A Brief Review of Communications

- Communication system: transmit information from point A to point B
  - analog/digital
  - wireline/wireless
  - single user/multiuser
- Communication network: multiple As and Bs (or multi-user)
- Extensive applications
  - Internet
  - WiFi
  - cell phones
  - TV/radio broadcast
  - GPS
- Active R&D: 5G
Block Diagram of a Communication System

Source \rightarrow Transmitter (Tx) \rightarrow Channel

Destination \rightarrow Receiver (Rx)

**Figure:** A high-level view of a communication system
A Digital Communication System

Figure: A high-level view of a digital communication system
A Wireless Communication System\textsuperscript{12}

Figure: Block diagram of a wireless communication system


\textsuperscript{2}D. Tse, P. Viswanath, Fundamentals of Wireless Communications, Cambridge University Press, 2005
A Wireless Communication System

Major challenges, due to the wireless propagation channel

- out of designer’s control
- low SNR (large path loss, 100s dB)
- multipath propagation → fading
  - frequency selectivity (delay spread)
  - time selectivity/variability (Doppler spread)
  - inaccurate/unavailable channel state information
- interference
- limited/expensive bandwidth

How to combat?
A Wireless Communication System

How to combat major challenges?

• frequency/time domain processing: at their limits
  • modulation
  • coding
  • filtering

• space-domain processing: ”last frontier”

• via multiple (”smart”) antennas

• current active R&D: 5G
Modern Wireless Communication Systems

- Key objectives of 5G
  - 1000× rate
  - wide availability
  - low latency
  - multiple services
- How? Key technologies:
  - massive MIMO
  - mmWaves
  - NOMA
  - HetNet

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Digital Communications$^{567}$

- modulation
  - BPSK
  - QPSK
  - QAM
- signalling
  - sinc
  - raised-cosine
  - etc.
- optimum receiver: ML
  - matched filter
  - sampler
  - decision device

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Digital Communications: Key Performance Metrics

- transmission rate, [bit/s]
- error rate/probability, BER/SER
- fading: outage probability
Digital Communications: fundamental limits

- from information theory\(^8\)
- single user: channel capacity: [bit/s] or [bit/ch. use]
- fading: outage capacity
- benchmark for actual system performance
- optimal system design (Tx, Rx)
- much less is known about networks

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Digital Communications: channel model

- AWGN channel (discrete-time)

\[ y_k = x_k + \xi_k \]  (1)

- \( x_k \) = Tx signal
- \( y_k \) = Rx signal
- \( \xi_k \) = noise (i.i.d. Gaussian)
Fundamental Limit: Channel Capacity

• largest transmission rate s.t. power & reliability constraints

\[ R < C = \Delta f \log(1 + \gamma) \text{ [bit/s]} \] (2)

• \( \Delta f \) = channel bandwidth
• \( \gamma = \frac{P_x}{P_\xi} = \text{SNR} \)
• power constraint: \( \sigma_x^2 \leq P_x \)
• reliability constraint: arbitrary-low error probability

• equivalently, spectral efficiency:

\[ C = \log(1 + \gamma) \text{ [bit/s/Hz]} \] (3)
Fundamental Limit: Channel Capacity

Claude Shannon, Farther of Information Theory:

Figure: The constellation capacity of M-PAM

Fundamental Limit: Channel Capacity

- $R = C$ is not possible, but $R$ can be close to $C$
- In practice,

$$R = \log(1 + \frac{\gamma}{\Gamma}) \text{ [bit/s/Hz]} \quad (4)$$

where $\Gamma > 1$ is the SNR gap to capacity
- $\Gamma \rightarrow 1$ for good (capacity-approaching) system
- Depends on modulation and coding\textsuperscript{10,11}

SNR-Gap-to-Capacity in Practice

BER vs. SNR/bit

- **BPSK**
- **R=1 b/s**
- **R=4 b/s**
- **16 QAM**
- **gap**
Progress Towards the Capacity

Progress toward the Shannon limit

The original turbo codes: about 0.7 dB from capacity


Irregular LDPC codes: about 0.1 dB from capacity


How about 0.01 dB from capacity? And 0.001 dB?


Conclusion: For all practical purposes, Shannon’s puzzle has been now solved and Shannon’s promise has been achieved!

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A.Vardy, What’s New and Exciting in Algebraic and Combinatorial Coding Theory? Plenary Talk at ISIT-06.
Channel Capacity: two fundamental resources

From the capacity expression,

\[ C = \Delta f \log(1 + \gamma) \text{ [bit/s]} \]  \hspace{1cm} (5)

\( C \) can be increased by increasing

1. bandwidth \( \Delta f \) (expensive in wireless)
2. power \( P_x \), via the SNR \( \gamma = P_x / P_\xi \)
3. anything else?
Spectral/Power Efficiency: Fundamental Tradeoff

- power efficiency: \( \gamma_b = \frac{\text{SNR(energy)}}{\text{bit}} \)
- spectral efficiency: \( \frac{R}{\Delta f} \) [bit/s/Hz]
- the tradeoff:

\[
\gamma_b \geq \frac{2^{R/\Delta f} - 1}{R/\Delta f} \geq \ln 2 = -1.6 \text{ dB } \tag{6}
\]
Spectral/Power Efficiency: Fundamental Tradeoff

![Graph showing spectral efficiency vs. SNR](image)

- Achievable condition: 
  \[ \gamma_b = \frac{2^{R/\Delta f} - 1}{R / \Delta f} \geq -1.6 \text{dB} \]
Fundamental Tradeoff in Practice

Figure 9.6  Bandwidth-efficiency plane.

Legend
- Coherent MPSK, $P_B = 10^{-5}$
- Noncoherent orthogonal MFSK, $P_B = 10^{-5}$
- Coherent QAM, $P_B = 10^{-5}$
## Practical Example: Spectral Efficiency of LTE/4G

<table>
<thead>
<tr>
<th>CQI index</th>
<th>Modulation</th>
<th>Coding rate</th>
<th>Spectral efficiency (bps/Hz)</th>
<th>SINR estimate (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>QPSK</td>
<td>0.0762</td>
<td>0.1523</td>
<td>-6.7</td>
</tr>
<tr>
<td>2</td>
<td>QPSK</td>
<td>0.1172</td>
<td>0.2344</td>
<td>-4.7</td>
</tr>
<tr>
<td>3</td>
<td>QPSK</td>
<td>0.1885</td>
<td>0.3770</td>
<td>-2.3</td>
</tr>
<tr>
<td>4</td>
<td>QPSK</td>
<td>0.3008</td>
<td>0.6016</td>
<td>0.2</td>
</tr>
<tr>
<td>5</td>
<td>QPSK</td>
<td>0.4385</td>
<td>0.8770</td>
<td>2.4</td>
</tr>
<tr>
<td>6</td>
<td>QPSK</td>
<td>0.5879</td>
<td>1.1758</td>
<td>4.3</td>
</tr>
<tr>
<td>7</td>
<td>16QAM</td>
<td>0.3691</td>
<td>1.4766</td>
<td>5.9</td>
</tr>
<tr>
<td>8</td>
<td>16QAM</td>
<td>0.4785</td>
<td>1.9141</td>
<td>8.1</td>
</tr>
<tr>
<td>9</td>
<td>16QAM</td>
<td>0.6016</td>
<td>2.4063</td>
<td>10.3</td>
</tr>
<tr>
<td>10</td>
<td>64QAM</td>
<td>0.4551</td>
<td>2.7305</td>
<td>11.7</td>
</tr>
<tr>
<td>11</td>
<td>64QAM</td>
<td>0.5537</td>
<td>3.3223</td>
<td>14.1</td>
</tr>
<tr>
<td>12</td>
<td>64QAM</td>
<td>0.6504</td>
<td>3.9023</td>
<td>16.3</td>
</tr>
<tr>
<td>13</td>
<td>64QAM</td>
<td>0.7539</td>
<td>4.5234</td>
<td>18.7</td>
</tr>
<tr>
<td>14</td>
<td>64QAM</td>
<td>0.8525</td>
<td>5.1152</td>
<td>21.0</td>
</tr>
<tr>
<td>15</td>
<td>64QAM</td>
<td>0.9258</td>
<td>5.5547</td>
<td>22.7</td>
</tr>
</tbody>
</table>

**Figure:** Spectral efficiency of LTE/4G cell phones
Practical Example: IEEE 802.11n (WiFi)

<table>
<thead>
<tr>
<th>MCS Index</th>
<th>Type</th>
<th>Coding Rate</th>
<th>Spatial Streams</th>
<th>Data Rate (Mbps) with 20 MHz CH</th>
<th>Data Rate (Mbps) with 40 MHz CH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>800 ns</td>
<td>400 ns (SGI)</td>
</tr>
<tr>
<td>0</td>
<td>BPSK</td>
<td>1 / 2</td>
<td>1</td>
<td>6.50</td>
<td>7.20</td>
</tr>
<tr>
<td>1</td>
<td>QPSK</td>
<td>1 / 2</td>
<td>1</td>
<td>13.00</td>
<td>14.40</td>
</tr>
<tr>
<td>2</td>
<td>QPSK</td>
<td>3 / 4</td>
<td>1</td>
<td>19.50</td>
<td>21.70</td>
</tr>
<tr>
<td>3</td>
<td>16-QAM</td>
<td>1 / 2</td>
<td>1</td>
<td>26.00</td>
<td>28.90</td>
</tr>
<tr>
<td>4</td>
<td>16-QAM</td>
<td>3 / 4</td>
<td>1</td>
<td>39.00</td>
<td>43.30</td>
</tr>
<tr>
<td>5</td>
<td>64-QAM</td>
<td>2 / 3</td>
<td>1</td>
<td>52.00</td>
<td>57.80</td>
</tr>
<tr>
<td>6</td>
<td>64-QAM</td>
<td>3 / 4</td>
<td>1</td>
<td>58.50</td>
<td>65.00</td>
</tr>
<tr>
<td>7</td>
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<td>5 / 6</td>
<td>1</td>
<td>65.00</td>
<td>72.20</td>
</tr>
<tr>
<td>8</td>
<td>BPSK</td>
<td>1 / 2</td>
<td>2</td>
<td>13.00</td>
<td>14.40</td>
</tr>
<tr>
<td>9</td>
<td>QPSK</td>
<td>1 / 2</td>
<td>2</td>
<td>26.00</td>
<td>28.90</td>
</tr>
<tr>
<td>10</td>
<td>QPSK</td>
<td>3 / 4</td>
<td>2</td>
<td>39.00</td>
<td>43.30</td>
</tr>
<tr>
<td>11</td>
<td>16-QAM</td>
<td>1 / 2</td>
<td>2</td>
<td>52.00</td>
<td>57.80</td>
</tr>
<tr>
<td>12</td>
<td>16-QAM</td>
<td>3 / 4</td>
<td>2</td>
<td>78.00</td>
<td>86.70</td>
</tr>
<tr>
<td>13</td>
<td>64-QAM</td>
<td>2 / 3</td>
<td>2</td>
<td>104.00</td>
<td>115.60</td>
</tr>
<tr>
<td>14</td>
<td>64-QAM</td>
<td>3 / 4</td>
<td>2</td>
<td>117.00</td>
<td>130.00</td>
</tr>
<tr>
<td>15</td>
<td>64-QAM</td>
<td>5 / 6</td>
<td>2</td>
<td>130.00</td>
<td>144.40</td>
</tr>
<tr>
<td>16</td>
<td>BPSK</td>
<td>1 / 2</td>
<td>3</td>
<td>19.50</td>
<td>21.70</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>31</td>
<td>64-QAM</td>
<td>5 / 6</td>
<td>4</td>
<td>260.00</td>
<td>288.90</td>
</tr>
</tbody>
</table>

Figure: Data rates of WiFi routers
The SNR

The SNR is

\[ \text{SNR} = \gamma = \frac{P_x}{P_\xi} \] (7)

\( P_x \) = signal power at the receiver (Rx),
\( P_\xi = kTF \Delta f \) = Rx noise power,
\( k = 1.38 \cdot 10^{-23} \) [J/K] = Boltzman constant,
\( \Delta f \) = bandwidth [Hz],
\( F \) = Rx noise figure (typically a few dB)
\( T \) = Rx temperature [deg. K]
$P_x$: from the link budget

The Rx signal power $P_x$ is

$$P_x = P_t \frac{G_t G_r}{L_p}$$  \hspace{1cm} (8)

$P_t = \text{Tx signal power}$,
$G_t, G_r = \text{Tx and Rx antenna gains}$,
$L_p = \text{propagation channel path loss (large, 50...150 dB or even more)}$.

Fade margin and other losses can be added too (to make it worse (:}
SNR: impact of antennas

- via antenna gain $G$
- isotropic antenna: $G = 1$
- nearly isotropic: $G$ is close to 1
- highly-directional antenna: large $G \gg 1$
- antenna array of $N$ elements (antennas): $G = N$ in many cases

$G_t$ can be accounted for via effective isotropic radiated power (EIRP):

$$P_E = P_t \cdot G_t$$  \hspace{1cm} (9)
Impact of Antennas on the Capacity

• With the old-fashioned use of directional antennas,

\[ C = \Delta f \log(1 + G_t G_r \gamma_{iso}) \]  \hspace{1cm} (10)

\[ \gamma_{iso} = \text{Rx SNR with isotropic antennas (when } G_t = G_r = 1). \]
• but it increases with \( G_t, G_r \) only logarithmically (very slow)
• Can we do better???
Impact of Antennas: Historical Perspective

- What is the best way to use antenna arrays?
- SISO
- MISO/SIMO
- MIMO
SISO: single antennas at both ends

- single-antenna systems: $N = 1 = G_t = G_r$

\[ C = \log(1 + \gamma) \text{ [bit/s/Hz]} \]  \hspace{1cm} (11)

where $\gamma = \gamma_{iso}$

- SE is not large (unless the SNR is very large)
- fading degrades performance
- simple design
Antenna array at one end (beamforming)

\[ C = \log(1 + N\gamma) \]  \hspace{1cm} (12)

- SE is larger, but not much (only logarithmic in \( N \))
- fading can be reduced
- more complex design (\( N \) antennas + circuitry)
Multiple Antennas at Both Ends: Old Fashion

- Try old-fashioned use of antenna arrays at both ends
- \( T_x + R_x \) beamforming: \( G_t = G_r = N > 1, \)
  \[
  C = \Delta f \log(1 + N^2 \gamma)
  \]  
  (13)
- larger SE, but still only logarithmic in \( N \rightarrow \) very slow increase
- fading can be reduced
- more complex design (2 \( \cdot \) \( N \) antennas + 2 \( \cdot \) \( N \) circuitry)
Multiple Antennas at Both Ends: Old Fashion

Can we do better?
True MIMO: launch multiple bit streams!

- Multi-stream transmission (not beamforming):

\[ C = \log |I + \gamma \mathbf{HH}^+| \]  

(14)

where \( \mathbf{H} \) is the channel matrix.
True MIMO: launch multiple bit streams!

- Under favorable propagation,
  \[ C = N \log(1 + \gamma/N) \]  \hspace{1cm} (15)
  i.e. almost linear in \( N \to \) much faster increase!
- Much larger SE with large \( N! \)
- large \( N \to \) massive MIMO = key technology for 5G.
True MIMO: launch multiple bit streams!

\[
- n \cdot \log_2 \left(1 + \frac{SNR}{n}\right)
\]

MIMO

\[
\log_2 \left(1 + SNR \cdot n^2\right)
\]

conventional array

\[
\log_2 (1 + SNR)
\]

SISO

MIMO

convent.

SISO

Capacity, bit/s/Hz

Number of antennas

1 \cdot 10^3

100

10

1

1 \cdot 10^3

100

10

1

S. Loyka

Lecture 2, ELG7177: MIMO Communications
Summary

• brief review of communications
• wireless & digital communications
• key performance metrics
• fundamental limits
• impact of antennas