State Space Modeling and Control of the DC-DC Multilevel Boost Converter

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Abstract

The dc-dc multilevel boost converter topology was recently proposed with features such as high voltage gain without using an extreme duty ratio, few components and self dc-link balancing. This paper presents the state space modeling and a linear controller based on the pole placement technique for the dc-dc multilevel boost converter. Nonlinear and linear models are derived. Simulation results obtained with the averaging model in Matlab and with the simulated circuit in Synopsys Saber software are presented to prove the principle of the proposition.

1. Introduction

When an extremely low or high voltage gain is required in dc-dc power conversion, traditional topologies sacrifice *(i)* the switching frequency and/or *(ii)* the system size because of the extremely low or high duty cycle or because of the transformer requirement [1-2].

The use of high switching frequency results in a small converter for equivalent current and voltage ripples [1-4] and this fact motivates the use of several hundred of kilohertz [1], but the natural delay in real switches limits the switching frequency when the duty ratio is extreme. A solution to this is the employment of transformers; however a transformer with a large turn ratio is undesirable because it enhances the transformer non-idealities [1-4].

In emerging applications such as renewable energy generation systems the low dc voltage of a renewable energy sources have to be boosted before being inverted to connect it to the grid.

Several topologies exist for implementing a transformer-less dc-dc converter with high efficiency and high boost ratios, all of them with high complexity compared with the conventional single switch converters [1-5].

Initially introduced in [6-7] the dc-dc multilevel boost converter *MBC* combines the boost converter and the switched capacitor function to provide an output of several capacitors in series with the same voltage and self voltage balancing by using a static voltage multiplier structure [8], see Fig. 1.

The main features and advantages of this topology are: it can control the voltage by pulse width modulation and self balance (all levels have the same voltage) with few components, only one driven switch, one inductor, 2N-1 diodes and 2N-1 capacitors are needed for an N-times MBC. The number of levels can be increased adding capacitors and diodes, and then it is possible to achieve modular implementations. It has continuous input current, a large conversion ratio without using an extremely high duty cycle and without employing a transformer.

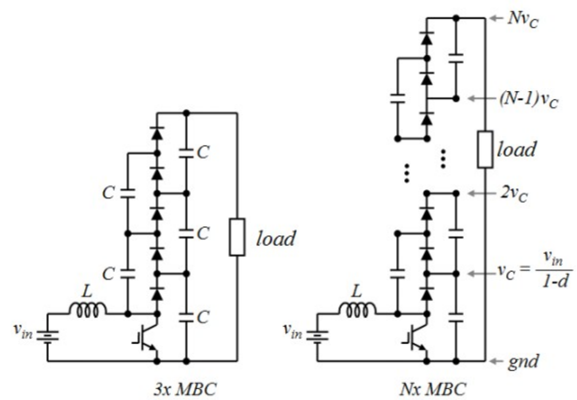

Fig. 1. 3x and Nx dc-dc multilevel boost converters.

The principle of operation has been studied in [6-7] and it has been shown in [9] that it can be used to feed a diode clamped multilevel converter providing a self voltage balancing of the dc-link. There are some works that are reported in the literature that deal with modeling and control of other topologies of high voltage gain dc-dc converter [10-11]. However the modeling and control of the multilevel boost converter has not been reported in the literature.

This paper presents the state space modeling of the MBC and a controller based on the pole placement technique. Both nonlinear and linear models are presented. Simulation results are presented to demonstrate the proposed theory.

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2. The Multilevel Boost Converter

This section presents a briefly description of the multilevel boost converter operation, for a more detailed explanation see [6-7]. Fig. 2 shows a 2x MBC. The lowest part of the converter is the conventional dc-dc boost converter.

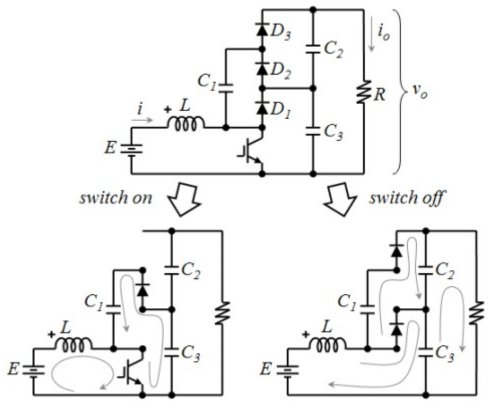

Fig. 2. 2x MBC and equivalent circuits according with the switching state.

When a switched converter is working in steady state in continuous conduction mode *CCM*, the current in all inductors and the voltage in all capacitors keep constant in average over a switching cycle, that means the voltage across all inductors, the current in all capacitors is equal to zero in average over one switching cycle. The average voltage in the inductor over one switching cycle can be expressed as (1):

$$\langle v_L \rangle_T = \frac{1}{T}\left[t_{on}E + t_{off}(E - v_{C3})\right] = 0 \tag{1}$$

Where t_{on} and t_{off} are the time when the switch is closed and opened respectively, and T is the total switching period, it can be seen in Fig. 2, when the switch is closed, the inductor voltage is equal to the input voltage E, and when the switch is open, the inductor voltage is equal to the input voltage minus the voltage in capacitor C_3. Defining the duty ratio d as the time when the switch is *on* over the switching period, the next expressions can be written.

$$d = \frac{t_{on}}{T} \; ; \qquad (1-d) = \frac{t_{off}}{T} \tag{2}$$

And (1) can be re-written as (3):

$$\langle v_L \rangle_T = dE + (1-d)(E - v_{C3}) = 0 \tag{3}$$

From (3), the voltage in capacitor C_3 can be expressed as:

$$v_{C3} = E\frac{1}{1-d} \tag{4}$$

That means the first stage of the MBC works exactly as a boost converter. The diodes-capacitors arrangement works as a voltage multiplier [8], when the switch is closed, if the capacitor C_1 has lower voltage than C_3 the diode D_2 would close connecting C_1 and C_3 in parallel, and C_3 clamps the voltage in C_1, when the switch opens and the inductor current closes D_1, if the voltage in C_2 is lower than the voltage in C_1 the diode D_3 would close changing C_2, this action clamps the voltage in all capacitors to be the same and provides a self balancing capability. For example if a diode clamped multilevel converter is connected as a load, the MBC would maintain balanced the voltage in all dc levels [9].

The number of levels can be increased by adding more capacitors and diodes with the same structure, see Fig. 1.

3. Modeling of the Multilevel Boost Converter

The steady state equations of the multilevel boost converter are very similar to the steady state equations of the conventional boost converter. The average output voltage equation of the conventional multilevel boost converter is multiplied by the number of levels of the multilevel boost converter. The average output voltage equation of the multilevel boost converter is

$$v_o = \frac{N}{1-d}E \tag{5}$$

Where N is the number of capacitors in the dc output link, E is the input voltage and d is the duty ratio. Therefore, the average output current can be written as:

$$i_o = \frac{1-d}{N}i \tag{6}$$

Where i is the input inductor current and i_O is the output current, see Fig. 2. From these equations, the average dynamic model of the multilevel boost converter can be written as (7) and (8):

$$L\frac{di}{dt} = -(1-d)v_o + NE \tag{7}$$

$$C_{eq}\frac{dv}{dt} = (1-d)i - \frac{N}{R}v_o \tag{8}$$

In this model, the state variables are the output voltage and the inductor current. On the other hand the input voltage and the duty ratio can be considered as inputs. This model can be written as (9):

$$\frac{d}{dt}\begin{bmatrix} i \\ v_o \end{bmatrix} = \begin{bmatrix} 0 & -\left(\frac{1}{L}\right) \\ \frac{1}{C_{eq}} & -\frac{N}{RC_{eq}} \end{bmatrix}\begin{bmatrix} i \\ v_o \end{bmatrix} + \begin{bmatrix} \frac{1}{L}v_o & \frac{N}{L} \\ -\frac{1}{C_{eq}}i & -0 \end{bmatrix}\begin{bmatrix} d \\ E \end{bmatrix} \tag{9}$$

Since the state variables are multiplied by the input d, the multilevel boost converter is a nonlinear system. In this case, linearization of the averaged model about an operating point (equilibrium state) is performed. The main idea is to control the output voltage.

In order to obtain an incremental average model, the operating point is calculated from the steady state equations (10) and (11) where 200V is the desired voltage output, the input voltage is 40V and the output current is 2A.

$$d_s = \frac{v_{os} - NE_s}{v_{os}} = \frac{(200v) - (2)(40v)}{(200v)} = 0.6 \qquad (10)$$

$$i_s = \frac{N}{1-d_s} i_{os} = \frac{(2)}{(1-0.6)}(2A) = 10amps \qquad (11)$$

Therefore the desired operating point is:

$$v_{os} = 200; \quad i_s = 10; d_s = 0.6; \qquad E_s = 40$$

The standard linear time-varying state equation is (12):

$$\frac{d}{dt}x = Ax + Bu \qquad (12)$$

And the averaged incremental linear model of the multilevel boost converter can be expressed as (13).

$$\frac{d}{dt}\begin{bmatrix} \bar{i} \\ \bar{v}_o \end{bmatrix} = \begin{bmatrix} 0 & -\left(\frac{1-d_s}{L}\right) \\ \frac{(1-d_s)}{C_{eq}} & -\frac{N}{RC_{eq}} \end{bmatrix}\begin{bmatrix} \bar{i} \\ \bar{v}_o \end{bmatrix} + \begin{bmatrix} \frac{v_{os}}{L} & \frac{N}{L} \\ -\frac{i_s}{C_{eq}} & 0 \end{bmatrix}\begin{bmatrix} \bar{d} \\ \bar{E} \end{bmatrix} \qquad (13)$$

By substituting the values of the desired operating point, the numerical matrices A and B are obtained. The desired equilibrium is located at the origin of the average incremental state coordinates.

4. Controller

In this case, the input voltage E is considered as an unknown disturbance. The controller must compensate the possible variations of this voltage. Therefore, the system can be rewritten as (14):

$$\frac{d}{dt}\begin{bmatrix} \bar{i} \\ \bar{v}_o \end{bmatrix} = \begin{bmatrix} 0 & \left(\frac{d_s-1}{L}\right) \\ \frac{(1-d_s)}{C_{eq}} & \frac{-N}{RC_{eq}} \end{bmatrix}\begin{bmatrix} \bar{i} \\ \bar{v}_o \end{bmatrix} + \begin{bmatrix} \frac{v_{os}}{L} \\ \frac{-i_s}{C_{eq}} \end{bmatrix}\bar{d} + \begin{bmatrix} \left(\frac{N}{L}\right) \\ 0 \end{bmatrix}\bar{E} \qquad (14)$$

In this form the system has the structure in (15):

$$\frac{d}{dt}\bar{x}(t) = A\bar{x}(t) + B\bar{u}(t) + B_1\bar{w}(t) \qquad (15)$$

The average incremental state $\bar{x}$ is defined as (16):

$$\bar{x} = x - X_s \qquad (16)$$

Where x is the state of the original nonlinear system and X_s is the average equilibrium state corresponding to the average equilibrium input d_s and $\bar{w}(t)$ is the incremental disturbance voltage.

For this second order system, the controllability is determined by the fact:

$$rank\ \mathcal{C} = rank\ [B, AB] = 2 \qquad (17)$$

In order to compensate the variations in parameters and disturbances, a state integrator is included. The state equation of the integrator is (18):

$$\frac{d}{dt}x_1(t) = \bar{v}_o(t) - \bar{r}(t) \qquad (18)$$

Where $\bar{r}(t)$ is the incremental set point. In order to control the output voltage of the multilevel boost converter in the desired operating point, the incremental set point is equal to zero.

The state space is then extended for including the integrator state equation, this is

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ \bar{i} \\ \bar{v}_o \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & \left(\frac{d_s-1}{L}\right) \\ 0 & \frac{(1-d_s)}{C} & \left(\frac{-N}{RC}\right) \end{bmatrix}\begin{bmatrix} x_1 \\ \bar{i} \\ \bar{v}_o \end{bmatrix} + \begin{bmatrix} x_1 \\ \bar{i} \\ \bar{v}_o \end{bmatrix}\bar{d}$$

$$-\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\bar{r} + \begin{bmatrix} 0 \\ \frac{N}{L} \\ 0 \end{bmatrix}\bar{E} \qquad (19)$$

In this case, the control law for the linearized system is defined as (20):

$$\bar{u}(t) = -K \cdot \bar{x}(t) \qquad (20)$$

And in this context is (21):

$$\bar{d}(t) = -K_1 x_1(t) - K_i \bar{i}(t) - K_{vo}\bar{v}_o(t) \qquad (21)$$

Where there are some gains related to each one of the state variables. These gains can be calculated by following the procedure presented in [12-13] or by using the command place in Matlab. The actual input applied to the original nonlinear system is:

$$d(t) = d_s + \bar{d}(t) \qquad (22)$$

or

$$d(t) = d_s - K_1 x_1 - K_i(i - i_s) - K_{vo}(v_o - v_{os}) \qquad (23)$$

It is well known that each s-plane pole has a particular time constant. Therefore, a pole can be obtained depending on the value of the associated time constant. The time constant of the system can be chosen depending on the requirements of speed response for different applications.

In this case, the time constant vector is arbitrary chosen in (24):

$$\tau = [0.00066265, 0.0006626, 0.00066256] \qquad (24)$$

Therefore, the poles are the inverse of each value in expression (24), this is

$$s_{1,2,3} = [-1509.1, -1509.2, -1509.3] \qquad (25)$$

In order to define a stable behavior, poles are negative (at the left half of the s-plane). The closed-loop gains in (23) can be calculated by following the procedure presented in [12-13] or by using the command place in Matlab. Both methods require the poles expressed in (25). The calculated gains for this application are

$$K_I=2.5781; \quad K_i=0.0068318; \quad K_{vo}= 0.0025383 \qquad (26)$$

5. Simulation Results

The circuit shown in Fig. 3(a) was simulated with the control system shown in Fig. 3(b). It can be seen than the state variables i and v_o are used to get the variables $\bar{i}$ and $\bar{v_o}$ used in the linear controller.

A limiter is used in the duty ratio to clamp the value between 0.01 and 0.99 before the comparison with the carrier triangular signal *carr*, the comparison between the duty ratio and the carrier signal gives the firing signal *s* for the switch. A perturbation is introduced with a switch that connects and disconnects an additional load with a frequency of 5Hz.

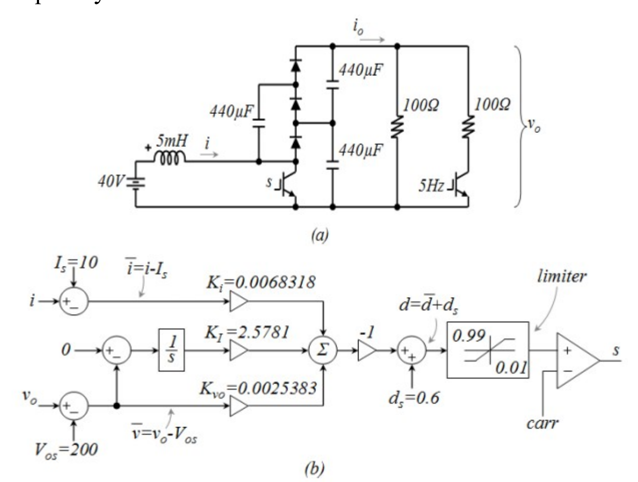

Fig. 3. (a) simulated circuit (b) control system for the MBC.

Fig. 4 shows the duty ratio defined by the controller. Fig. 3 was simulated in Synopsys Saber software and the averaging model explained in section 3 was simulated in Matlab. Both simulation results are shown in the Figures.

Fig. 5 and Fig. 6 show transient traces of the input current and the output voltage respectively during the initialization of the circuit.

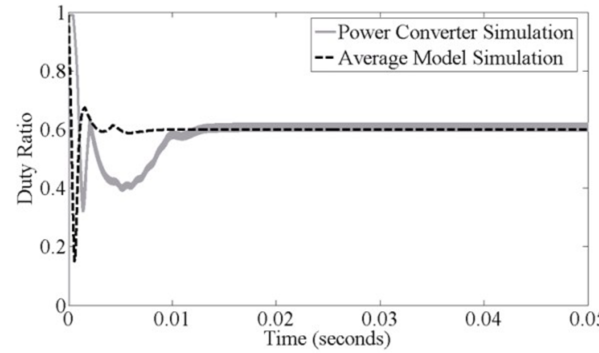

Fig. 4. Closed-loop duty ratio during the initialization.

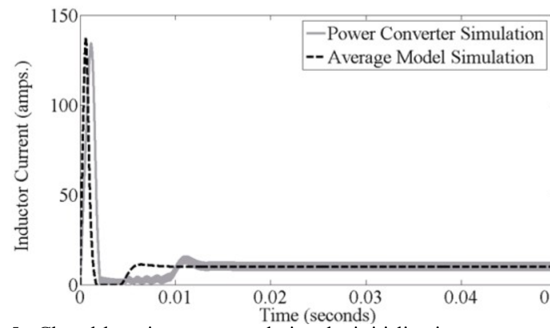

Fig. 5. Closed-loop input current during the initialization.

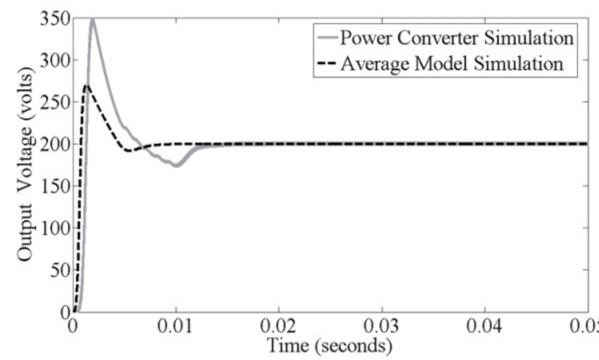

Fig. 6. Closed-loop output voltage during the initialization.

The system was perturbed by changing the value of the resistive load from 100ohms to 50ohms, this is illustrated in Fig. 3a. Figures 7 and 8 show the variations of the input current and the output voltage.

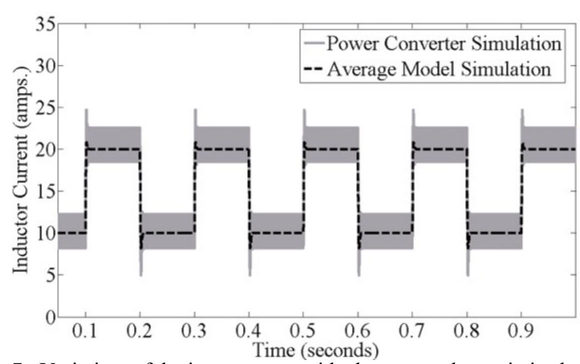

Fig. 7. Variations of the input current with changes on the resistive load.

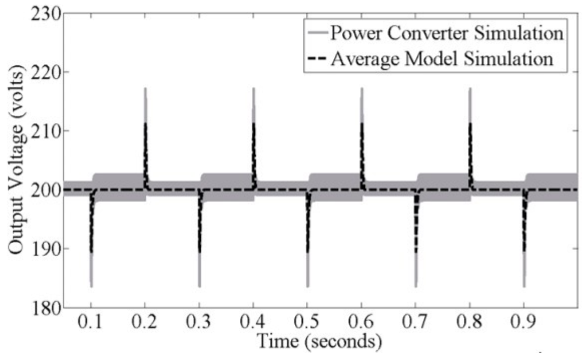

Fig. 8. Variations of the output voltage with changes on the resistive load.

Another perturbation for the same system was an input voltage variation, from 40 to 30 volts and vice versa, Fig. 9 shows the schematic of the simulation. Fig. 10 and Fig. 11 show the input inductor current and the output voltage with the input voltage perturbation.

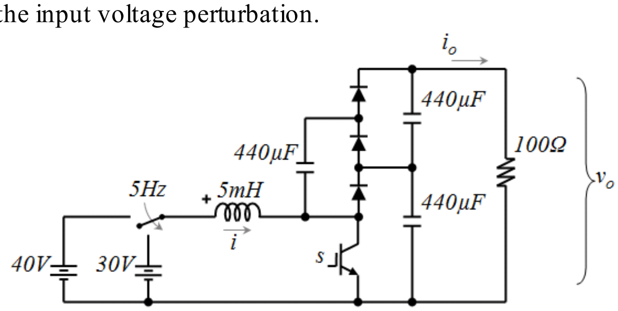

Fig. 9. Input voltage perturbation.

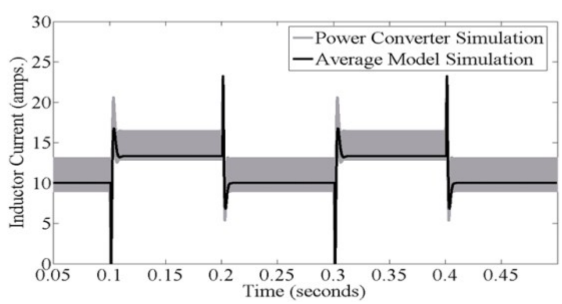

Fig. 10. Variations of the input current with input voltage perturbations.

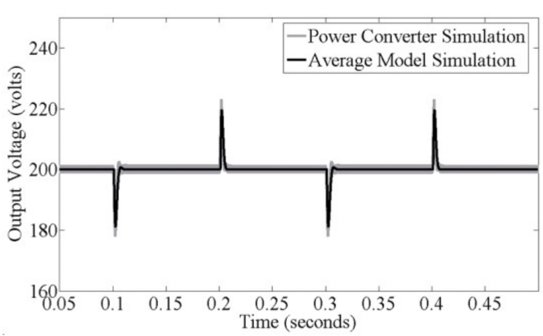

Fig. 11. Variations of the output voltage with input voltage perturbations.

5. Conclusions

A second order state space averaging model of the multilevel boost converter *MBC* topology was presented for a 2x MBC, and a linear controller was proposed and simulated with the averaging model in Matlab and with a switching circuit simulation in Saber, a severe perturbation was introduced to the system with a highly non-linear load (a switching load) with acceptable voltage regulation and dynamic performance. Future work will be realized in the control of this recently proposed topology.

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