# ELG4157/SYS5100
# Control Systems Engineering

## Chapter 2
Modeling in the Frequency Domain

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A System!

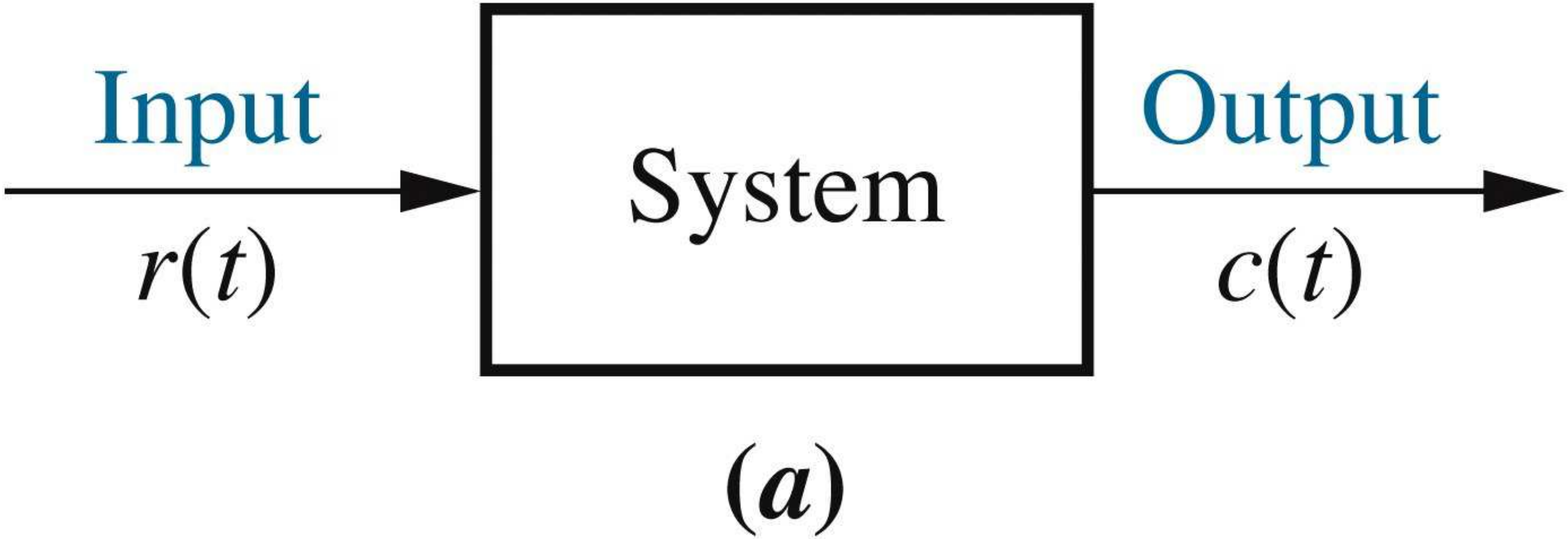


Figure 2.1a

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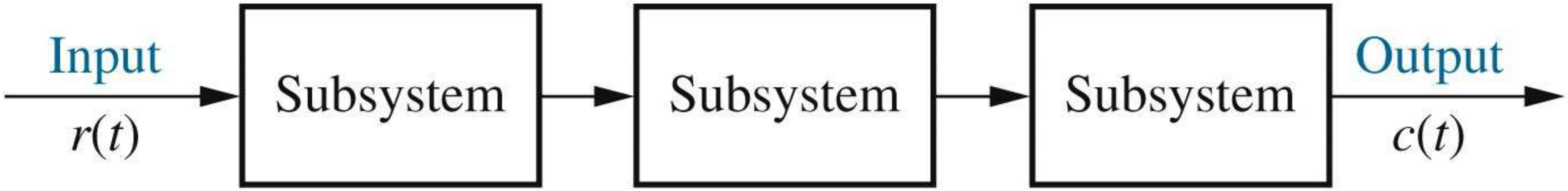
# Interconnection of Subsystems




***(b)***

Note: The input, $r(t)$, stands for *reference input*.
The output, $c(t)$, stands for *controlled variable*.

**Figure 2.1b**

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# Laplace Transform

- Laplace Transforms: method for solving differential equations, converts differential equations in time $t$ into algebraic equations in complex variable $s$.
- Transfer Functions: another way to represent system dynamics, via the $s$ representation gotten from Laplace transforms, or excitation by $e^{st}$.

**TABLE 2.1** Laplace transform table

| Item no. | $f(t)$ | $F(s)$ |
|---|---|---|
| 1. | $\delta(t)$ | $1$ |
| 2. | $u(t)$ | $\frac{1}{s}$ |
| 3. | $tu(t)$ | $\frac{1}{s^2}$ |
| 4. | $t^n u(t)$ | $\frac{n!}{s^n + 1}$ |
| 5. | $e^{-at}u(t)$ | $\frac{1}{s + a}$ |
| 6. | $\sin \omega t u(t)$ | $\frac{\omega}{s^2 + \omega^2}$ |
| 7. | $\cos \omega t u(t)$ | $\frac{s}{s^2 + \omega^2}$ |

**Table 2.1**

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**TABLE 2.2** Laplace transform theorems

| Item no. | Theorem | Name |
|---|---|---|
| 1. | $\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$ | Definition |
| 2. | $\mathscr{L}[kf(t)] = kF(s)$ | Linearity theorem |
| 3. | $\mathscr{L}[f_1(t) + f_2(t)] = F_1(s) + F_2(s)$ | Linearity theorem |
| 4. | $\mathscr{L}[e^{-at}f(t)] = F(s+a)$ | Frequency shift theorem |
| 5. | $\mathscr{L}[f(t-T)] = e^{-sT}F(s)$ | Time shift theorem |
| 6. | $\mathscr{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$ | Scaling theorem |
| 7. | $\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0-)$ | Differentiation theorem |
| 8. | $\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - f'(0-)$ | Differentiation theorem |
| 9. | $\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$ | Differentiation theorem |
| 10. | $\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right] = \frac{F(s)}{s}$ | Integration theorem |
| 11. | $f(\infty) = \lim_{s\to 0} sF(s)$ | Final value theorem[1] |
| 12. | $f(0+) = \lim_{s\to\infty} sF(s)$ | Initial value theorem[2] |

[1]For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts, and no more than one can be at the origin.

[2]For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (that is, no impulses or their derivatives at $t = 0$).

Table 2.2

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# Transfer Function

- Method to represent system dynamics, via *s* representation from Laplace transforms.
- Transfer functions show flow of signal through a system, from input to output.
- Method gives system dynamics representation equivalent to
  - Ordinary differential equations.
  - State equations.

# The Transfer Function

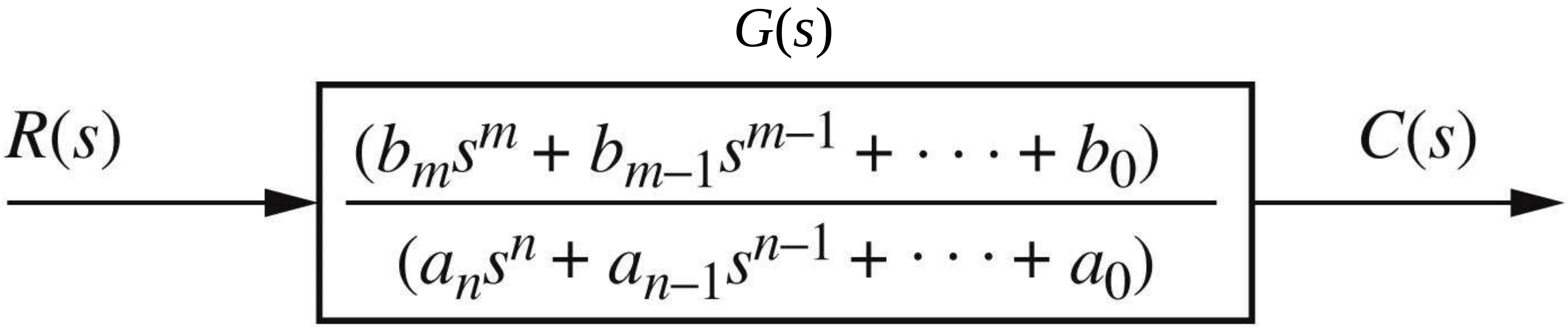


**Figure 2.2**
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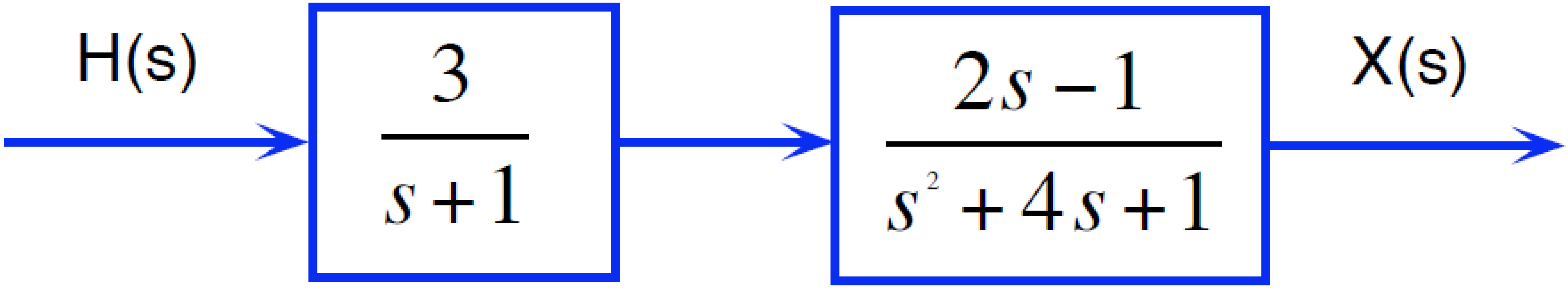
$$C(s) = R(s)G(s)$$




$$\frac{3}{s+1}$$ $$\frac{2s-1}{s^2+4s+1}$$

- Transfer function:

$$G(s) = \frac{X(s)}{H(s)} = G_1(s)G_2(s) = \frac{3}{s+1} \cdot \frac{2s-1}{s^2+4s+1}$$

$$G(s) = \frac{3(2s-1)}{(s+1)(s^2+4s+1)} = \frac{6s-3}{s^3+5s^2+5s+1}$$

- Poles = roots of denominator (values of *s* such that transfer function becomes infinite)

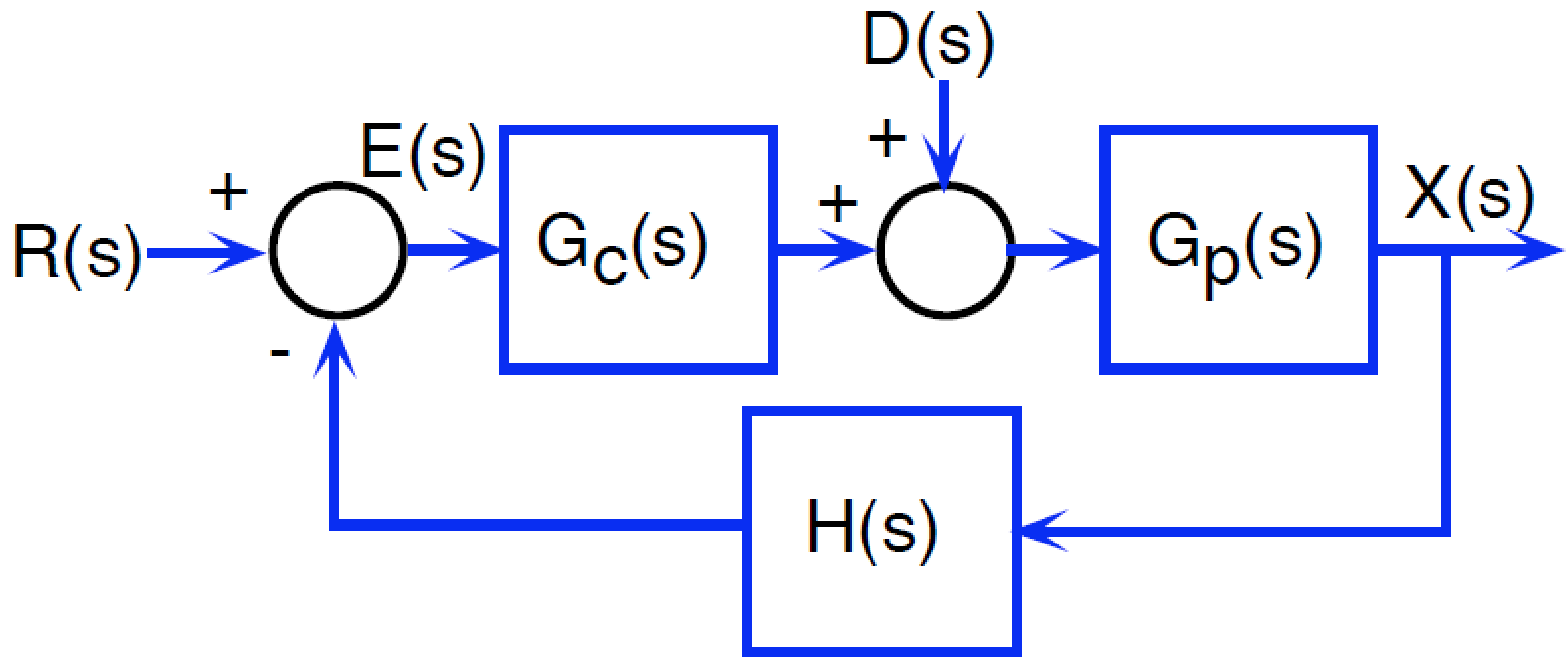
$$p_1 = -1, \quad p_2, p_3 = -2 \pm \sqrt{3}$$

- Zeros = roots of numerator (values of *s* such that transfer function becomes 0)

$$z_1 = 1/2$$

# Transfer Function

## Input Signal and Disturbance

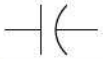
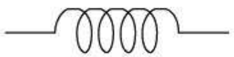


$$G_{cl}(s) = \frac{X(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)H(s)}$$

- Transfer function, disturbance (set $R(s) = 0$)

$$G_d(s) = \frac{X(s)}{D(s)} = \frac{G_p(s)}{1 + G_c(s)G_p(s)H(s)}$$

**TABLE 2.3** Voltage-current, voltage-charge, and impedance relationships for capacitors, resistors, and inductors

| Component | Voltage-current | Current-voltage | Voltage-charge | Impedance $Z(s) = V(s)/I(s)$ | Admittance $Y(s) = I(s)/V(s)$ |
|---|---|---|---|---|---|
|  Capacitor | $v(t) = \frac{1}{C}\int_0^1 i(\tau)d\tau$ | $i(t) = C\frac{dv(t)}{dt}$ | $v(t) = \frac{1}{C}q(t)$ | $\frac{1}{Cs}$ | $Cs$ |
|  Resistor | $v(t) = Ri(t)$ | $i(t) = \frac{1}{R}v(t)$ | $v(t) = R\frac{dq(t)}{dt}$ | $R$ | $\frac{1}{R} = G$ |
|  Inductor | $v(t) = L\frac{di(t)}{dt}$ | $i(t) = \frac{1}{L}\int_0^1 v(\tau)d\tau$ | $v(t) = L\frac{d^2q(t)}{dt^2}$ | $Ls$ | $\frac{1}{Ls}$ |

Note: The following set of symbols and units is used throughout this book: $v(t)$ – V (volts), $i(t)$ – A (amps), $q(t)$ – Q (coulombs), $C$ – F (farads), $R$ – Ω (ohms), $G$ – Ω (mhos), $L$ – $H$ (henries).

Table 2.3
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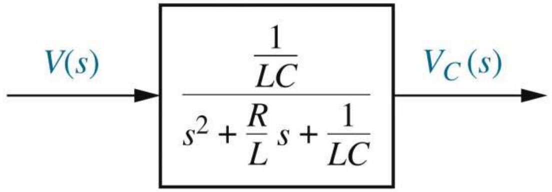


Figure 2.4
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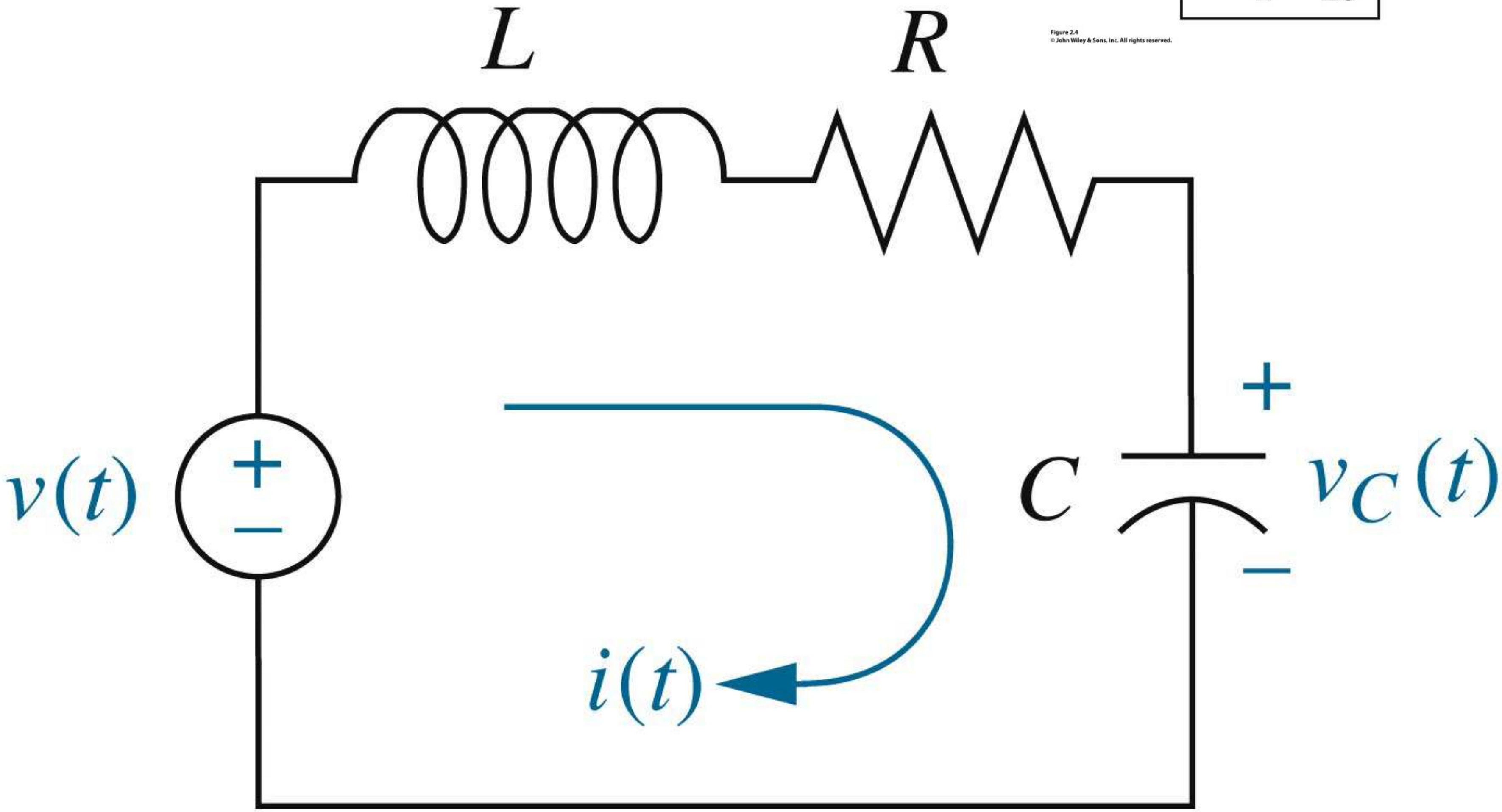


**Figure 2.3**
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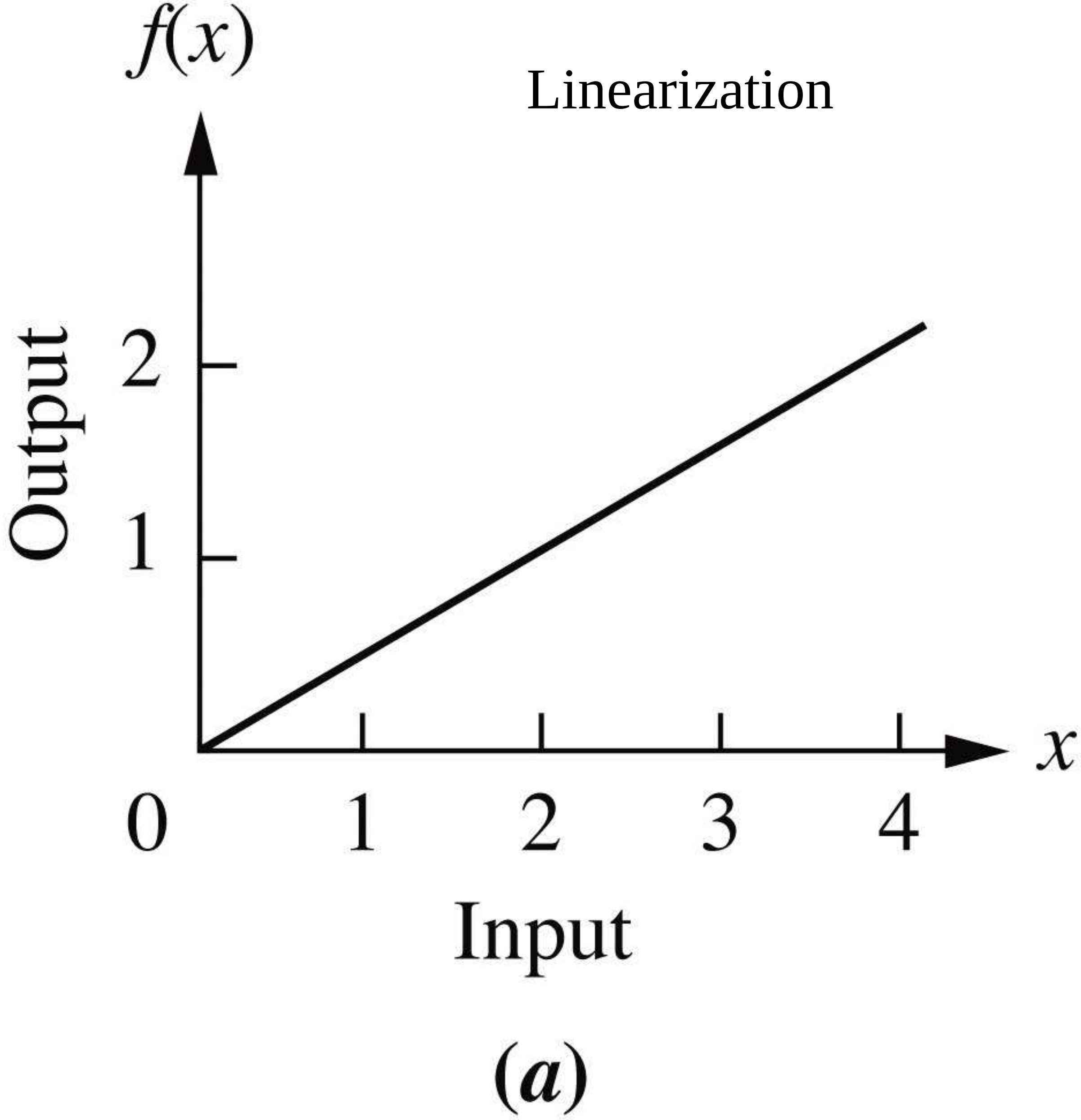


Figure 2.45a

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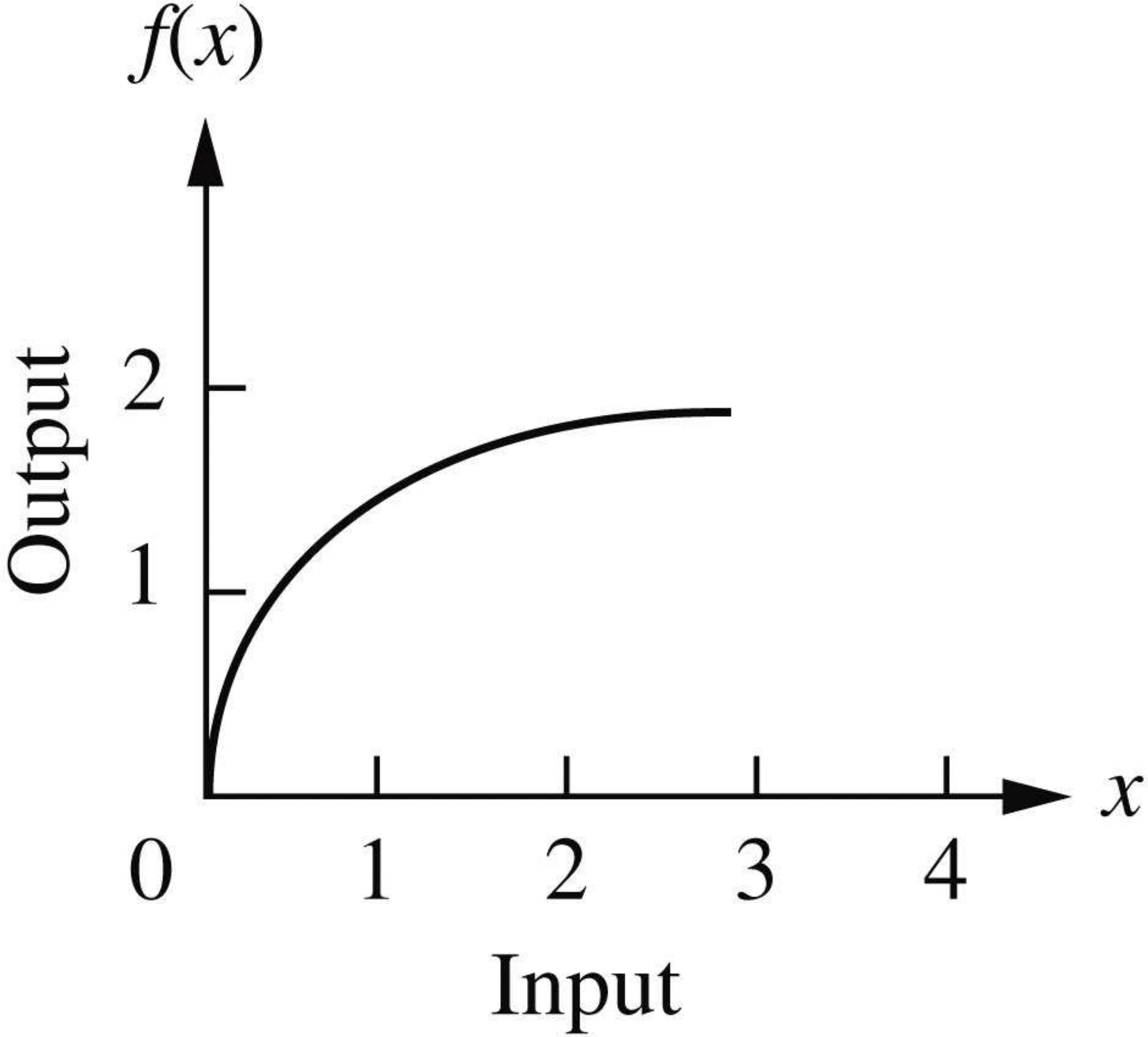


(*b*)

**Figure 2.45b**

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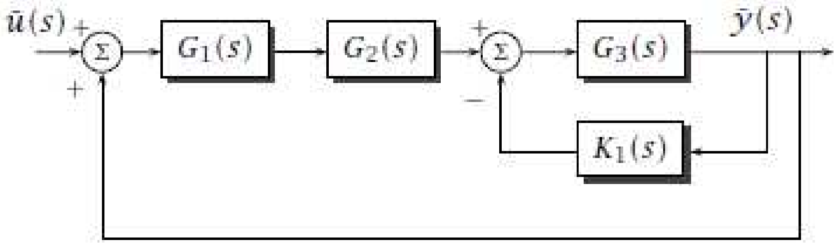
# Block Diagrams

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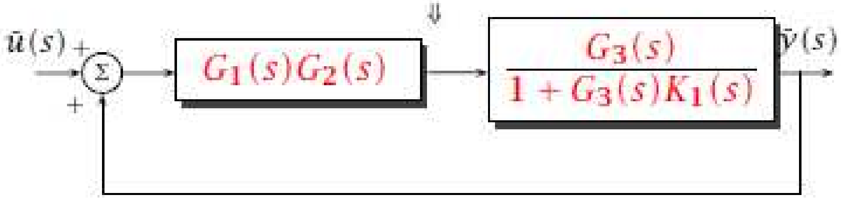


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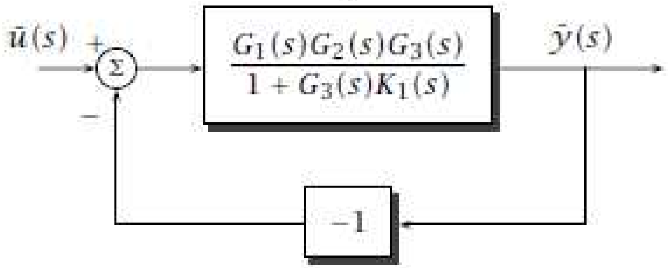


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