Assignment #3

Due: Feb. 5, 1pm, FTX 137  (the tutorial). Hard copy only, no email submissions.  Late entries will not be accepted!

1) Can the response of an LTI system to \( x(t) = \text{sinc}(t) \) be \( y(t) = \text{sinc}^2(t) \)? Justify your answer.

2) For a lowpass signal with a bandwidth of 6000 Hz, what is the minimum sampling frequency for perfect reconstruction of the signal? What is the minimum required sampling frequency if a guard band of 2000 Hz is required? What is the minimum required sampling frequency and the value of \( K \) for perfect reconstruction if the reconstruction filter has the following frequency response \( H(f) \):

\[
H(f) = \begin{cases}
K, & |f| < 7000 \\
K - K \frac{|f| - 7000}{3000}, & 7000 < |f| < 10000 \\
0, & \text{otherwise}
\end{cases}
\]

3) Let the signal \( x(t) = 10 \text{sinc}(10000t) \) be sampled with a sampling frequency of 3000 samples per second. Determine the most general class of reconstruction filters for perfect reconstruction of this signal.

4) A lowpass signal \( x(t) \) with bandwidth of 50Hz is sampled at the Nyquist rate and the resulting sampled values are \( x(nT_s) \). (a) Find \( x(0.007) \). (b) Is this signal power-type or energy-type? Find its power or energy content.

5) Problem 2.82 in the course text (8th edition).

6) Let \( x(t) \) be a signal with Nyquist rate \( \omega_0 \). Find the Nyquist rate for each of the following signals:

   a) \( \frac{d^2 x(t)}{dt^2} \), b) \( x(t) \cdot x^2(t-5) \), c) \( x(t) * x^2(t-5) \), where * denotes convolution, d) \( \int_{-\infty}^{t} x(\tau)d\tau \)

Optional (will not be marked)

7) Let the response of an LTI system to \( \Pi(t) \) be \( \Lambda(t) \). (a) Can you find the response of this system to \( x(t) = \cos 2\pi t \) from the information provided? (b) Show that \( h_1(t) = \Pi(t) \) and \( h_2(t) = \Pi(t) + \cos 2\pi t \) can both be impulse response of this system, and therefore having the response of a system to \( \Pi(t) \) does not uniquely determine the system. (c) Does the response of an LTI system to \( u(t) \) uniquely determine the system? How about the response to \( e^{-\alpha t} u(t) \) for some \( \alpha > 0 \)? In general, what conditions must the input \( x(t) \) satisfy so that the system can be uniquely determined by knowing its corresponding output?

8) Problem 7.39 in Signals and Systems by Oppenheim and Willsky (2nd ed.).

9) The lowpass signal \( x(t) \) with a bandwidth of \( W \) is sampled at the Nyquist rate and the signal \( x_1(t) = \sum_{n=-\infty}^{\infty} (-1)^n x(nT_s) \delta(t - nT_s) \) is generated. (a) Find the Fourier transform of \( x_1(t) \). (b) Can \( x(t) \) be reconstructed from \( x_1(t) \) by using an LTI system? Why? (c) Can \( x(t) \) be reconstructed from \( x_1(t) \) by using a linear time-varying system? How?

* Please give detailed solutions, not just final answers. Do not skip important steps.

Plagiarism (i.e. “cut-and-paste” from a student to a student, other forms of “borrowing” the material for the assignment) is absolutely unacceptable and will be penalized. Each student is expected to submit his own solutions. If two (or more) identical or almost identical sets of solutions are found, each student involved receives 0 (zero) for that particular assignment. If this happens twice, the students involved receive 0 (zero) for the entire assignment component of the course in the marking scheme and the case will be sent to the Dean’s office for further investigation.