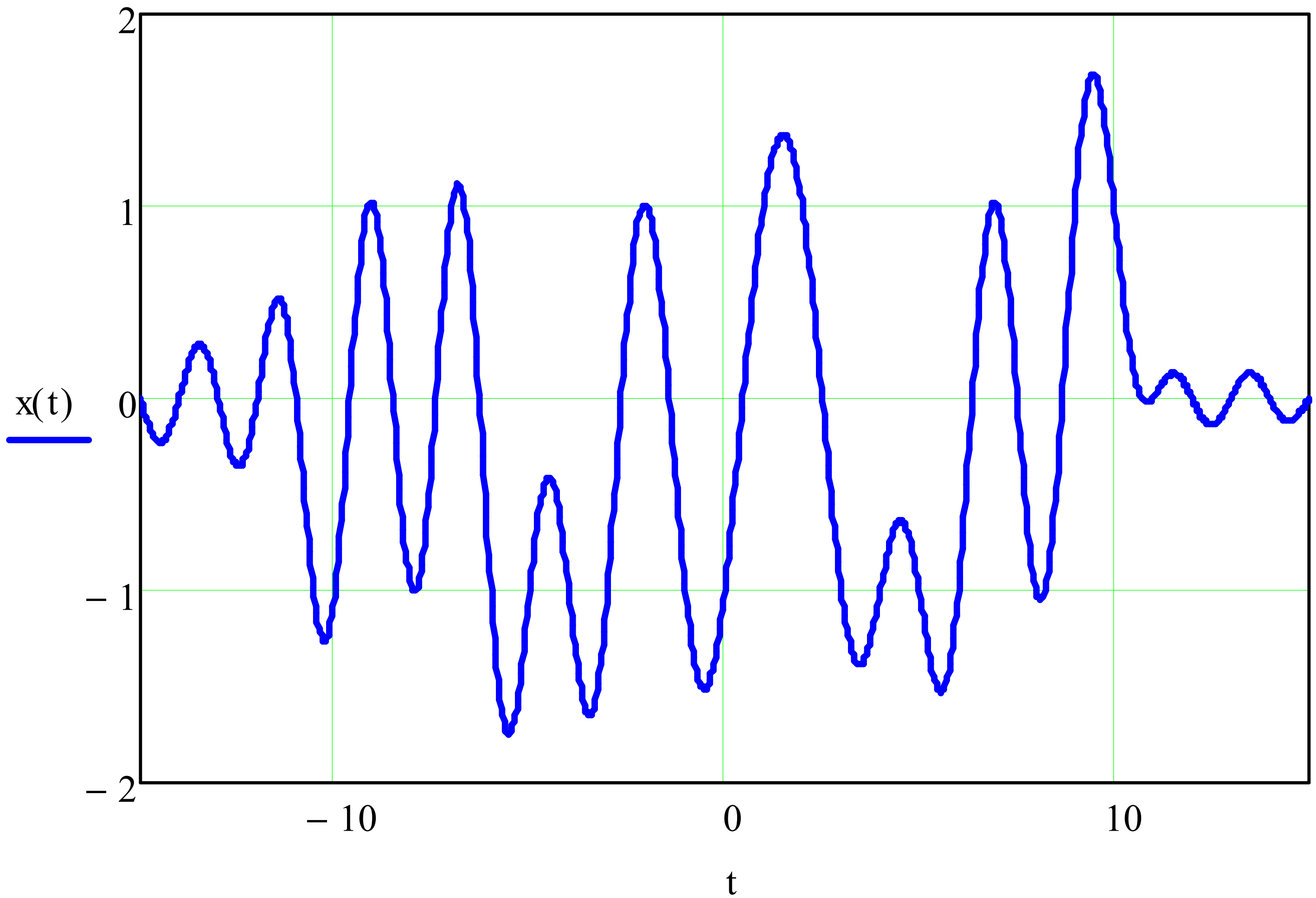


$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} a_n \mathrm{sinc}\left(\frac{t}{T_S} - n\right)}$$

$$\Delta f_x \le \frac{1}{2} R_s$$

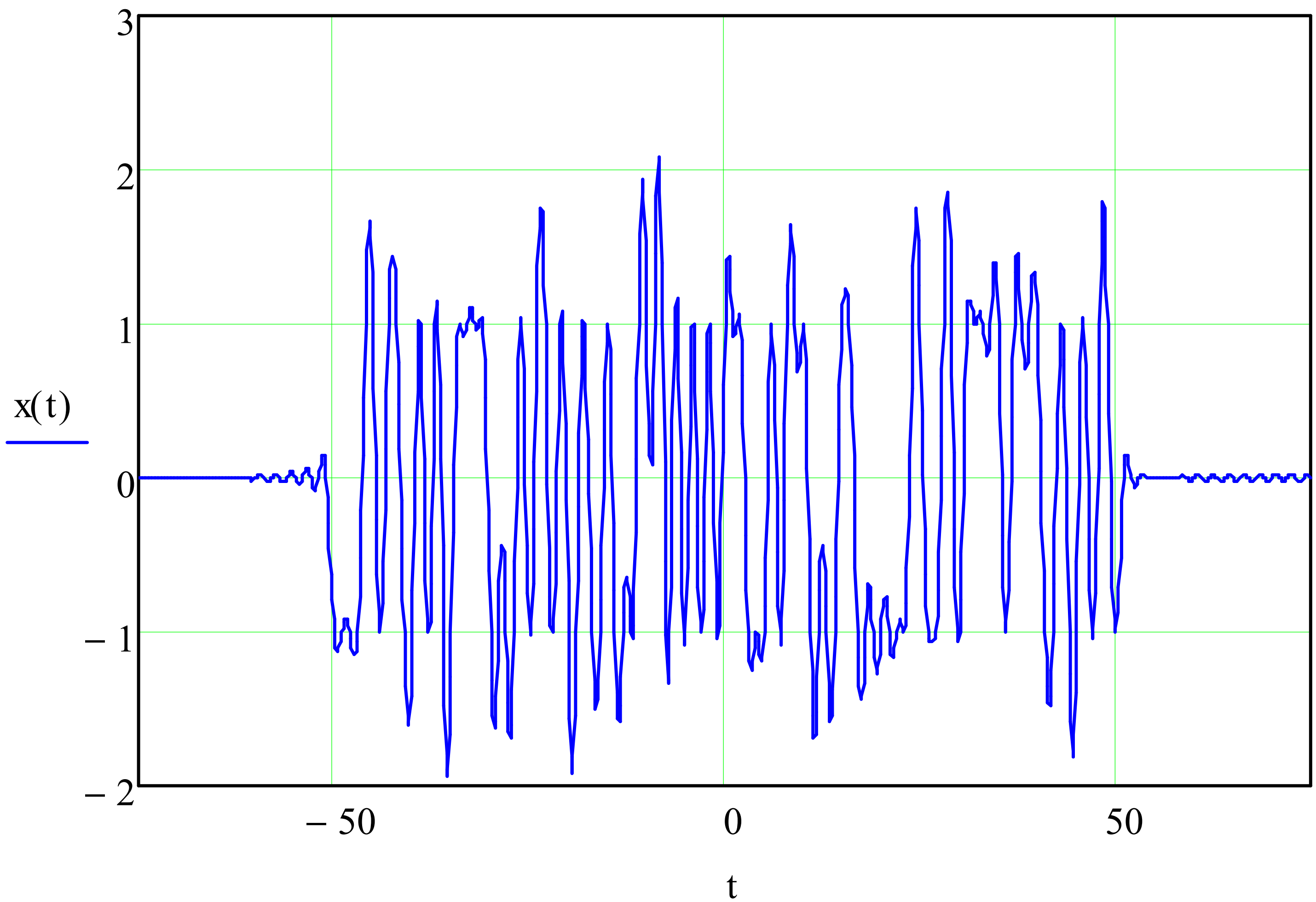
Example: random signal/process

- random data -> random signal $x(t) = \sum_{n=-10}^{10} a_n \operatorname{sinc}(t-n), \quad a_n = \pm 1$



Example: random signal/process

- random data -> random signal $x(t)=\sum_{n=-50}^{50} a_n \mathrm{sinc}(t-n), \quad a_n=\pm 1$



Summary

- Signal design for bandlimited channels.
- Eye diagram. Timing error, sensitivity to timing error, noise margin.
- Transmission over bandlimited channels and intersymbol interference.
- Nyquist criterion for zero ISI. Sinc pulse.
- Raised cosine pulse.
- **Homework**: Reading, Couch, 3.5, 3.6. Study carefully all the examples and make sure you understand and can solve them with the book closed. Do some end-of-chapter problems. Students' solution manual provides solutions for many of them.