ELG4156
State Space Averaging of DC to DC Converters
Introduction

• A few applications of interest of DC-DC converters are where 5V DC on a personal computer motherboard must be stepped down to 3V, 2V or less for one of the latest CPU chips.

• DC to DC converters are widely used in hybrid cars which is our main focus to alter DC energy from a particular level to other with minimum loss.

• The need for converters is in demand due to the fact that transformers are unable to operate on DC.

• A converter is not producing power. Whatever comes at the output has to come only from input. Efficiency cannot be made equal to 100%.
When the switch is closed, the voltage across the inductor is $VL = Vi - Vo$. The current through inductor linearly rises. The diode does not allow current to flow through it, since it is reverse-biased by voltage $V$.

For Off state, diode is forward biased and voltage is $VL = -Vo$ across inductor. The inductor current which was rising in ON case, now decreases.
Buck Modes of Operation

\[ d_1 T_S = \text{ON Period time} \]
\[ d_2 T_S = \text{OFF Period time} \]
\[ T_S = \text{Total time period for one cycle} \]
\[ i_{pk} = \text{peak value of inductor current after ON period} \]
\[ \bar{i}_L = \text{Average value of current} \]
\[ V_{in} = \text{input voltage} \]
Buck Converter During ON Mode
Write the State Space Model

From KVL \[ v_{in} - L \frac{di_L}{dt} - v_C = 0 \]

From KCL \[ \frac{v_C}{R} + C \frac{dv_C}{dt} - i_L = 0 \]

\[
\begin{bmatrix}
\frac{di_L}{dt} \\
\frac{dV_C}{dt}
\end{bmatrix} =
\begin{bmatrix}
i_L \\
v_C
\end{bmatrix} +
\begin{bmatrix}
0 & 1
\end{bmatrix}
\begin{bmatrix}
i_L \\
v_C
\end{bmatrix}
\]
Buck Converter During Off Mode
Write State Space Model

\[ \frac{di_L}{dt} + C \frac{dv_C}{dt} = 0 \]

\[ \frac{v_C}{R} + \frac{dv_C}{dt} = 0 \]

\[ \left[ \begin{array}{c} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{array} \right] = \left[ \begin{array}{c} i_L \\ v_C \end{array} \right] + \left[ \begin{array}{c} 0 \\ 0 \end{array} \right] v_{\text{in}} ; v_o = \left[ \begin{array}{c} 0 \\ 1 \end{array} \right] \left[ \begin{array}{c} i_L \\ v_C \end{array} \right] \]
During Discontinuous Conduction Mode

\[\text{From KVL} \quad \frac{di_L}{dt} = 0\]

\[\text{From KCL} \quad \frac{v_c}{R} + C \frac{dv_c}{dt} = 0\]

Write

\[
\begin{bmatrix}
\frac{di_L}{dt} \\
\frac{dv_c}{dt}
\end{bmatrix}
= \begin{bmatrix}
i_L \\
v_c
\end{bmatrix} + \begin{bmatrix}0 \\
0
\end{bmatrix} v_{in}; v_o = \begin{bmatrix}0 \\
0
\end{bmatrix} \begin{bmatrix}i_L \\
v_c
\end{bmatrix}
\]
Buck Modelling Analysis

• Averaging
• Inductor current analysis
• Duty-ratio constraint.

State space averaging techniques are employed to get a set of equations that describe the system over one switching period.

\[ \dot{X} = [A_1d_1 + A_2d_2 + A_3(1 - d_1 - d_2)]\bar{X} + [B_1d_1 + B_2d_2 + B_3(1 - d_1 - d_2)]u \]

\[ \bar{i}_L = \frac{i_{pk}}{2} \cdot (d_1 + d_2) \]
The Final Model

The state space averaged model for the above equation is

\[ \frac{d}{dt} \begin{bmatrix} \bar{L} \\ \frac{V_C}{V_C} \end{bmatrix} = \begin{bmatrix} K \end{bmatrix} \begin{bmatrix} \bar{L} \\ \frac{V_C}{V_C} \end{bmatrix} + \begin{bmatrix} \frac{d_1}{L} \\ \frac{d_1}{L} \end{bmatrix} V_{in} \]

\[ \frac{d}{dt} \begin{bmatrix} \bar{L} \\ \frac{V_C}{V_C} \end{bmatrix} = \begin{bmatrix} \frac{1}{d_1 + d_2} \\ 0 \end{bmatrix} V_{in} \]
Boost Converter

Write the State Space Model

On

Off