

# Design of Control Systems

## Case Studies

Motor Speed

Motor Position

## Design Cycle

Modeling; Identification; Design, and Implementation  
Plus Analysis and Simulation

## Techniques Used

Modeling; PID; Root Locus; Frequency Response;  
State Space

# Control of Transient Response

- One of the most important characteristics of control systems is their transient response.
- The transient response is the response of a system as a function of time. Since the purpose of control systems is to provide a desired response, the transient response of control systems often must be adjusted until it is satisfactory.
- If an open loop system does not provide a satisfactory response, then the process  $G(s)$ , must be replaced with a suitable process. It is also possible to alter the response of an open-loop system by inserting a suitable cascade controller preceding the process.
- However, the closed-loop system can often be adjusted to yield the desired response by adjusting the feedback loop parameters.

# Control Modes

There are many ways by which a control unit can react to an error and supply an output for correcting elements.

- **The two-step mode:** The controller is just a switch which is activated by the error signal and supplies just an on-off correcting signal. Example of such mode is the bimetallic thermostat.
- **The proportional mode (P):** This produces a control action that is proportional to the error. The correcting signal thus becomes bigger the bigger the error. Therefore, the error is reduced the amount of correction is reduced and the correcting process slows down. A summing operational amplifier with an inverter can be used as a proportional controller.
- **The derivative mode:** This produces a control action that is proportional to the rate at which the error is changing. When there is a sudden change in the error signal the controller gives a large correcting signal. When there is a gradual change only a small correcting signal is produced. An operational amplifier connected as a differentiator circuit followed by another operational amplifier connected as an inverter make an electronic derivative controller circuit.

- **The integral mode (I):** This produces a control action that is proportional to the integral of the error with time. Therefore, a constant error signal will produce an increasing correcting signal. The correction continues to increase as long as the error persists.
- **Combination of modes:** Proportional plus derivative modes (PD), proportional plus integral modes (PI), proportional plus integral plus derivative modes (PID). The term three-term controller is used for PID control.
- The controller may achieve these modes by means of pneumatic circuits, analog electronics involving operational amplifiers or by the programming of a microprocessor or computer.

# DC Motors

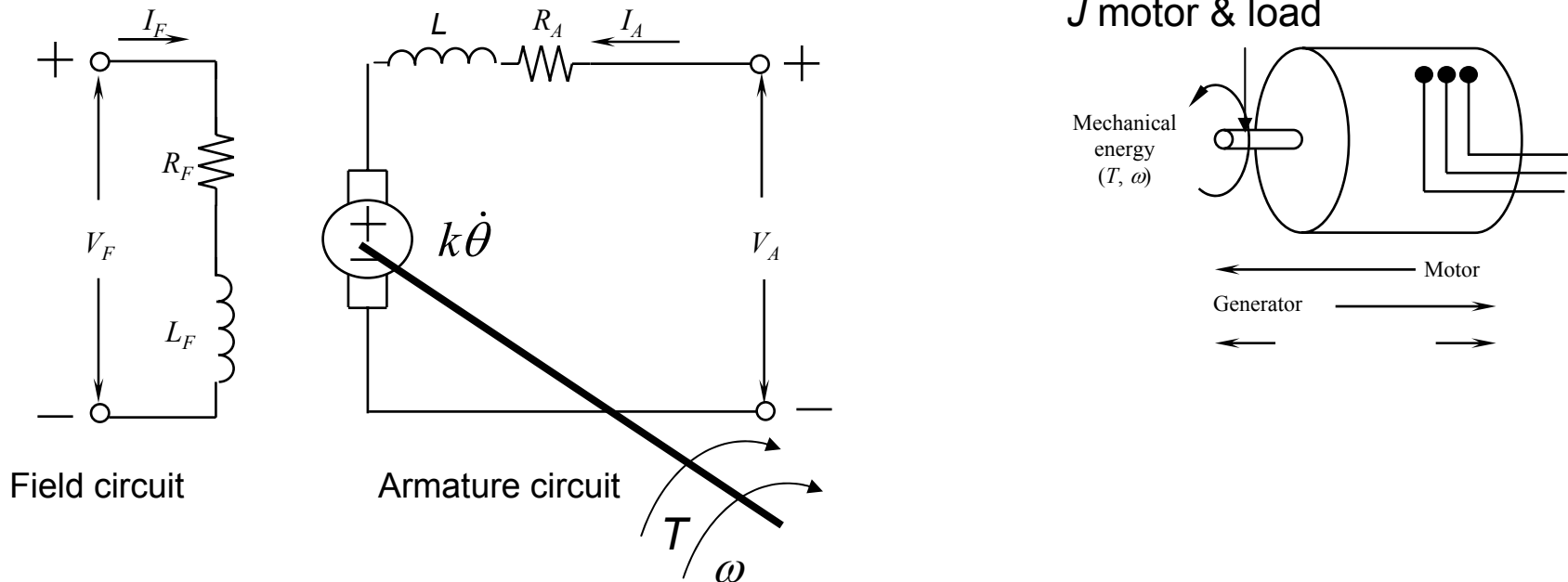
- Electric motors are frequently used as the final control element in positional or speed-control systems. Motors may be classified into two main categories: DC motors and AC motors. Most motors used in modern control systems are DC motors.
- There are five types of DC motors:
  - **Separately-excited DC motors:** It has a separate control of the armature and field currents.
  - **Shunt DC motors:** The armature and field coils are in parallel. It provides lower starting torque.
  - **Series DC motors:** The armature and field coils are in series. It has the highest starting torque.
  - **Compound DC motors:** It has two field windings, one in series with the armature and one in parallel.
  - **Permanent magnet DC motors:** This gives a constant value of flux density. The starting torque is proportional to the applied voltage and the torque decreases with increasing speed.

# Case 1: DC Motor Speed Modeling

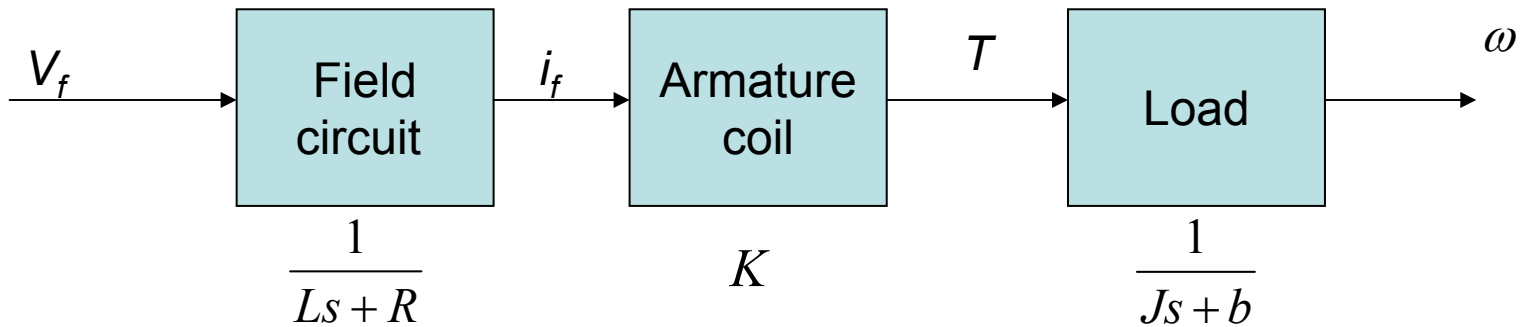
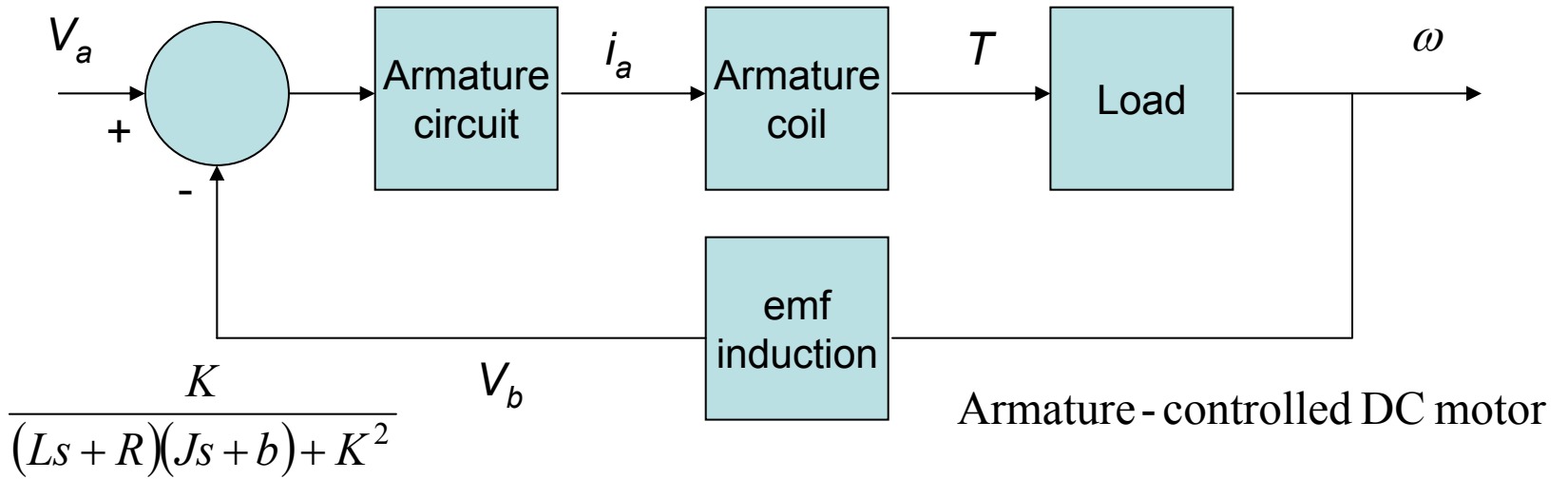
The DC motor has been the workhorse in industry for many reasons including good torque speed characteristics. It is a common actuator in control systems.

It directly provides rotary motion and, coupled with wheels or drums and cables, can provide translational motion. The electric circuit of the armature and the free body diagram of the rotor are shown in the following Figure.

We develop here the transfer function of a separately excited armature controlled DC motor.



# Armature-Controlled and Field-Controlled DC Motors



Field - controlled DC motor : See three subsystems in series

# Compensation

- The output from a system might be unstable or the response is too slow or there is much overshoot.
- Systems may have their responses to inputs altered by including compensators.
- A compensator is a block which is incorporated in the system so that it alters the overall transfer function of the system in such a way to obtain the required characteristics.

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{\frac{K}{s(s+1)}}{1 + \frac{K}{s(s+1)}} = \frac{K}{s(s+1) + K}$$

The denominator is therefore  $s^2 + s + K$ . This will have the roots

$$s = \frac{-1 \pm \sqrt{1 - 4K}}{2}. \text{ To be critically damped we must have } 1 - 4K = 0$$

and therefore the compensator must have the proportional gain of  $K = \frac{1}{4}$



# Physical Parameters

$$T = K_t i; \quad e = K_e \dot{\theta}$$

$$J\ddot{\theta} + b\dot{\theta} = Ki; \quad L \frac{di}{dt} + Ri = V - K\dot{\theta}$$

- Electrical resistance  $R = 1 \Omega$
- Electrical inductance  $L = 0.5 \text{ H}$
- Input voltage  $V$
- Electromotive force constant  $K = 0.01 \text{ nm/A}$
- Moment of inertia of the rotor  $J = 0.01 \text{ kg.m}^2/\text{s}^2$
- Damping ratio of the mechanical system  $b = 0.1 \text{ Nms}$
- Position of the shaft  $\theta$
- The rotor and shaft are assumed to be rigid
- The motor torque  $T$  is related to the armature current  $i$  by a constant  $K_t$
- The back (induced) emf,  $e$ , is related to the rotational velocity

# Speed Control

- **Speed Control by Varying Circuit Resistance:** The operating speed can only be adjusted downwards by varying the external resistance,  $R_{ext}$  which is connected in the armature circuit.

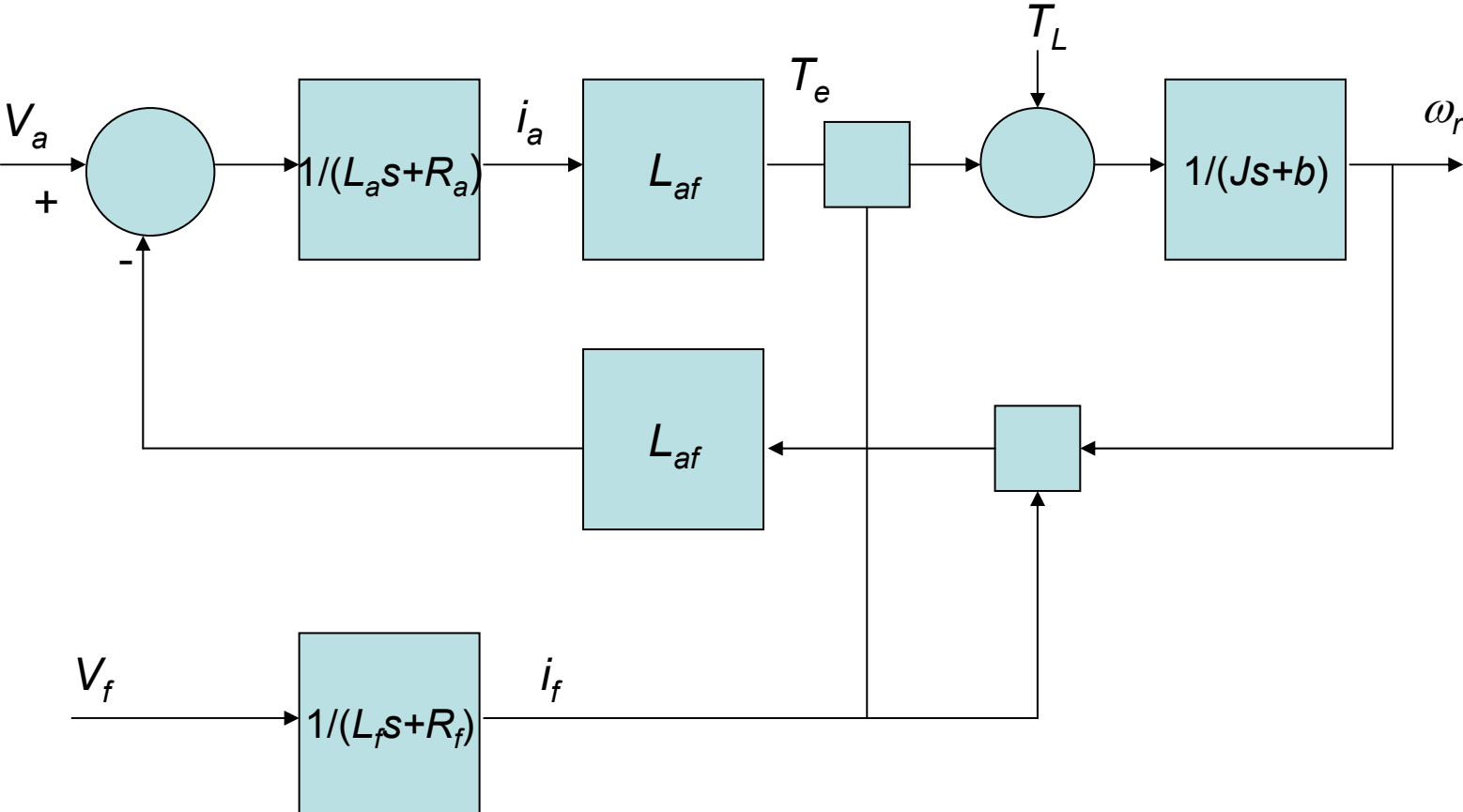
$$\omega_m = \frac{V_a - R_a I_a}{k_a \phi} = \frac{V_a}{k_a \phi} - \frac{(R_a + R_{ext})}{k_a^2 \phi^2} \text{ rad/s}$$

- **Speed Control by Varying Excitation Flux:**

$$\frac{\omega_{m1}}{\omega_{m2}} = \frac{\phi_2}{\phi_1}$$

- **Speed Control by Varying Applied Voltage:** Wide range of control 25:1; fast acceleration of high inertia loads.
- **Electronic Control:** High-speed, reliable, and inexpensive semiconductor devices have produced a dramatic change in the control of DC motors.

# Resistance, Excitation Flux, and Applied Voltage



# Circuitry Dynamics

$$\frac{di_a}{dt} = -\frac{R_a}{L_a}i_a - \frac{L_{af}}{L_a}i_f\omega_r + \frac{1}{L_a}V_a$$

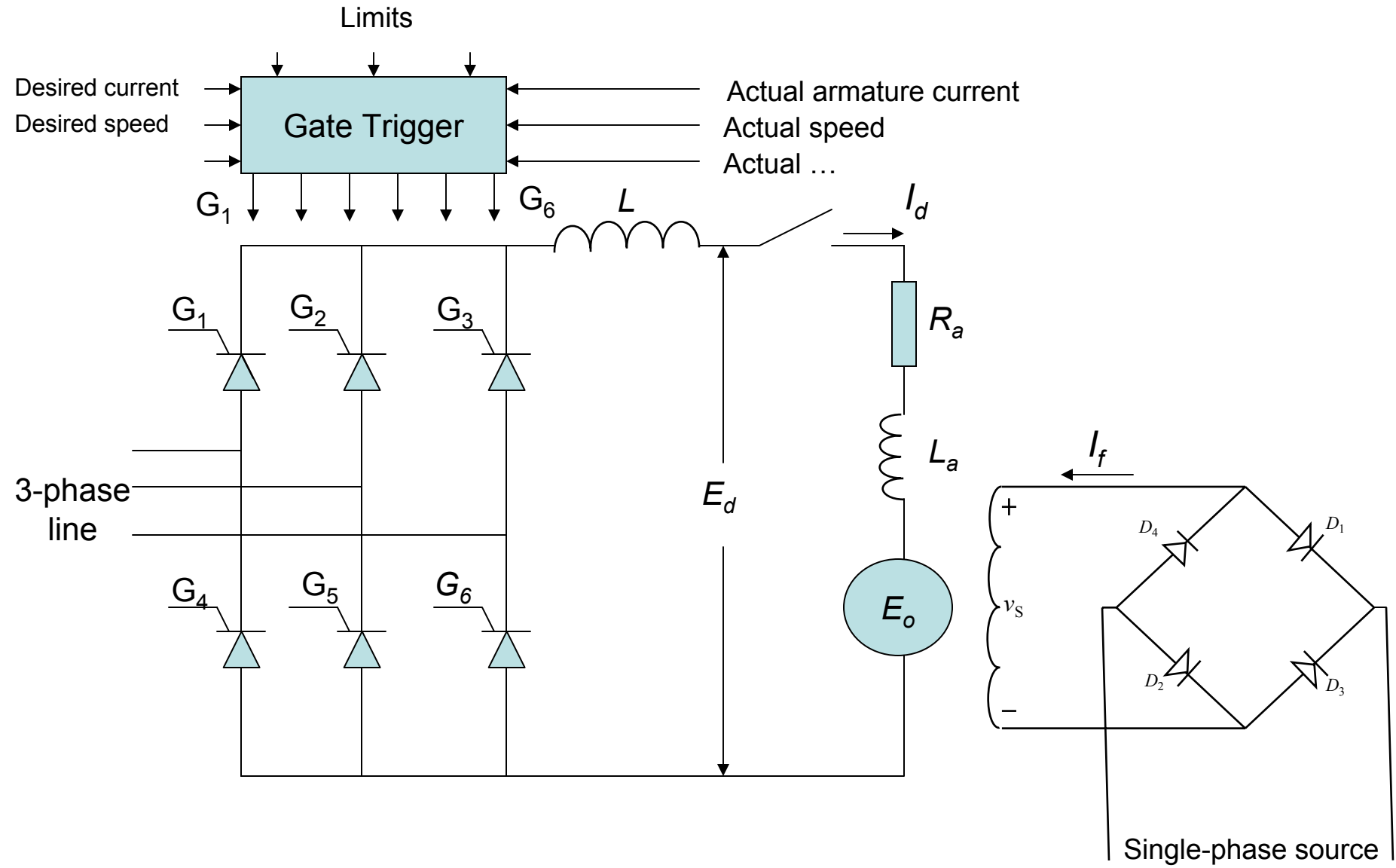
$$\frac{di_f}{dt} = -\frac{R_f}{L_f}i_f + \frac{1}{L_f}V_f$$

$$\frac{d\omega_r}{dt} = \frac{L_{af}}{J}i_a i_f - \frac{B_m}{J}\omega_r - \frac{T_L}{J}$$

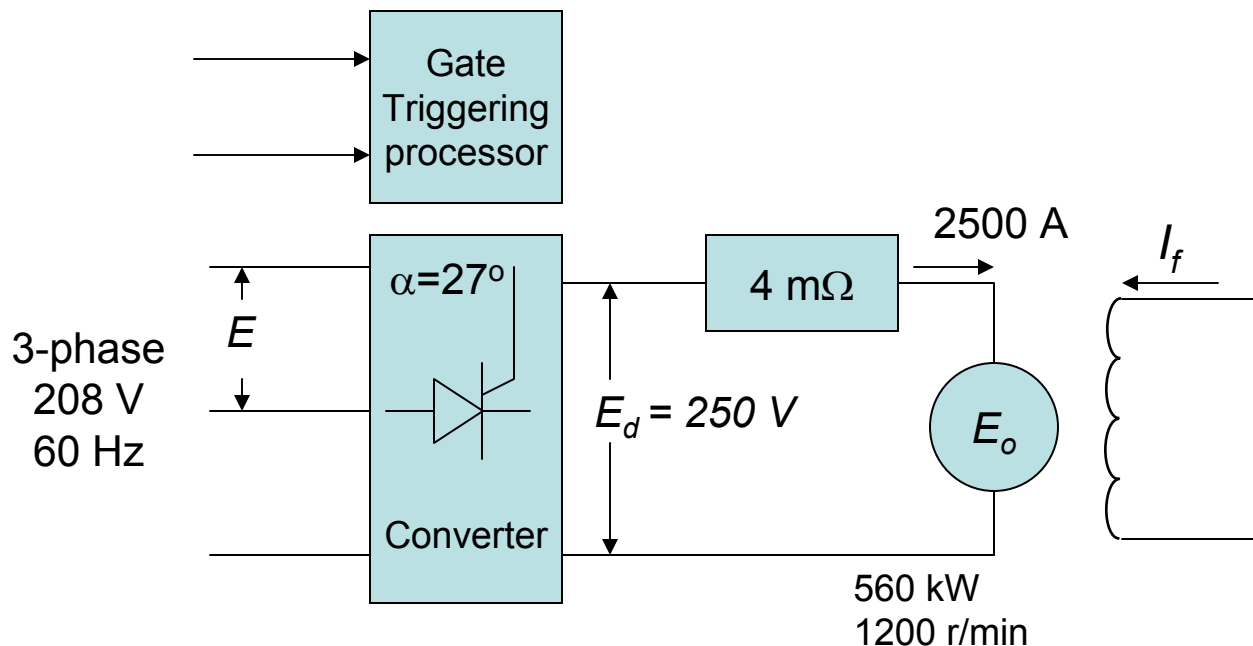
$$\omega_r = \frac{V_a - R_a i_a}{L_{af} i_f} = \frac{V_a}{L_{af} i_f} - \frac{R_a}{(L_{af} i_f)^2} T_e \text{ [steady - state]}$$

The above three equations which were found using Kirchhoff's voltage equation and Newton's second law of motion, as well as the Lagrange equations of motion.

# Armature Torque and Speed Control of a DC Motor using a Thyristor Converter



Example: A 750 hp, 250 V, 1200 r/min DC motor is connected to a 208 V, 3-phase, 60 Hz line using a 3-phase bridge converter as shown. The full-load armature current is 2500 A and the armature resistance is 4 mΩ and the armature resistance is 4 mΩ.



Find the required firing angle  $\alpha$  under rated full - load conditions.

Find the firing angle required so that the motor develops its rated torque at 400 r/min

(a) At full load the converter must develop a DC output of 250 V

$$E_d = 1.35 E \cos \alpha$$

$$250 = 1.35 \times 208 \cos \alpha$$

$$\alpha = 27^\circ$$

Armature IR drop at rated current =  $2500 \text{ A} \times 0.004 \Omega = 10 \text{ V}$

Counter - emf at 1200 r/min ( $E_o$ ) =  $250 - 10 = 240 \text{ V}$

(b) To develop rated torque at 400 r/min, the armature current must still be

be 2500 A. The emf at 400 r/min is :  $E_o = \left( \frac{400}{1200} \right) \times 240 = 80 \text{ V}$

Armature terminal voltage is  $E_d = 80 + 10 = 90 \text{ V}$

The converter must generator 90 V. To determine the firing angle, we have

$$E_d = 1.35 E \cos \alpha = 1.35 \times 208 \cos \alpha$$

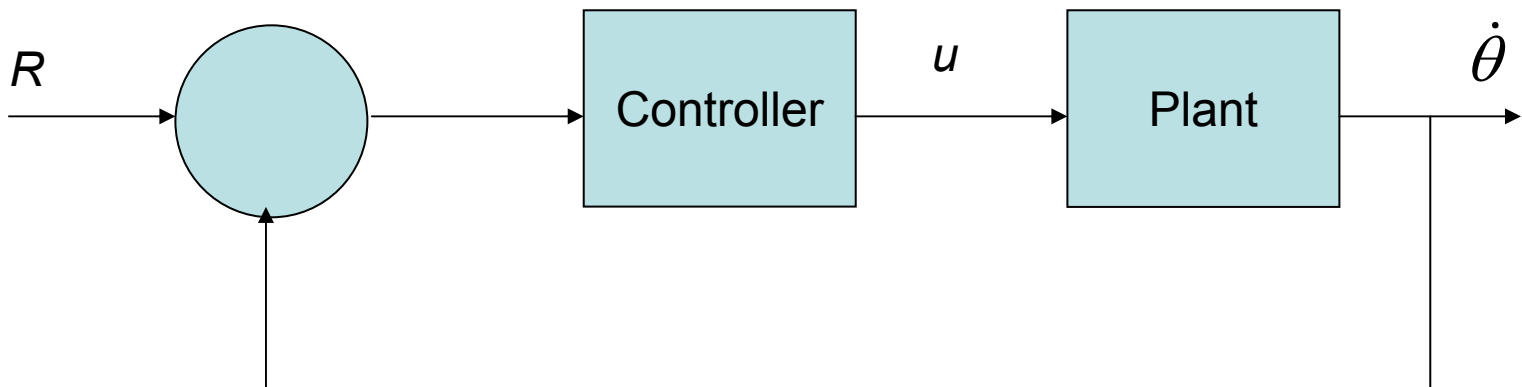
$$\alpha = 71^\circ \text{ (see the Figure in the next page)}$$

# Transfer Function

$$s(Js + b)\theta(s) = KI(s)$$

$$(Ls + R)I(s) = V - Ks\theta(s)$$

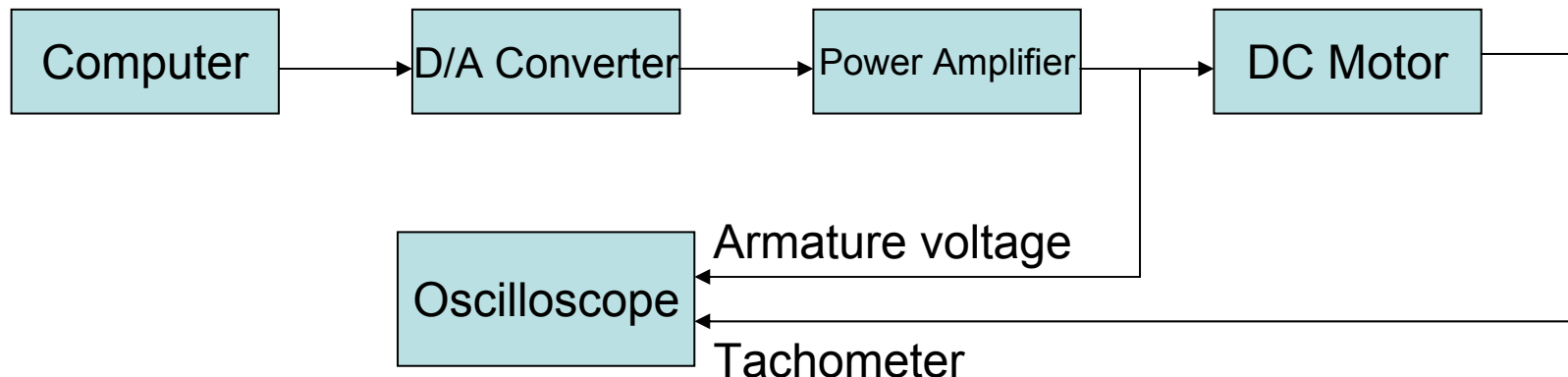
$$\frac{\theta}{V} = \frac{K}{(Js + b)(Ls + R) + K^2}$$





# Data Measurement

- Once we have identified the transfer function of the system we may proceed to the final two phases of the design cycle, the design of a suitable controller and the implementation of the controller on the actual system. In the case of speed control of the DC motor, the control will prove to be quite easy.
- An important point to be highlighted here is that if we have a good model of the plant to be controlled, and we already have identified the parameters of the model, then the design of the controller is easy.



# State Space

Chose the speed and current as the state variables and the voltage as an input. The output is chosen to be the speed.

$$\frac{d}{dt} \begin{bmatrix} \dot{\theta} \\ i \end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & \frac{K}{J} \\ -\frac{K}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} V$$

$$\dot{\theta} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ i \end{bmatrix}$$

# Design Needs and System Performance

- The uncompensated motor may only rotate at 0.1 rad/sec with an input voltage of 1 V. Since the most basic requirement of a motor is that it should rotate at the desired speed, the steady-state error of the motor speed should be less than 1%.
- The other performance requirement is that the motor must accelerate to its steady-state speed as soon as it turns on. In this case, we want it to have a settling time of 2 seconds for example. Since a speed faster than the reference may damage the equipment, we want to have an overshoot of less than 5%. If we simulate the reference input ( $r$ ) by a unit step input, then the motor speed output should have:
  - Settling time less than 2 seconds
  - Overshoot less than 5%
  - Steady-state error less than 1%

Use the MATLAB to represent the open loop response

# PID Design technique for DC Motor Speed Control

- Combining all three modes of control (proportional, integral and derivative) enables a controller to be produced which has no offset error and reduces the tendency for oscillations. Such a controller is called PID controller.

$$I_{out} = K_P e + K_I \int e dt + K_D \frac{de}{dt} + I_o$$

$$(I_{out} - I_o)(s) = K_P E(s) + \frac{1}{s} K_I E(s) + s K_D (s)$$

$$\text{Transfer function} = K_P e + \frac{1}{s} K_I + s K_D$$

- Design a PID controller and add it into the system.
- Recall the transfer function for a PID controller.
- See how the PID controller works in a closed-loop system using the previous Figure. The variable ( $e$ ) represents the tracking error, the difference between the desired input value ( $R$ ) and the actual output ( $y$ ). This error signal ( $e$ ) will be sent to the PID controller, and the controller computes both the derivative and the integral of this error signal.
- The signal ( $u$ ) just past the controller is equal to the proportional gain ( $K_p$ ) times the magnitude of the error plus the integral gain ( $K_I$ ) times the integral of the error plus the derivative gain ( $K_D$ ) times the derivative of the error.

# The Characteristics of P, I, and D Controllers

These correlations may not be exactly accurate, because  $K_p$ ,  $K_i$ , and  $K_D$  are dependent of each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the table should only be used as a reference when you are determining the values for  $K_p$ ,  $K_i$  and  $K_D$ .

Response	Rise Time	Overshoot	Settling Time	SS Error
$K_p$	Decrease	Increase	Small Change	Decrease
$K_i$	Decrease	Increase	Increase	Eliminate
$K_D$	Small Change	Decrease	Decrease	Small Change

# The PID Adjustment Steps

- Use a proportional controller with a certain gain. A code should be added to the end of m-file.
- Determine the closed-loop transfer function.
- See how the step response looks like.
- You should get certain plot.
- From the plot you may see that both the steady-state error and the overshoot are too large.
- Recall from the PID characteristics that adding an integral term will eliminate the steady-state error and a derivative term will reduce the overshoot. Let us try a PID controller with small  $K_I$  and  $K_D$ .
- The settling time is too long. Let us increase  $K_I$  to reduce the settling time.
- Large  $K_I$  will worsen the transient response (big overshoot). Let us increase  $K_D$  to reduce the overshoot.
- See the plot now and see if design requirements will be satisfied.

# Root Locus Design Method for DC Motor Speed Control

## Drawing the Open-Loop Root Locus

- The main idea of root locus design is to find the closed-loop response from the open-loop root locus plot. Then by adding zeros and/or poles to the original plant, the closed-loop response can be modified.
- We need the settling time and the overshoot to be as small as possible. Large damping corresponds to points on the root locus near the real axis. A fast response corresponds to points on the root locus far to the left of the imaginary axis.
- The system may be overdamped and the settling time will be about one second, so the overshoot and settling time requirements could be satisfied.
- The only problem we may see from the generated plot is the steady state error. If we increase the gain to reduce the steady-state error, the overshoot becomes too large. We need to add a lag controller to reduce the steady-state error.



# Adding a Lag Controller

- A first-order lag compensator can be designed using the root locus. A lag compensator in root locus form is given by

$$G(s) = \frac{s - z_o}{s - p_o}$$

- where the magnitude of  $z_o$  is greater than the magnitude of  $p_o$ . A phase-lag compensator tends to shift the root locus to the right, which is undesirable. For this reason, the pole and zero of a lag compensator must be placed close together (usually near the origin) so they do not appreciably change the transient response or stability characteristics of the system.

# How does the Lag Controller Shift the Root Locus to the Right?

- Recall finding the asymptotes of the root locus that lead to the zeros at infinity, the equation to determine the intersection of the asymptotes along the real axis is:

$$\alpha = \frac{\sum \text{poles} - \sum \text{zeros}}{\text{poles} - \text{zeros}}$$

- When a lag compensator is added to a system, the value of this intersection will be a smaller negative number than it was before. The net number of zeros and poles will be the same (one zero and one pole are added), but the added pole is a smaller negative number than the added zero. Thus, the result of a lag compensator is that the asymptotes' intersection is moved closer to the right half plane, and the entire root locus will be shifted to the right.

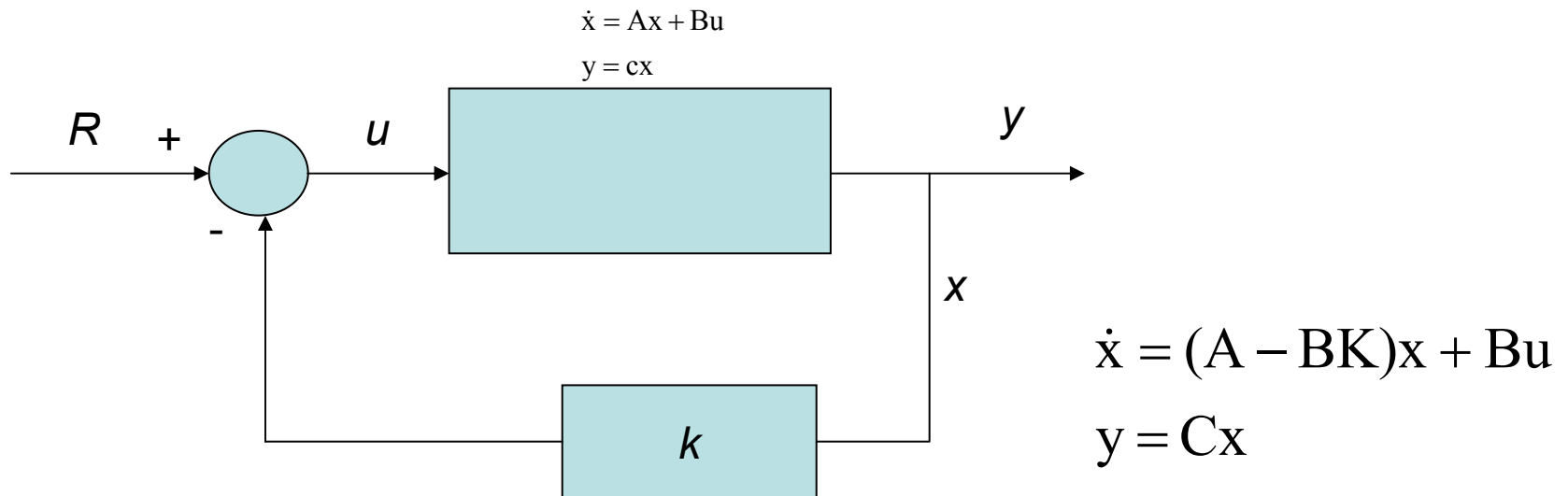
# Frequency Design Method for DC Motor Speed Control

- The idea of frequency-based design is to use the Bode plot of the open-loop transfer function to estimate the closed-loop response. Adding a controller to the system changes the open-loop Bode plot, therefore changing the closed-loop response. First you should draw the Bode plot for the original open-loop transfer function.
- Add proportional gain!
- Plot the closed-loop response.
- The settling time will be fast enough, but the overshoot and the steady state error will be too high. The overshoot can be reduced by reducing the gain a bit to get a higher phase margin, but this would cause the steady-state error to increase. A lag controller is probably needed.
- Add a lag controller to reduce the steady state error.
- At the same time, we should try to reduce the overshoot by reducing the gain.

- Lag controller should only minimally change the transient response because of its negative effect. If the phase-lag compensator is not supposed to change the transient response noticeably, what is it important for?
- The answer is that a phase-lag compensator can improve the system's steady-state response. It works in the following manner. At high frequencies, the lag controller will have unity gain. At low frequencies, the gain will be  $z_o/p_o$  which is greater than 1. This factor  $z_o/p_o$  will multiply the position, velocity, or acceleration constant ( $K_p$ ,  $K_v$ , or  $K_a$ ), and the steady state error will thus decrease by the factor  $z_o/p_o$ .

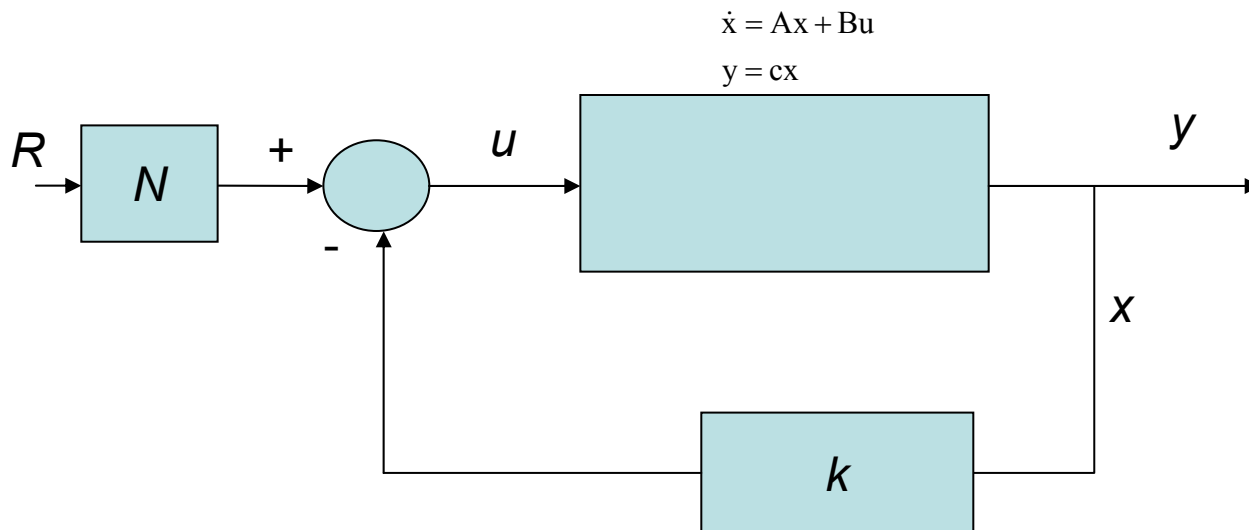
# A State-Space Controller for DC Motor Speed

- We have two state variables in this example. They are easy to measure. An ammeter may measure the current and a tachometer (DC generator) may measure the speed. The schematic for a full state feedback problem is shown.



the characteristic polynomial for this closed - loop system is the determinant of  
 $[sI - (A - BK)]$

- Run the mfile to see the plot.
- Add a reference point. From this plot we may see that the steady-state error is too large. In contrast to the other design methods, where we feed back the output and compare it to the reference to compute an error, here we are feeding back both states. We need to compute what the steady-state value of the states should be, multiply that by the chosen gain  $K$ , and use this new value as our reference for computing the input. This can be done in one step by adding a constant gain after the reference. See the plot after that.



- Since the matrices **A** and **BK** are both 2x2 matrices, there should be 2 poles for the system.
- By designing a full-state feedback controller, we can move these two poles anywhere we want them. We shall first try to place them at  $-5 + i$  and  $-5 - i$  (note that this corresponds to a  $\xi = 0.98$  which gives 0.1% overshoot and a sigma = 5 which leads to a 1 sec settling time).

# Project: Methods of Braking DC Motors

For DC machines, a changeover from motoring to braking may be accomplished by reversing the flow of the motoring armature current,  $I_a$ . In this project we shall examine braking methods on the second quadrant of the speed torque domain in which  $I_a$  and  $T_{em}$  are reversed.

- Use the following parameters: Rated power = 2 kW,  $V_a = 125$ -V,  $I_a = 16$  A,  $R_a = 0.25 \Omega$ ,  $R_f = 111 \Omega$  1750 rev/min, separately excited DC machine.
- Fix the excitation field at the rated condition.
- The inertia of the rotor  $J$  is  $0.5 \text{ kgm}^2$ .
- Ignore brush drop and armature reaction.
- Assume  $I_a$  never to exceed 40 A.



## Plugging

$$k_a \phi = \frac{V_a - R_a I_a}{\omega_m} = 60 \frac{125 - 0.25 \times 16}{2\pi \times 1750} = 0.68 \text{ N - m/A}$$

The developed torque,  $T_{em} = k_a \phi I_a = 10.72 \text{ N - m}$

The internal voltage  $E_b = k_a \phi \omega_m = 122.76 \text{ V}$

$$E_a - (-V_a) = I_a^m (R_{ext} + R_a)$$

Keep  $I_a$  within 250% of its rated value

$$R_{ext} = \frac{122.76 + 125}{2.5 \times 16} - 0.14 = 6 \Omega$$

# Dynamic Braking

Similar equation may be used to compute the limiting resistor for the dynamic braking method, except that  $V_a$  during braking will be zero.

Examine the braking performance of the plugging and dynamic braking methods assuming that the load torque behaves in a manner described by

$$T_{load} = -T_{mech} = 0.01 + 3.189e^{-4} \omega_m^2$$

# Regenerative Braking

Use the SIMULINK of a simple closed loop speed system that can be used to demonstrate the raising, holding, and lowering operations.

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