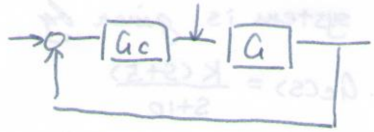
12.1 Consider a system of the form




where $G(s) = \frac{1}{s+1}$

using the ITAE performance method for a step input determine the required $G_c(s)$. Assume $\omega_n = ~~30~~\ 20$. Determine the step response with and without a prefilter $G_p(s)$.

Answer: Try a PI controller, given by

$$G_c = K_D + \frac{K_I}{S} \qquad T(s) = \frac{SK_D + K_I}{S^2 + (1+K_D)S + K_I}$$

The ITAE characteristic equation is

$$q(s) = s^2 + 1.4\,\omega_n s + \omega_n^2 \quad \text{and} \quad s^2 + (1+K_D)s + K_I = 0.$$

where $\omega_n = 20$. then $K_D = 27$. $K_I = 400$.

without the prefilter, the close-loop system transfer function is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{27s + 400}{s^2 + 28s + 400}$$

with a prefilter

$$T(s) = \frac{400}{s^2 + 28s + 400} \quad \text{where} \quad G_p(s) = \frac{14.8}{s + 14.8}$$

$G_p$ eliminate the ~~zeros~~ zeros of $T(s)$. and bring the overall numerator to 400.

12.3. A close-loop ~~system~~ unity feedback system has the loop transfer function $G_cG = \frac{18}{s(s+b)}$

where $b$ is ~~nom~~ normally equal to 3. Determine $S_b^T$

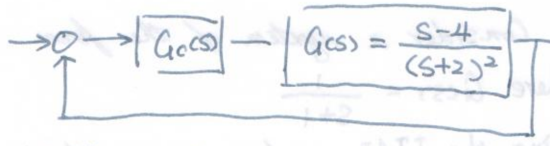
Answer: $T(s) = \frac{G_cG}{1+G_cG} = \frac{18}{1+s(s+b)}$

the sensitivity function is.

$$S^T(s) = [1 + G_cG]^{-1} = \frac{s^2 + bs}{s^2 + bs + 18}$$

$$S_b^T = \frac{\partial T}{\partial p} \cdot \frac{P}{T} = \frac{-bs}{s^2 + bs + 18}$$

P12.3 a system is given by.




where $G_c(s) = \frac{K(s+5)}{s+10}$

a. find the range of K for a stable system

b. select a gain so that the steady-state error of the system is zero for a step input.

Answer: the open-loop tf is

$$GG_c = \frac{K(s+5)(s-4)}{(s+10)(s+2)^2}$$

so the characteristic equation is

$$1 + GG_c = 0 = 1 + \frac{K(s+5)(s-4)}{(s+10)(s+2)^2}$$

or $s^3 + (14+K)s^2 + (44+K)s + 40 - 20K = 0$

Routh-criterion

| | | |
|---|---|---|
| $s_0$ | $1$ | $44+K$ |
| $s_1$ | $14+K$ | $40-20K$. |
| $s_2$ | $\frac{(14+K)(44+K)-(40-20K)}{14+K}$ | |
| $s_3$ | $40-20K$. | |

$$\Rightarrow \begin{cases} 40-20K > 0 \\ \frac{(14+K)(44+K)-(40-20K)}{14+K} > 0 \end{cases}$$

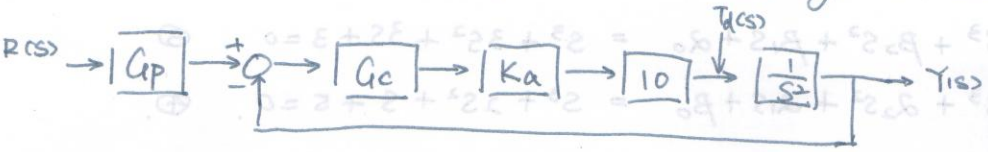
$\Rightarrow -8.25 < K < 2$

b. the steady-state error is

$$e_{ss} = \frac{2}{2-K}.$$

the steady-state error cannot be zero. for any stable K.
Choose. K close to $-8.25$ to minimize the tracking error.

12.8 A motor and load with negligible friction and a voltage-to-current amplifier $K_a$ is used in the feedback control system shown in Fig.




a designer selects a PID controller $G_c = K_P + \frac{K_I}{s} + K_D s$

where. $K_P = 5$, $K_I = 500$. and $K_D = 0.0475$

a. determine the appropriate value of $K_a$ so that the phase margin of the system is $40°$.

answer: the open-loop transfer function

$$G G_c(s) = \frac{10 K_a (5s + 500 + 0.0475 s^2)}{s^3} = L(s).$$

$$L(j\omega) = \frac{10 K_a (5\omega j + 500 - 0.0475\omega^2)}{j\omega\, j\omega\, j\omega}$$

$$|L(j\omega)| = \frac{10 K_a \sqrt{25\omega^2 + (500 - 0.0475\omega^2)^2}}{\omega^3} = 1$$

$$180° - \left(90° \times 3 - \arctan \frac{\cancel{25}\, 5\omega}{500 - 0.0475\omega^2}\right) = 40°$$

$\Rightarrow \omega = 155.953.$

$\Rightarrow K_a = \underset{\text{computer.}}{\underline{374.5}}.\ \underset{\text{mine}}{\underline{372.4}}.$

12.9. A unity feedback system has a nominal characteristic equation

$q(s) = s^3 + 3s^2 + 3s + 4 = 0.$

the coefficients vary as follows:

$2 \le a_2 \le 3 \quad 1 \le a_1 \le 3 \quad 3 \le a_0 \le 5$ ; Determine whether the system is stable for these uncertain coefficients.

Answer: $\alpha_0 = 3 \quad \beta_0 = 5$

$\alpha_1 = 1 \quad \beta_1 = 3$

$\alpha_2 = 2 \quad \beta_2 = 3$

$q(s) = s^3 + \alpha_2 s^2 + \beta_1 s + \beta_0 = s^3 + 2s^2 + 3s + 5 = 0$ ①

$q(s) = s^3 + \beta_2 s^2 + \alpha_1 s + \alpha_0 = s^3 + 3s^2 + s + 3 = 0$ ②

$q(s) = s^3 + \beta_2 s^2 + \beta_1 s + \alpha_0 = s^3 + 3s^2 + 3s + 3 = 0$ ③

$q(s) = s^3 + \alpha_2 s^2 + \alpha_1 s + \beta_0 = s^3 + 2s^2 + s + 5 = 0$. ④.

① 

| | | |
|---|---|---|
| $s^3$ | 1 | 3 |
| $s^2$ | 2 | 5 |
| $s^1$ | $\frac{1}{2}$ | |
| $s^0$ | 5 | |

② 

| | | | | |
|---|---|---|---|---|
| $s^3$ | 1 | 1 | 1 | 1 |
| $s^2$ | 3 | 3 | 3 | 3 |
| $s^1$ | 0 | | 6 | 0 |
| | | | 3 | |
| $s^0$ | | | | |

③ 

| | | |
|---|---|---|
| $s^3$ | 1 | 3 |
| $s^2$ | 3 | 3 |
| $s^1$ | 2 | |
| $s^0$ | 3 | |

$3s^2+3 = F(s)$ $\quad \frac{dF(s)}{ds} = 6s + 0$

④ 

| | | |
|---|---|---|
| $s^3$ | 1 | 1 |
| $s^2$ | 2 | 5 |
| $s^1$ | $-3/2$ | |
| $s^0$ | 5 | |

the forth polynominal is not stable.
$\Rightarrow$ the system is not stable for the uncertain parameters.

12.11 A plant has a transfer function $G(s) = \frac{25}{s^2}$
we want to use a negtive unity feedback with a PID controller (and a prefilter). The goal is to achieve a peak time of 1 second with ITAE - type performance. ~~Predict the system~~ for a step input.

Answer: the characteristic equation is

$$1 + G_c G(s) = 0 = 1 + \frac{K_D s^2 + K_I + K_P s}{s} \cdot \frac{25}{s^2} = 0$$

$$s^3 + 25 K_D s^2 + 25 K_P s + 25 K_I = 0 .$$

based on ITAE :

$$s^3 + 1.75\,\omega_n s^2 + 2.15\,\omega_n^2 s + \omega_n^3 = 0$$

$$\omega_n T_p \approx 4 \quad \Rightarrow \omega_n = 4$$

$$25 K_I = 64 \qquad K_I = 2.56$$

$$25 K_P = 2.15 \times 16 \quad \Rightarrow \quad K_P = 1.376$$

$$25 K_D = 1.75 \times 4 \qquad K_D = 0.28$$