E13.4. Given a function $Y(s) = \frac{5}{s(s+2)(s+10)}$ use a partial expansion fraction of Y(s) and Z-transform to find Y(Z) with T=0.1s.

Answer:

The partial expansion of Y(s) is

$$Y(s) = \frac{5}{s(s+2)(s+10)} = \frac{0.25}{s} + \frac{0.0625}{s+10} - \frac{0.3125}{s+2}$$

Based on the Z-transform:

$$\frac{1}{S} \overset{Z}{\Rightarrow} \frac{Z}{Z-1}, \qquad \frac{1}{S+A} \overset{Z}{\Rightarrow} \frac{Z}{Z-e^{-AT}}$$

$$Y(Z) = \frac{0.25Z}{Z-1} + \frac{0.0625Z}{Z-e^{-10T}} - \frac{0.3125Z}{Z-e^{-2T}} = \frac{0.25Z}{Z-1} + \frac{0.0625Z}{Z-0.135} - \frac{0.3125Z}{Z-0.67}$$

E13.8. Determine whether the closed loop system with T(Z) is stable when

$$T(Z) = \frac{Z}{Z^2 + 0.2Z - 0.4}$$

Answer: system is stable. Because both poles of the transfer function are located in the unit circle in the Z-plant.

E3.11 A system has a process transfer function $G_p(s) = \frac{100}{S^2+100}$

(a) Determine G(Z) for $G_p(s)$ preceded by a zero-order hold with T=0.05s
(b) Determine whether the digital system is stable.

Answer:

(a) The transfer function of the system is $G(s) = G_o G_p(s) = \frac{1-e^{-st}}{s}\frac{100}{S^2+100}$
Expanding into partial expansion yields
$$G(Z) = (1-Z^{-1})\mathcal{Z}\left[\frac{1}{s} - \frac{s}{s^2+100}\right] = (1-Z^{-1})\left[\frac{Z}{Z-1} - \frac{Z(Z-\cos(10T)Z)}{Z^2 - 2\cos(10T)Z + 1}\right]$$
When T=0.05s, we have
$$G(Z) = \frac{0.1224(Z+1)}{Z^2 - 1.7552Z + 1}$$
(b) The system is marginally stable since the system poles $Z = -0.8776 \pm 0.4794j$ are on the unit circle in Z-plant.

E13.12 Find the Z-transform of $X(s) = \frac{s+1}{s^2+5s+6}$ when the sampling period is 1s

Answer:

Using the partial expansion:

$$X(s) = \frac{s+1}{s^2+5s+6} = -\frac{1}{s+2} + \frac{2}{S+3}$$

So, we have

$$X(Z) = -\frac{Z}{Z-e^{-2T}} + \frac{2Z}{Z-e^{-3T}} = -\frac{Z}{Z-0.135} + \frac{2Z}{Z-0.0498}$$

E3.13. The characteristic equation of a sampled system is $Z^2 + (K-2)Z + 0.75 = 0$. Find the range of K so that the system is stable.

Answer: when all the poles of system are located in the unit circle in Z-plant, the system is stable.

The roots of the characteristic equation are $Z = \frac{-(K-2) \pm \sqrt{(K-2)^2-3}}{2}$

(a) Have two real roots

$$\begin{cases} (K-2)^2 - 3 \geq 0 \\ -1 < \dfrac{-(K-2) \pm \sqrt{(K-2)^2-3}}{2} < 1 \end{cases} \Rightarrow \begin{cases} K \leq 0.268 \; or \; K \geq 3.732 \\ 0.25 < K < 3.75 \end{cases}$$

(b) Have two complex roots

$$\begin{cases} (K-2)^2 - 3 < 0 \\ (K-2)^2 + 3 - (K-2)^2 < 4 \end{cases} \Rightarrow 0.268 < K < 3.732$$

So the range of K is $0.25 < K < 3.75$