Assignment #1

Due: Jan. 22, 1pm, FTX 137 (the tutorial). Late entries will not be accepted! No email submissions.

1) Sketch on the same graph the following functions (see Note 1!) and indicate their values at the principal points: \[ \sin x, \quad \sin \left( x + \frac{\pi}{4} \right), \quad \sin^{1023} x \]

2) Find graphically the convolution of the functions \( x(t) \) and \( h(t) \) and sketch the result:

3) Noting that \( \text{Re}[z_1 z_2] = \text{Re}[z_1] \text{Re}[z_2] - \text{Im}[z_1] \text{Im}[z_2] \), how can \( \cos(\alpha + \beta) = \text{Re}\left[e^{j(\alpha+\beta)}\right] \) be expressed? Give the final answer in a simple form, without complex-valued functions.

4) A system input-output relation is expressed by the following operator: \( y(t) = \mathcal{L}[x(t)] = x^*(t) \), where * means complex conjugate. Is this system: (a) linear, (b) time-invariant, (c) casual, (d) stable?

5) Find the Fourier series expansion for the following signals below. First sketch each signal. Definition of \( \Lambda(t) \) is given in the course textbook.

   a) \( x(t) = \sum_{n=-\infty}^{\infty} \Lambda(t-n) \); b) \( x(t) = \cos t + \cos 2.5t \); c) \( x(t) = \sum_{n=-\infty}^{\infty} (-1)^n \delta(t-nT) \); d) \( x(t) = |\cos 2\pi f_0 t| \)

6) Let \( x_n \) and \( y_n \) represent the Fourier Series coefficients of \( x(t) \) and \( y(t) \), respectively. Assuming the period of \( x(t) \) is \( T_0 \), express \( y_n \) in terms of \( x_n \) in each of the following cases:

   a) \( y(t) = x(t-t_0) \); b) \( y(t) = x(t)e^{j2\pi \phi t} \); c) \( y(t) = x(at), a \neq 0 \); d) \( y(t) = \frac{d}{dt} x(t) \).

7) Show that for all periodic physical signals that have finite power, the coefficients of the Fourier series expansion \( c_n \) tend to zero as \( n \to \infty \).

Optional (will not be marked)

8) Using the Parseval theorem, prove the following identity:

\[
\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}
\]

9) Using the result of Problem 8, prove the following identity:

\[
1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots + \frac{1}{(2n+1)^2} + \cdots = \frac{\pi^4}{96}
\]

10) Find the period of the following signal: \( x(t) = \sin(t) + \sin(\pi t) \)

* All sketching of functions is to be done by hand. No graphing calculators or computers may be used!
* Please give detailed solutions, not just final answers. Do not skip important steps

Plagiarism (i.e. “cut-and-paste” from a student to a student, other forms of “borrowing” the material for the assignment) is absolutely unacceptable and will be penalized. Each student is expected to submit his own solutions. If two (or more) identical or almost identical sets of solutions are found, each student involved receives 0 (zero) for that particular assignment. If this happens twice, the students involved receive 0 (zero) for the entire assignment component of the course in the marking scheme and the case will be send to the Dean’s office for further investigation.