MODERN CONTROL ENGINEERING
LAB 3

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FULL STATE FEEDBACK

Consider a SISO system in its ss representation:

$$\dot{\underline{x}} = A\underline{x} + Bu$$
$$y = C\underline{x}$$

The system response – governed by $\lambda(A)$

We want to use K to change the position of these eigenvalues!

We assume $u = r - Kx$ where r is some reference signal and the gain matrix $K \in \Re^{1xn}$

If $r = 0$ this controller is caller regulator.

We obtain:

$$\dot{\underline{x}} = A\underline{x} + Bu = A\underline{x} + B(r - K\underline{x}) = (A - BK)\underline{x} + Br = A_{CL}\underline{x} + Br$$
$$y = C\underline{x}$$

<u>Objective</u>: Pick K such that A_{CL} has the desired eigenvalues!

<u>Condition</u>: States x are accessible and the system is controllable.

This are the first steps that need to be checked before designing a full state controller.

<u>Problem 1</u>. Consider the following system:

$$\dot{\underline{x}} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}\underline{x} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}u$$
$$y = \begin{pmatrix} 1 & 0 \end{pmatrix}\underline{x}$$

a. Design a full state controller such that the closed loop system has the following poles: $p_1 = -5, p_2 = -6$

Solve first analytically and then with the use of MATLAB.

b. Redo the operations for the case in which the A matrix has changed to:

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$

c. Check the tracking performances in the case of $r(t) = u(t)$ (unit step). Construct a SIMULINK diagram for our control system.

Do we have zero steady state error for the unit-step reference signal?

Conclusion: The output does not track our unit-step reference signal. How do we solve this problem??

One obvious solution is to scale the reference input $r(t)$ so that $u = \bar{N}r - K\underline{x}$, where $\bar{N}$ is an extra-gain.

The open-loop system is:

$$\dot{\underline{x}} = A\underline{x} + Bu$$

$$y = C\underline{x} + Du$$

For a unit-step input: $r = r_{SS}u(t)$

At steady state: $\dot{\underline{x}} = 0 \Rightarrow \underline{x} = \underline{x}_{SS}, u = u_{SS}$. For good tracking we want:

$$y = y_{SS} = r_{SS}$$

Solve for:

$$\begin{pmatrix} \underline{x}_{SS} \\ u_{SS} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} \begin{pmatrix} \underline{0} \\ r_{SS} \end{pmatrix}$$

Let us define: $x_{SS} = N_x r_{SS}, u_{ss} = N_u r_{SS}$. Also:

$$u = \bar{N}r - K\underline{x} \Rightarrow u_{SS} = \bar{N}r_{SS} - K\underline{x}_{SS} \Rightarrow \bar{N}r_{SS} = u_{SS} + K\underline{x}_{SS}$$

Again,

$$u = u_{SS} - K(\underline{x} - \underline{x}_{SS}) = N_u r_{SS} - K(x - N_x r_{SS}) = (N_u + KN_x)r_{SS} - K\underline{x}$$

Such that:

$$\bar{N} = N_u + KN_x$$

Introduce this gain in the SIMULINK Model and check the tracking again!!