Modeling Fluid Systems

The prevalent use of fluid (hydraulic) circuitry in machines tool applications, aircraft control systems, and similar operations occurs because of such factors such as accuracy, flexibility, fast starting and stopping, simplicity of operation, and high horsepower-to-weight ratio. A combination of electronic and hydraulic systems is widely used because it combines the advantages of both electronics control and hydraulic power. Note that most hydraulic systems are nonlinear. Sometimes, however, it is possible to linearize nonlinear systems so as to reduce their complexity and permit solutions that are adequately precise for the purpose of design and analysis.

In fluid flow systems there are three basic building blocks which may be considered while modeling such systems as shown in Figure 1. The input is the volumetric rate of flow q and the output is the pressure difference ($p_1 - p_2$). Fluid systems fall into two categories: hydraulic, where the fluid is a liquid and is deemed to be incompressible; and pneumatic, where it is a gas which can be compressed and shows a density change.

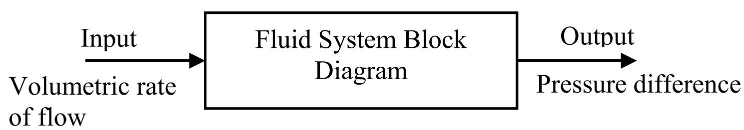

Figure 1 Fluid system.

Hydraulic Resistance

Hydraulic resistance is the resistance to flow which occurs as a result of a liquid flowing through valves or changes in a pipe diameter as shown in Figure 1. Such case may also be true when we consider the flow through a short pipe connecting two tanks. The relationship between the change in flow rate of liquid q through the resistance element and the resulting pressure difference ($p_1 - p_2$) is given as

$$R = \frac{\text{Pressure difference}}{\text{Change in flow rate, m}^3\text{/s}} = \frac{p_1 - p_2}{q} \qquad (1)$$

Where R is a constant called the **hydraulic resistance**. The higher the resistance the higher the pressure difference for a given rate of flow. Equation (1), like that for the electrical resistance and Ohm's law, assumes a linear relationship. Such hydraulic linear resistances occur with orderly flow through capillary tubes and porous plugs but nonlinear resistances occur with flow through sharp-edge orifices or if the flow is turbulent.

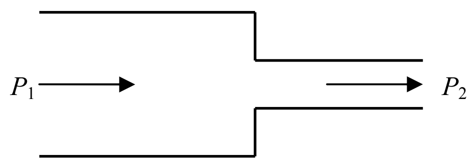

Figure 2 Hydraulic resistance.

Hydraulic Capacitance

Hydraulic capacitance is the term used to describe energy storage with a liquid where it is stored in the form of potential energy as shown in Figure 3. A height of liquid in the container shown in Figure 3 (called pressure head) is one form of such storage

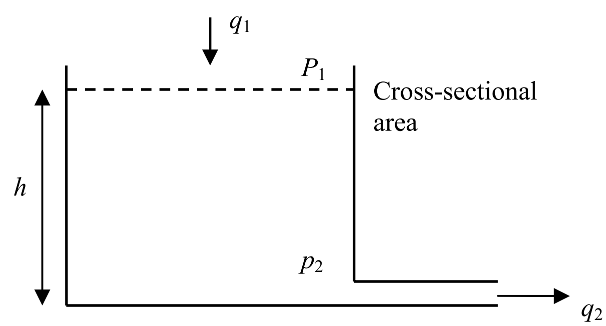

Figure 3 Hydraulic capacitance.

The capacitance C of a tank is defined as the change in quantity of stored liquid necessary to cause a unit change in the potential (head). The potential is the quantity that indicates the energy level of the system

	$C = \dfrac{\text{Change in liquid stored, m}^3}{\text{Change in head, m}}$	

It should be noted that the capacity (m^3) and the capacitance (m^2) are different. The capacitance of the tank is equal to the cross-sectional area. If this is constant, the capacitance is constant for any head.

For such a capacitance, the rate of change of volume V in the tank (dV/dt) is equal to the difference between the volumetric rate at which liquid enters the container q_1 and the rate at which it leaves the container q_2

$$q_1 - q_2 = \frac{dV}{dt} \quad (2)$$

But $V = Ah$, where A is the cross-sectional area of the container and h is the height of liquid in it. Hence

$$q_1 - q_2 = \frac{d(Ah)}{dt} = A\frac{dh}{dt} \quad (3)$$

The pressure difference between the input and output is p, where $p = h\rho g$ with ρ being the liquid density and g the acceleration due to gravity. Therefore, if the liquid is assumed to be incompressible (density does not change with pressure), then Equation (3) can be written as

$$q_1 - q_2 = A\frac{d(p/\rho g)}{dt} = \frac{A}{\rho g}\frac{dp}{dt} \quad (4)$$

The hydraulic capacitance C is defined as

$$C = \frac{A}{\rho g} \quad (5)$$

Therefore

$$q_1 - q_2 = C\frac{dp}{dt} \quad (6)$$

Integration of Equation (6) gives

$$p = \frac{1}{C}\int (q_1 - q_2)\,dt \quad (7)$$

Hydraulic Inertance

Hydraulic inertance is the equivalent of inductance in electrical systems or a spring in mechanical systems. To accelerate a fluid and to increase its velocity a force is required. Consider a block of liquid of mass *m* as shown in Figure 4. The net force acting on the liquid is

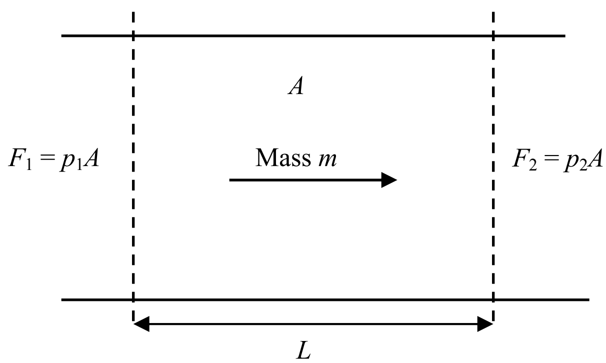

Figure 4 Hydraulic inertance.

$$F_1 - F_2 = p_1A - p_2A = (p_1 - p_2)A \qquad (8)$$

Where $(p_1 - p_2)$ is the pressure difference and A the cross-sectional area. This net force causes the mass to accelerate with an acceleration a, and so

$$(p_1 - p_2) = ma \qquad (9)$$

However, a is the rate of change of velocity dv/dt, therefore

$$(p_1 - p_2)A = m\frac{dv}{dt} \qquad (10)$$

The mass of liquid concerned has a volume of AL, where L is the length of the block or the distance between the points in the liquid where the pressures p_1 and p_2 are measured. If the liquid has a density ρ then $m = AL\rho$, and

$$(p_1 - p_2)A = AL\rho\frac{dv}{dt} \qquad (11)$$

But the volume rate of flow $q = Av$, hence

	$(p_1 - p_2)A = L\rho \frac{dq}{dt}$ $p_1 - p_2 = I\frac{dq}{dt}$	(12)

Where the hydraulic inertance I is defined as

	$I = \frac{L\rho}{A}$	(13)

Table 1 shows the basic characteristics of the hydraulic system building blocks.

Table 1 Hydraulic building blocks

Building block	Describing equation	Energy
Resistance	$q = \frac{p_1 - p_2}{R}$	$P = \frac{1}{R}(p_1 - p_2)^2$
Capacitance	$q = C\frac{d(p_1 - p_2)}{dt}$	$E = \frac{1}{2}C(p_1 - p_2)^2$
Inertance	$q = \frac{I}{L}\int (p_1 - p_2)dt$	$E = \frac{1}{2}Iq^2$

Modeling Simple Hydraulic System

Figure 5 shows a simple hydraulic system as the liquid enters and leaves a container. Such system can be considered to consist of a capacitor, the liquid in the container, with a resistor, the valve. Inertance may be neglected since flow rates change only very slow. For the capacitor we can write

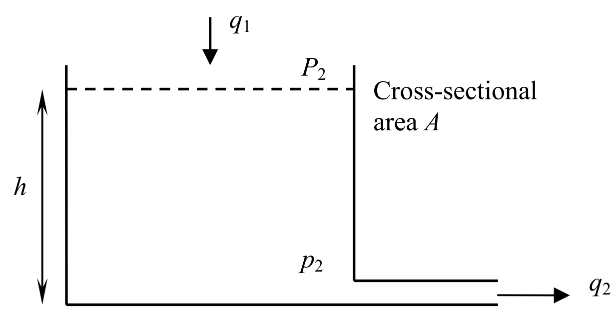

Figure 5 A hydraulic system.

$$q_1 - q_2 = C\frac{dp}{dt} \tag{14}$$

Integration of Equation (14) gives

$$p = \frac{1}{C}\int (q_1 - q_2)dt \tag{15}$$

If we define the resistance for liquid flow in a restriction, such as the open pipe at the tank exit or an orifice in the exit line, as the change in pressure difference in the tank necessary to cause a change in flow rate, then the rate at which liquid leaves the container q_2 equals the rate at which it leaves the valve. Therefore for the resistance R we have

$$R = \frac{p_1 - p_2}{q_2} \tag{16}$$

The pressure difference $(p_1 - p_2)$ is the pressure due to the height of liquid in the container and is thus $h\rho g$ with ρ being the liquid density and g the acceleration due to gravity. Therefore

$$q_2 = \frac{h\rho g}{R} \tag{17}$$

Similarly, we may define the capacitance, C, of a tank to be the change in quantity of stored liquid necessary to cause a unit change in potential, and so substituting for q_2 in Equation (17) gives

	$q_1 - \frac{h\rho g}{R} = C\frac{d(h\rho g)}{dt}$		(18)

And, since $C = A/\rho g$

	$q_1 = A\frac{dh}{dt} + \frac{\rho g h}{R}$		(19)

Equation (19) describes the relationship between the height of liquid in the container and the rate of input of liquid into the container.