Switched Mode Power Supply (SMPS)
Control, Modeling, and Simulation

Why Use a Switching Regulator?
SMPS

• Various types of voltage regulators, used in Linear Power Supplies (LPS), are dissipative regulator, as they have a voltage control element usually transistor or zener diode which dissipates power equal to the voltage difference between an unregulated input voltage and a fixed supply voltage multiplied by the current flowing through it.

• The switching regulator acts as a continuously variable power converter and hence its efficiency is negligibly. Hence the switching regulator is ‘non-dissipative regulator’.

• In a SMPS, the active device that provides regulation is always operated in cut-off or in saturation mode.
Buck Switching Regulator

Maximum Duty Cycle: \[ D = \frac{V_{OUT}}{V_{IN(\text{max})} \times \eta} \]

Inductor: \[ L = \frac{V_{OUT} \times (V_{IN} - V_{OUT})}{\Delta I_L \times f_S \times V_{IN}} \]

Output Capacitor: \[ C_{OUT(\text{min})} = \frac{\Delta I_L}{8 \times f_S \times \Delta V_{OUT}} \]
Control Design Using Pole Placement

[Diagram of control system with blocks labeled B, 1/s, C, A, K, L, r, u, x, y, and arrows indicating the flow of signals.]
Voltage Mode Control of Buck Converter

\[ L \quad 50 \, \mu\text{H} \]
\[ C \quad 500 \, \mu\text{F} \]
\[ R \quad 3 \, \Omega \]
\[ f_s = 100 \, \text{kHz} \]

\[ v_{g(t)} \quad 28 \, \text{V} \]

Transistor gate driver

Pulse-width modulator
\[ V_M = 4 \, \text{V} \]

Compensator

Sensor gain

Error signal

\[ v_e \]

\[ v_{\text{ref}} \quad 5 \, \text{V} \]
Current Mode Control of Buck Converter
Boost SMPS

\[
D = 1 - \frac{V_{IN(min)} \times \eta}{V_{OUT}}
\]

\[
L = \frac{V_{IN} \times (V_{OUT} - V_{IN})}{\Delta I_L \times f_s \times V_{OUT}}
\]

\[
C_{OUT(min)} = \frac{I_{OUT(max)} \times D}{f_s \times \Delta V_{OUT}}
\]
Feedback Control of Boost Converter
Buck Modes of Operation

\[ d_1 T_S = \text{ON Period time} \]
\[ d_2 T_S = \text{OFF Period time} \]
\[ T_S = \text{Total time period for one cycle} \]
\[ i_{pk} = \text{peak value of inductor current after ON period} \]
\[ \bar{i}_L = \text{Average value of current} \]
\[ V_{in} = \text{input voltage} \]
State Space Modeling

• If the system is linear, then the derivatives of the state variables are expressed as linear combinations of the system independent inputs and state variables themselves.

• The physical state variables of a system are usually associated with the storage of energy.

• For a typical converter circuit, the physical state variables are the inductor currents and capacitor voltages.
Buck Converter During ON Mode
Write the State Space Model

From KVL\[ v_{in} - L \frac{di_L}{dt} - v_C = 0 \]

From KCL\[ \frac{v_C}{R} + C \frac{dv_C}{dt} i_L = 0 \]

Write\[ \begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -L & -C \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} v_{in} \end{bmatrix} \]

\[ v_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} \]
Buck Converter During Off Mode
Write State Space Model

From KVL \[ v_C + L \frac{di_L}{dt} = 0 \]

From KCL \[ i_L - \frac{v_C}{R} - C \frac{dv_C}{dt} = 0 \]

\[
\begin{bmatrix}
\frac{di_L}{dt} \\
\frac{dv_C}{dt}
\end{bmatrix}
= 
\begin{bmatrix}
i_L \\
v_C
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0
\end{bmatrix} \cdot v_{in} \\
V_o = \begin{bmatrix}
0 \\
1
\end{bmatrix} \cdot \begin{bmatrix}
i_L \\
v_C
\end{bmatrix}
\]
During Discontinuous Conduction Mode

From KVL\[\frac{di_L}{dt} = 0\]

From KCL\[\frac{v_c}{R} + C \frac{dv_c}{dt} = 0\]

Write\[\begin{bmatrix}
\frac{di_L}{dt} \\
\frac{dv_c}{dt}
\end{bmatrix} = \begin{bmatrix}
i_L \\
v_c
\end{bmatrix} + \begin{bmatrix}0 \ v_{in}\end{bmatrix} ; v_o = \begin{bmatrix}0 & 0\end{bmatrix} \begin{bmatrix}i_L \\
v_c
\end{bmatrix}\]
Buck Modelling Analysis

• Averaging

• Inductor current analysis

• Duty-ratio constraint.

State space averaging techniques are employed to get a set of equations that describe the system over one switching period.

\[ \dot{X} = [A_1 d_1 + A_2 d_2 + A_3 (1 - d_1 - d_2)]\bar{X} + [B_1 d_1 + B_2 d_2 + B_3 (1 - d_1 - d_2)]u \]

\[ \bar{i}_L = \frac{i_{pk}}{2} \cdot (d_1 + d_2) \]
The Final Model

The state space averaged model for the above equation is

\[
\frac{d}{dt} \begin{bmatrix} \bar{l}_L \\ \bar{v}_C \end{bmatrix} = \begin{bmatrix} \frac{d}{dt} \begin{bmatrix} \bar{l}_L \\ \bar{v}_C \end{bmatrix} = \begin{bmatrix} K \begin{bmatrix} \bar{l}_L \\ \bar{v}_C \end{bmatrix} + \begin{bmatrix} d_1 \\ L \end{bmatrix} v_{in} \\ \frac{d}{dt} \begin{bmatrix} \bar{l}_L \\ \bar{v}_C \end{bmatrix} + \begin{bmatrix} d_1 \\ L \end{bmatrix} v_{in} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} d_1 \\ L \end{bmatrix} v_{in}
\]