Convex Optimization Tools for Non-Convex Problems

- Apply locally if the problem is locally convex and if an optimum exists, i.e. give local solutions [1][2].
- Inspection of all local solutions reveals a global one.

4-step Method of Brinkhuis and Tikhomirov [2]:
1. Establish an existence of a global solution, e.g. if the objective is a continuous function and the constraint set is closed and bounded (thus compact), the existence of a solution follows from Weierstrass theorem.
2. Find necessary conditions: KKT conditions are necessary for optimality in many cases, see e.g. [3][1] for details, so that a global minimum is a solution of KKT conditions.
3. Find all solutions of KKT conditions.
4. By inspection, find the global minimum.

KKT conditions are necessary if all functions are continuously differentiable and one of the following holds:

1. Linear equalities, convex inequalities + Slater (see Proposition 3.3.9 in [3]).
2. Linear equalities, concave inequalities (Proposition 3.3.7 in [3]; no Slater is required here).
3. Gradients of all equality and active inequality constraints are linearly independent at a local minimum (no linearity, no convexity/concavity is required here, see Proposition 3.3.1 in [3]).