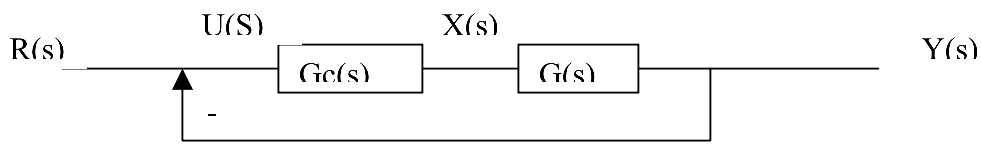
Chapter 13: Design of a digital controller

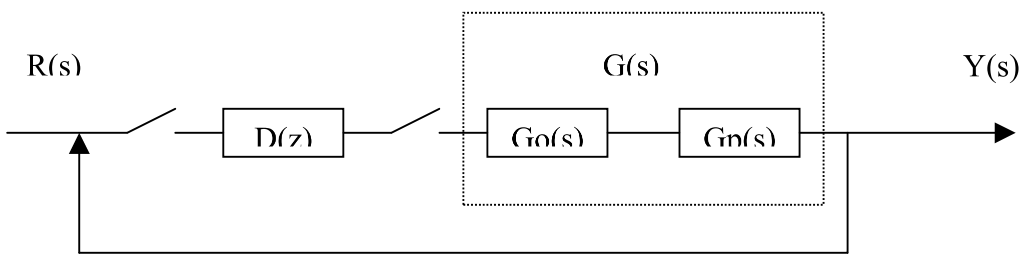
Method 1: a. design the controller Gc(s) in s domain;

b. convert the Gc(s) to z domain D(z).

Method 2: a. convert the given s domain plant G(s) into z domain;

b. design the digital controller in z domain directly.





This week we explain the first method.

Step b: convert s domain Gc(s) to z domain D(z):

Section 13.11, if $Gc(s) = k_1 + \frac{k_2}{s} + k_3 s = \frac{X(s)}{U(s)}$, then

$$U(s) = (k_1 + \frac{k_2}{s} + k_3 s)X(s),$$

if U(s) =X(s), U(z)=X(z);

if $U(s) = \frac{1}{s}X(s)$, then $u(t) = \int_0^t x(\tau)d\tau$, $\longrightarrow$

in discrete notation, $u(k) = u(kT) = u((k-1)T) + Tx(kT)$ $\longrightarrow$ $U(z) = z^{-1}U(z) + TX(z)$

i.e. $U(z) = \frac{T}{1 - z^{-1}}X(z)$

if $U(s) = sX(s)$, then $u(t) = \frac{dx(t)}{d(t)}$, in discrete notation, $u(kT) = \frac{x(kT) - x((k-1)T)}{T}$,

$$\longrightarrow \quad U(z) = \frac{X(z) - z^{-1}X(z)}{T} = \frac{1 - z^{-1}}{T}X(z)$$

so, $D(z) = Z(Gc(s)) = (k1 + k2\frac{Tz}{z-1} + k3\frac{z-1}{Tz}X(s)$

section 13.8: if $Gc(s) = k\frac{s+a}{s+b}$, corresponding $D(z) = C\frac{z-A}{z-B}$, with

$A = e^{-aT}$, $B = e^{-bT}$, $C = k\frac{a}{b}\frac{1-B}{1-A}$

Step a: design the controller Gc(s) in s domain

Gc(s) can be designed using the method explained in Chapter 10,11, and 12.

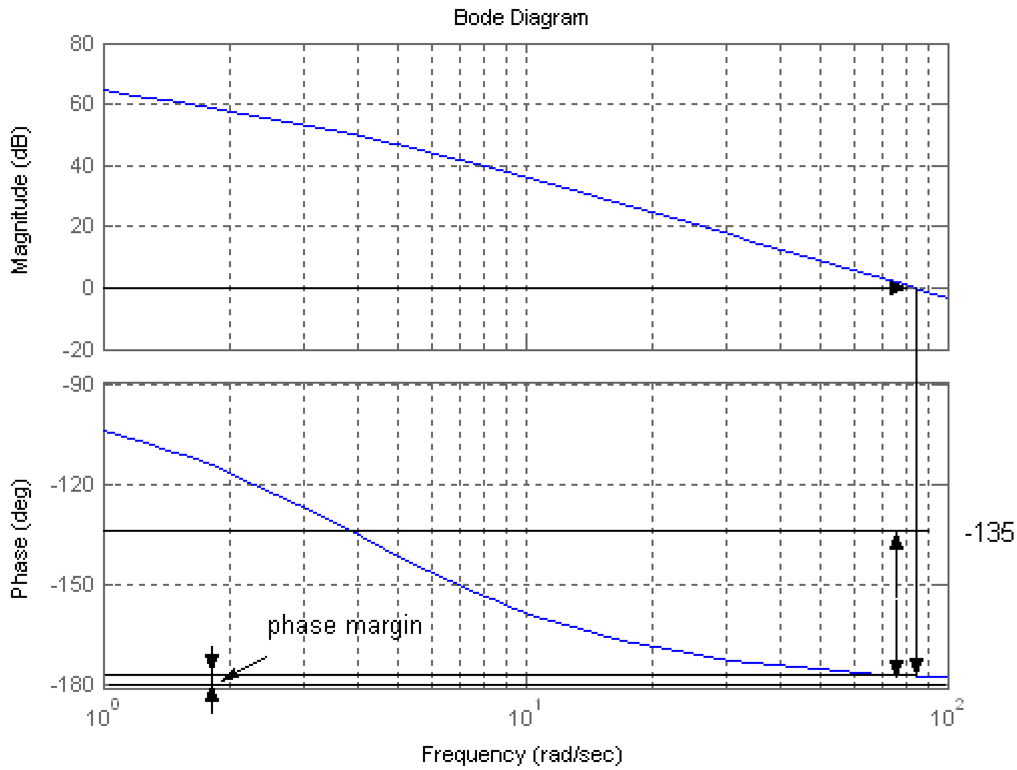
Review the design of a phase lead controller in Chapter 10 and explain the example 13.5 in details.

1. review the concept of phase margin (Chapter 7):

Definition: The phase margin is the amount of phase shift of the GH(jw) at unity magnitude that will result in a marginally stable system with intersection of the –1+j0 point on the Nyquist diagram.

It means phase margin is the phase shift needed to 180 or –180 at the frequency where the magnitude of GH(jw) is one, i.e. 0 dB, or it is 180 or –180 minus the phase 180 at the frequency where the magnitude of GH(jw) is 0dB . Using the Bode plot to explain the phase margin is shown in the following figure.

For example: $GH(s) = \frac{1740}{s(0.25s+1)}$

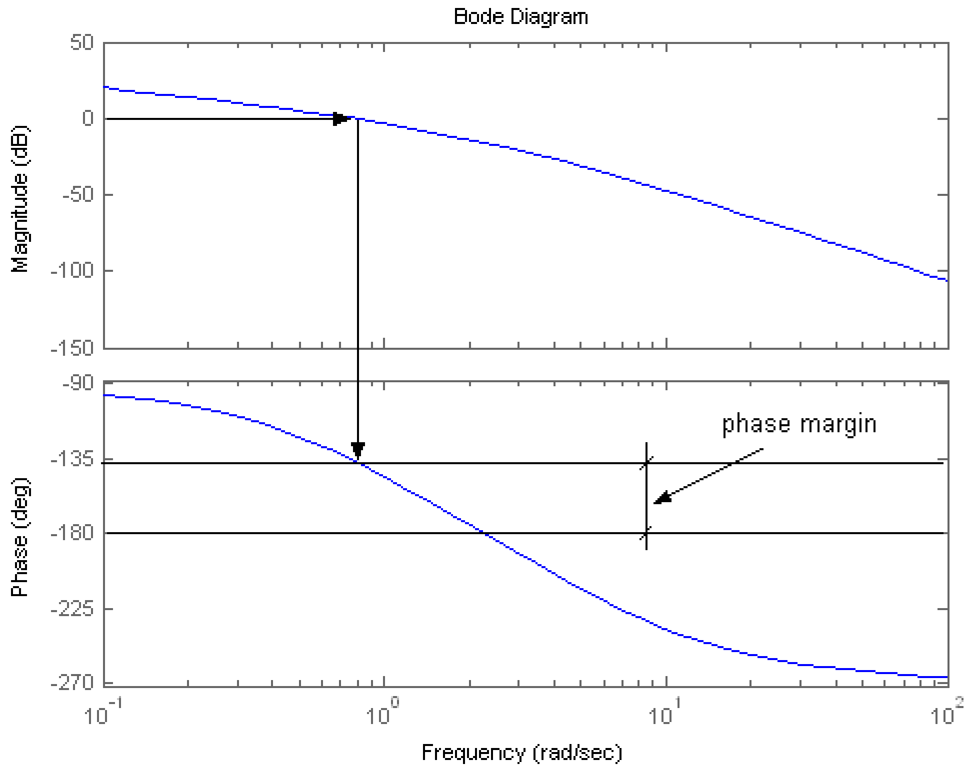


pm=2.7466=(-177.3)-180.   usually, we define phase margin within (-180 180).

the preferred closed-loop phase margin is greater than 45, we need phase compensation about at lest 45-2=43.

Another example from section 9.4

$$GH(s) = \frac{1}{s(s+1)(0.2s+1)} = \frac{1}{0.2s^3 + 1.2s^2 + s}$$



2. review the phase-lead design using phase margin (Section 10.3 and 10.4):

$$Gc(s) = k\frac{s-a}{s-b}\text{, i.e., } Gc(j\omega) = k\frac{j\omega + z}{j\omega + p} = k\frac{z}{p}\frac{j\frac{\omega}{z}+1}{j\frac{\omega}{p}+1} = k\frac{1}{\alpha}\frac{1+j\omega\alpha\tau}{1+j\omega\tau}\text{, where}$$

$$a = \frac{p}{z},\ \tau = \frac{1}{p}.$$

The magnitude is $|Gc(j\omega)| = \frac{k}{a}\sqrt{\frac{1+(\omega\alpha\tau)^2}{1+(\omega\tau)^2}}$, (Eq 1)

The phase is $\angle Gc(j\omega) = \tan^{-1}(\omega\alpha\tau) - \tan^{-1}(\omega\tau)$ (Eq 2)

Or we can write the phase in another formular,

$$Gc(j\omega) = k\frac{1}{\alpha}\frac{1+j\omega\alpha\tau}{1+j\omega\tau} = \frac{k}{\alpha}\frac{(1+j\omega\alpha\tau)(1-j\omega\tau)}{1-(\omega\tau)^2} = \frac{k}{a}\frac{1+\omega^2\alpha\tau^2 + j(\omega\alpha\tau - \omega\tau)}{1-(\omega\tau)^2}$$

The phase is $\angle Gc(j\omega) = \tan^{-1}\dfrac{\omega\alpha\tau - \omega\tau}{1+\omega^2\alpha\tau^2}$ (Eq 3)

The bode diagram of the general phase-lead compensator is shown in textbook figure 10.3, or reference to the figure in the last page of this notes. The maximum value of the phase lead occurs at a frequency ω_m, and $\omega_m = \sqrt{zp} = \dfrac{1}{\tau\sqrt{\alpha}}$.

It should also be noted that at ω_m, the Gc(s) add additional gain about

$$20\log_{10}\alpha/2 = 10\log_{10}\alpha.$$

At frequency ω_m, the corresponding maximum phase is

$\Phi_m = \angle Gc(j\omega)|_{\omega=\omega_m} = \tan^{-1}\dfrac{\omega\alpha\tau - \omega\tau}{1+\omega^2\alpha\tau^2} = \dfrac{\alpha-1}{2\sqrt{\alpha}}$, or it is equivalent to $\sin\Phi m = \dfrac{\alpha-1}{\alpha+1}$ (Eq 4).

So if we can derive the requested phase compensation Φ_m, then the can be calculated from (Eq 4).

Design step is expressed in Section 10.4:

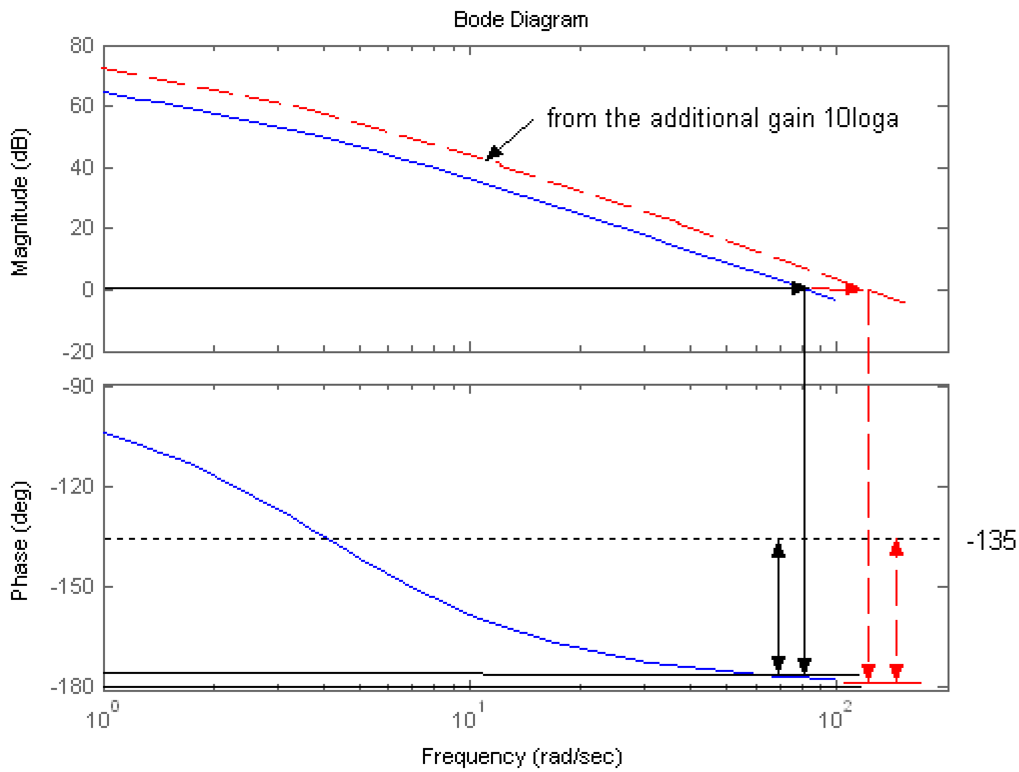
1. plot the Bode diagram of the uncompensated G(s), find the phase margin of G(s);
2. calculate the maximum phase lead Φ_m,
3. using Eq 4calculated α;
4. calculate $10\log_{10}\alpha$ and determine the frequency where the uncompensated magnitude curve is equal to -$10\log_{10}\alpha$. **Because the compensation network provide a gain of $10\log_{10}\alpha$ at ω_m.** This frequency is the new 0 dB crossover frequency and ω_m simultaneously.
5. using ω_m and α to calculate z and p using $a = \dfrac{p}{z}$, $\tau = \dfrac{1}{p}$. $\omega_m = \sqrt{zp}$, i.e.

   $p = \omega_m\sqrt{\alpha}$ and $z = p/a$.

6. k is calculated to yield $|GcG| = 1$

Example 13.5: $GH(s) = \dfrac{1740}{s(0.25s+1)}$

As shown previously, the phase margin of the uncompensated system is 2.7, to achieve the closed-loop phase margin greater than 45, we need additional phase lead Φ_m >45-2.7;



the preferred closed-loop phase margin is greater than 45, we need phase compensation about at lest 45-2=43. But considering that $Gc(s) = \dfrac{s-a}{s-b}$ adds additional gain about $10\log_{10}\alpha$ to the system, to be safely, we need larger Φ_m as illustrated in above figure. So we chose =46.

$\alpha = \dfrac{1+\sin\Phi_m}{1-\sin\Phi_m}$=6.25. At the end, we need verify the performance, if it does not achieve required performance, we need to pick a larger Φ_m

$10\log_{10}\alpha$ =7.9588 dB

$$|Gp(j\omega)| = \frac{1740}{\sqrt{(0.25\omega^2)^2 + \omega^w}} = 10^{-8/20} = 0.4$$, then we get $\omega = 125$

so z=50, p=312.

$$Gc(s) = k\frac{s+50}{s+312},$$

$$GcGp(s) = k\frac{s+50}{s+312}\frac{1740}{s(0.25s+1)},$$

magnitude is $$|k\frac{j\omega+50}{j\omega+312}\frac{1740}{(-0.25\omega^2 + j\omega)}| = k\frac{\sqrt{\omega^2+2500}}{\sqrt{\omega^2+312^2}}\frac{1740}{\sqrt{(0.25\omega^2)^2+\omega^2}} = 1$$

i.e. $k\frac{\sqrt{\omega^2+2500}}{\sqrt{\omega^2+312^2}}0.4 = 1$, $k = \frac{1}{2.5\bullet 0.44} = 5.67$