Assignment #2

Due: Jan. 29, 1pm, FTX 137 (the tutorial). Hard copy only, no email submissions. Late entries will not be accepted!

1) Find the Fourier transform of the Dirac delta function \( \delta(t) \) by considering it as the following limit of functions: (a) \( \delta(t) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \Pi(t/\varepsilon) \); (b) \( \delta(t) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \text{sinc}(t/\varepsilon) \). Assume that in these cases the Fourier transform of a limit of functions is the limit of the Fourier transform of the functions.

2) Determine the Fourier transform of the following signals:

3) Prove the convolution property of the Fourier transform.

4) Use the convolution theorem to show that \( \text{sinc}(t) \ast \text{sinc}(t) = \text{sinc}(t) \). Can you prove this directly?

5) Let \( x(t) \) be an arbitrary signal and define \( x_a(t) = \sum_{n=-\infty}^{\infty} x(t - n T_s) \). A) Show that \( x_a(t) \) is a periodic signal. B) How can you write \( x_a(t) \) in terms of \( x(t) \) and \( x_a(t) = \sum_{n=-\infty}^{\infty} \delta(t - n T_s) \)? C) Find the Fourier transform of \( x_a(t) \) in terms of the Fourier transform of \( x(t) \).

6) Classify the following signals into energy-type, power-type and neither energy-type nor power-type signals. For energy-type and power-type signals, find the energy or the power content of the signal:

   1) \( x_1(t) = e^{-\alpha t} \cos(\beta t) \cdot u(t) \); 2) \( x_2(t) = e^{-2t} \cos(t) \); 3) \( x_3(t) = \text{sinc}(t) \); 4) \( x_4(t) = A \cos(2\pi f_1 t + B \sin(2\pi f_2 t) \).

Optional (will not be marked)

7) By computing the Fourier series coefficient for the periodic signal \( \sum_{n=-\infty}^{\infty} \delta(t - n T_s) \), show that

\[ \sum_{n=-\infty}^{\infty} \delta(t - n T_s) = T_s^{-1} \sum_{n=-\infty}^{\infty} \exp\left(jn2\pi T_s^{-1}\right) \]. Using this result, prove that for any signal \( x(t) \) and any \( T_s \), the following identity holds

\[ \sum_{n=-\infty}^{\infty} x(t - n T_s) = T_s^{-1} \sum_{n=-\infty}^{\infty} X(n / T_s) \exp\left(jn2\pi T_s / T_s\right) \]. From this, conclude the following relation known as Poisson’s sum formula

\[ \sum_{n=-\infty}^{\infty} x(n T_s) = T_s^{-1} \sum_{n=-\infty}^{\infty} X(n / T_s) \].

8) Determine whether these signals are energy-type or power-type. In each case, find the energy or the power-spectral density and also the energy or power content of the signal:

   1) \( x(t) = e^{-\alpha t} \cos(\beta t) u(t) ; \alpha, \beta > 0 \); 2) \( x(t) = \text{sinc}(t) \); 3) \( x(t) = \sum_{n=-\infty}^{\infty} \lambda(t - 2n) \); 4) \( x(t) = u(t) \); 5) \( x(t) = 1/t \).

*All sketching of functions is to be done by hand. No graphing calculators or computers may be used!
* Please give detailed solutions, not just final answers. Do not skip important steps

*Plagiarism (i.e. “cut-and-paste” from a student to a student, other forms of “borrowing” the material for the assignment) is absolutely unacceptable and will be penalized. Each student is expected to submit his own solutions. If two (or more) identical or almost identical sets of solutions are found, each student involved receives 0 (zero) for that particular assignment. If this happens twice, the students involved receive 0 (zero) for the entire assignment component of the course in the marking scheme and the case will be send to the Dean’s office for further investigation.