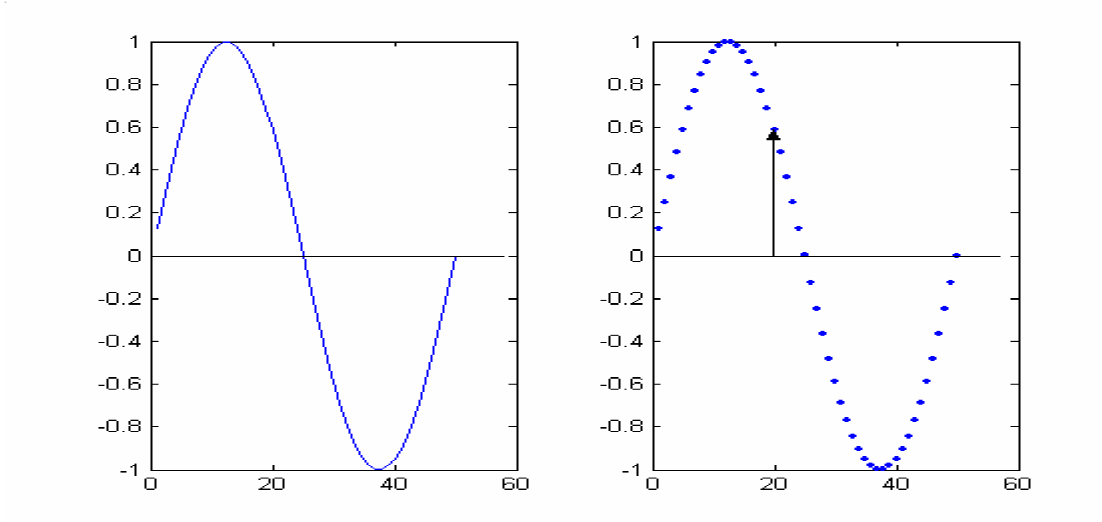
Chapter 13: Digital control systems

1. AD /DA converter



continues signal $r(t) = \sin(2\pi \ ft)$, discrete signal $r(nT) = \sin(2\pi fnT)$, $n = 1,2,3...$, a sequence of values occurred at time *nT*.
The output signal of an ideal sampler is $r^*(t) = r(nT)\delta(t - nT)$, $\delta(t - nT)$ is the impulse happened at time instant $t - nT = 0$,i.e. $t = nT$

2. s transform and z transform (Chapter 2 and Chapter 13): to linearize the non-linear operation, for example, in s domain serial connection

$$F(s) = \int_{0^-}^{\infty} f(t)e^{-st}dt \qquad \xrightarrow{\text{define } z = e^{sT}\text{, integral to summation}} \qquad X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

$$f(t) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} F(s)e^{st}ds \qquad\qquad x(n) = \frac{1}{2\pi j}\oint_c X(z)z^{n-1}dz$$

Table 13.1 summary of s and z transform

3. operations in z domain

transfer function in z domain is defined as $G(z) = \dfrac{Y(z)}{R(z)}$;

and the closed loop feedback system is $T(z) = \dfrac{Y(z)}{R(z)} = \dfrac{G(z)H(z)}{1 \pm G(z)H(z)}$

4. stability analysis in the z-plane

$$z = e^{sT} = e^{(\sigma + j\omega)T} = e^{\sigma T}e^{j\omega T} = e^{\sigma T}(\cos(\omega T) + j\sin(\omega T)) = e^{\sigma T}\angle\omega T$$

In the left-hand s-plane, $\sigma < 0$, therefore the related magnitude of z varies between 0 and 1. So the discrete system is stable if all the poles of the closed-loop transfer function *T(z)* **lie within the unit circle of the z-plane**.

5. examples:

$$x(t) = \delta(t),\ x(n) = \delta(nT),\ X(z) = \sum_{-\infty}^{\infty} x(n)z^{-n} = \sum_{-\infty}^{\infty} \delta(nT)z^{-n} = \delta(0)z^{-0} + 0 = 1$$

$$x(t) = \delta(t - kT),\ x(n) = \delta(nT - kT),$$

$$X(z) = \sum_{-\infty}^{\infty} x(n)z^{-n} = \sum_{-\infty}^{\infty} \delta(nT - kT)z^{-n} = \delta(kT - kT)z^{-k} + 0 = z^{-k}$$

$$x(t) = u(t),\ x(n) = u(nT),\ X(z) = \sum_{-\infty}^{\infty} x(n)z^{-n} = \sum_{-\infty}^{\infty} u(nT)z^{-n} = \sum_{0}^{\infty} 1z^{-n} = \frac{1}{1 - z^{-1}}$$

$$x(t) = e^{-at},\ x(n) = e^{-anT},\ X(z) = \sum_{-\infty}^{\infty} x(n)z^{-n} = \sum_{-\infty}^{\infty} e^{-anT} z^{-n} = \frac{1}{1 - e^{-aT} z^{-1}}$$

example 13.3

$$G(s) = G_0(s)G_p(s) = \frac{1 - e^{-sT}}{s}\frac{1}{s(s+1)} = (1 - e^{-sT})(\frac{k_1}{s^2} + \frac{k_2}{s} + \frac{k_3}{s+1})$$

$G(z) = (1 - z^{-1})\mathrm{Z}(k_1 t + k_2 u(t) + k_3 e^{-t})$, by checking the transform summarized in Table 13.1, we get

$$G(z) = (1 - z^{-1})(k_1 \frac{Tz}{(z+1)^2} + k_2 \frac{z}{z-1} + k_3 \frac{z}{z - e^{-T}}).$$

Where $k_1 = \frac{1}{s \cdot s(s+1)} s^2 |_{s=0} = \frac{1}{s+1}|_{s=0} = 1$

$$k_2 = \frac{\partial}{\partial s}(\frac{1}{s \cdot s(s+1)} s^2)|_{s=0} = \frac{-1}{(s+1)^2}|_{s=0} = -1$$

$$k_3 = \frac{1}{s \cdot s(s+1)}(s+1)|_{s=-1} = \frac{1}{s^2}|_{s=-1} = 1$$

6. The root locus of digital systems (Chapter 7 and Section 13.10)

For a system with the transfer function $T(s) = \frac{Y(s)}{R(s)} = \frac{p(s)}{q(s)}$

The roots of the characteristic equation *q(s)* determine the modes of the response of the system. The root locus is the path of the roots of the characteristic equation traced out in the s-plane as a system parameter k is changed.

First, whatever the system transfer function is, the root locus traced out the root path with respect to only one parameter we are interested. So the characteristic equation should be

rearrange into the form $\Delta(s) = q(s) = 1 + kP(s)$, where k is the parameter we are interested. And the corresponding transfer function is $T(s) = \dfrac{kP(s)}{1 + kP(s)}$. The polynomial P(s) is called root locus function. When we use Matlab to plot the root locus of system T(s), we need to use the specification as sys=P(s), **rlocus**(sys).
If the system T(s) includes several parameters, like *k1,k2,k3…*, we need to assume the relation among the parameters according to the given information and only concern the design on one parameter.
In Chapter 7, the 12-step method of s-plane root locus is explained. Also in Chapter 13, the similar method is introduced for z-plane root locus.
In chapter 7, **Table 7.7**, root locus plots for typical transfer functions are summarized.

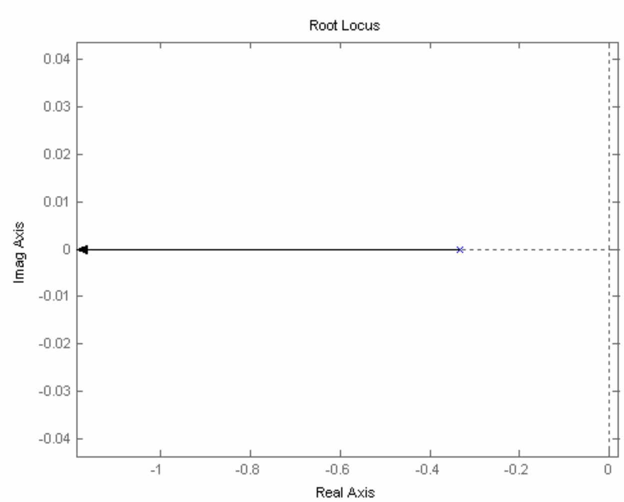
For example, the first : $\dfrac{k}{s\tau_1 + 1}$, $\qquad T(s) = \dfrac{\dfrac{k}{s\tau_1 + 1}}{1 + \dfrac{k}{s\tau_1 + 1}} = \dfrac{k}{s\tau_1 + 1 + k}$

k=0, root is $s = -\dfrac{1}{\tau_1}$; k=1, root is $s = -\dfrac{2}{\tau_1}$

k=10, root is $s = -\dfrac{11}{\tau_1}$; k=∞, root is $s = -\infty$

when k increases from 0 to infinity, the root starts from $s = -\dfrac{1}{\tau_1}$ to $s = -\infty$.

For example, with $\tau_1 = 3$, k=0, root is $s = -\dfrac{1}{\tau_1} = -0.3333$



Any point on the root locus is a root of the characteristic equation corresponding to a specified parameter k.

Example 12.13, 13.9, and 13.11.