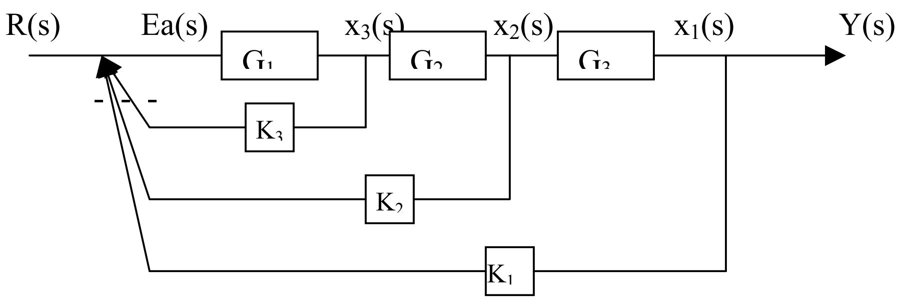
DGD notes:

How to get the characteristic equation form given information:

1. given the transfer function of the system $G(s) = \frac{p(s)}{q(s)}$: the denominator polynomial $q(s)$ is the characteristic equation.
2. given the estate variable model of the system, $\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}$. The characteristic equation is $\det([sI - A])$.
3. given the block diagram of the control system with control parameter K:

example of Ap11.1



$$G_1 = \frac{2k}{s+4}, \quad G_2 = \frac{2}{s+1}, \quad G_3 = \frac{1}{s}$$

method a: using the algebra of the diagram variables:

$$Ea(s) = R(s) - u(s) = R(s) - (k3\frac{2k}{s+4} + k2\frac{2k}{s+4}\frac{2}{s+1} + k1\frac{2k}{s+4}\frac{2}{s+1}\frac{1}{s})Ea(s)$$

$$Ea(s)(1 + (k3\frac{2k}{s+4} + k2\frac{2k}{s+4}\frac{2}{s+1} + k1\frac{2k}{s+4}\frac{2}{s+1}\frac{1}{s})) = R(s)$$

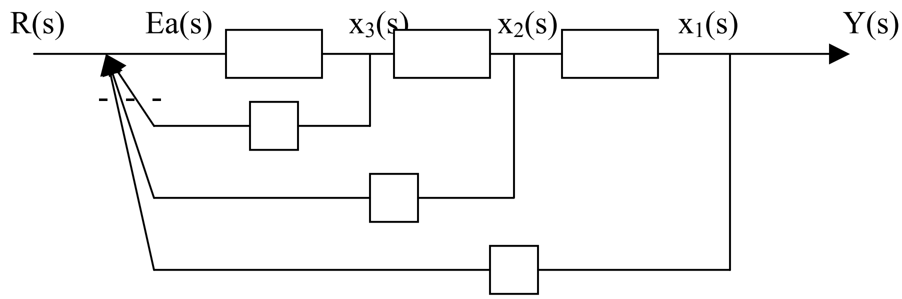
$$Y(s) = (\frac{2k}{s+4}\frac{2}{s+1}\frac{1}{s})Ea(s)$$

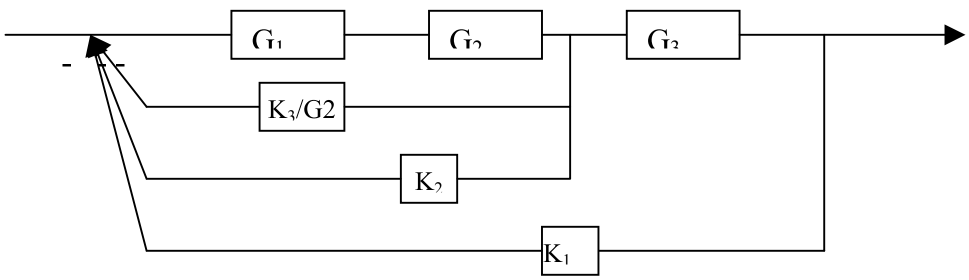
so, $R(s) = (1 + k3\frac{2k}{s+4} + k2\frac{2k}{s+4}\frac{2}{s+1} + k1\frac{2k}{s+4}\frac{2}{s+1}\frac{1}{s})\frac{(s+4)(s+1)s}{4k}Y(s)$

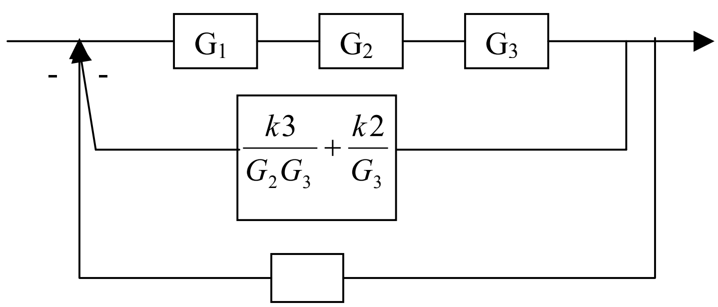
and the transfer function is $T(s) = \frac{Y(s)}{R(s)} = \frac{4k}{(s+4)(s+1)s + 4kk_3(s+1)s + 2k_2s + 4kk_1}$

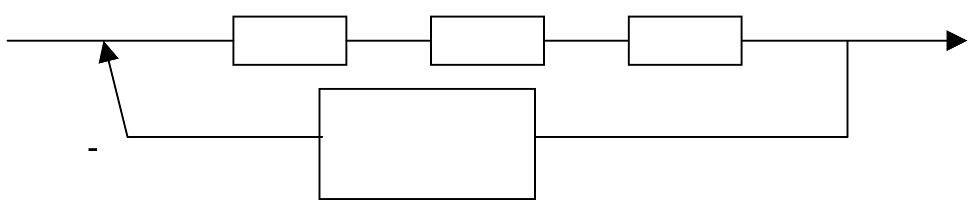
method b:

use table 2.6









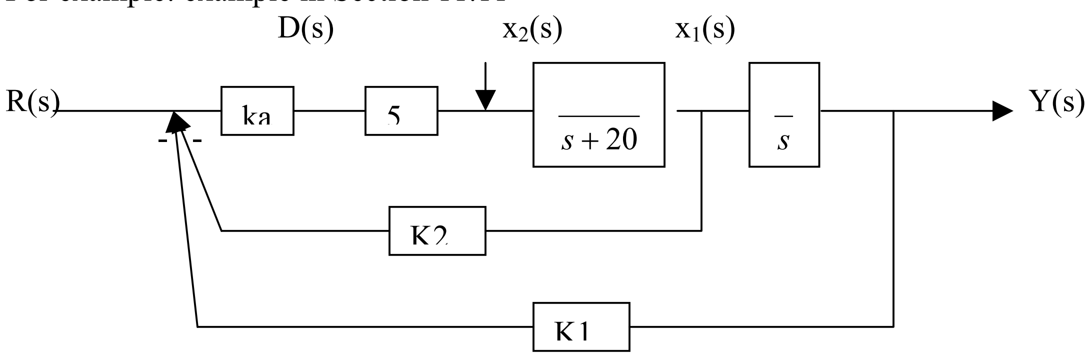
$$\frac{k3}{G_2G_3}+\frac{k2}{G_3}+k1$$

$$T(s)=\frac{Y(s)}{R(s)}=\frac{G}{1+GH}=\frac{G_1G_2G_3}{1+G_1G_2G_3(k_1+\frac{k_2}{G_3}+\frac{k_3}{G_2G_3})}$$

$$=\frac{G_1G_2G_3}{1+G_1G_2G_3k_1+G_2G_3k_2+G_1k_3}=\frac{4k}{(s+4)(s+1)s+4kk_3(s+1)s+2k_2s+4kk_1}$$

method c:
using signal-flow graph:
$T(s) = \dfrac{Y(s)}{R(s)} = \dfrac{p}{\Delta}$, p the path relate the output to input, in this problem $p = G_1 G_2 G_3$,
$\Delta = 1 - \sum Ln.....$, $\sum Ln$, sum of all different loop gain,
$\sum Ln = -k_3 G_1 - k_2 G_1 G_2 - k_1 G_1 G_2 G_3$,

For example: example in Section 11.11



Method b:

$$T(s) = \frac{Y(s)}{R(s)} = \frac{ka \cdot 5 \cdot \frac{1}{s+20} \cdot \frac{1}{s}}{1 + k2 \cdot ka \cdot 5 \cdot \frac{1}{s+20} + k1 \cdot ka \cdot 5 \cdot \frac{1}{s+20} \cdot \frac{1}{s}}$$

$$\frac{5 \cdot ka}{(s+20)s + 5k2 \cdot ka \cdot s + 5k1 \cdot ka} = \frac{5 \cdot ka}{s^2 + (20 + 5ka \cdot k2)s + 5k1 \cdot ka}$$

use state variable model:

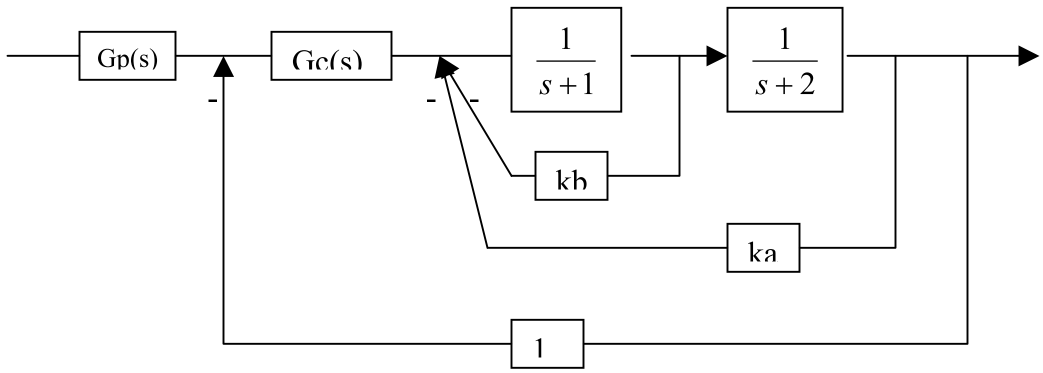
$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -20 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 5ka \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases}$$, let $u = \begin{bmatrix} -k1 & -k2 \end{bmatrix} x$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -20 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 5ka \end{bmatrix} \begin{bmatrix} -k1 & -k2 \end{bmatrix} x(t) = \begin{bmatrix} 0 & 1 \\ -5k1 \cdot ka & -5k2 \cdot ka - 20 \end{bmatrix} x(t),$$

characteristic equation $\Delta = \det([sI - H]) = \det(\begin{bmatrix} s & -1 \\ 5k1 \cdot ka & s + 5k2 \cdot ka + 20 \end{bmatrix}$

$$= s(s + 5k2 \cdot ka + 20) + 5k1 \cdot ka = s2 + (20 + 5k2 \cdot ka)s + 5k1 \cdot ka .$$

Ex12.11



$$T(s) = \frac{\omega_n^3}{s^3 + a\omega_n s^2 + \beta\omega_n^2 s + \omega_n^3}.$$

$$T(s) = Gp(s)\frac{Gc(s)\dfrac{\frac{1}{s+1}\frac{1}{s+2}}{1+kb\frac{1}{s+1}+ka\frac{1}{s+1}\frac{1}{s+2}}}{1+Gc(s)\dfrac{\frac{1}{s+1}\frac{1}{s+2}}{1+kb\frac{1}{s+1}+ka\frac{1}{s+1}\frac{1}{s+2}}} =$$

$$= Gp(s)\frac{\dfrac{s^2k_3+k_1s+k_2}{s}\dfrac{1}{(s+1)(s+2)+kb(s+2)+ka}}{1+\dfrac{s^2k_3+k_1s+k_2}{s}\dfrac{1}{(s+1)(s+2)+kb(s+2)+ka}}$$

$$= Gp(s)\frac{k_3s^2+k_1s+k_2}{s^3+(3+ka+kb+k_3)s^2+(2+2kb+ka+k1)s+k2}$$

if the response desired is deadbeat (table 10.2), then use a third order transfer function as

$T(s) = \frac{\omega_n^{\ 3}}{s^3 + a\omega_n s^2 + \beta\omega_n s + \omega_n^{\ 3}}$, with α=1.90, β=2.20, $Ts = 4.04/\omega_n$, for a settling time $Ts = 4.04/\omega_n < 0.5$, we can choose $\omega_n = 8.08$, then we can solve the parameter k.

if the performance is specified by ITAE, use table 5.6, the optimal coefficients of the characteristic equation for ITAE is $s^3 + 1.75\omega_n s^2 + 2.15\omega_n s + \omega_n^{\ 3}$. ω_n can be found from given information and Figure 5.30 (c), then we can solve the problem. Also see example 12.8. 3 third order system normalized Ts=8, normalized Tp=4.

Chapter 12: design o f P, PI, PD, and PID controllers

1) Structure of P, PI, PID controllers. P controller: $Gc(s) = K$;

PI controller: $Gc(s) = K_1 + \frac{K_2}{s}$;

PD controller: $Gc(s) = K_1 + K_3 s$

PID controller: $Gc(s) = K_1 + \frac{K_2}{s} + K_3 s$.

2) Robust control system and system sensitivity $S_a^T = \frac{\partial T / T}{\partial a / a} = (\frac{\partial T}{\partial a}) * \frac{a}{T}$,

Example: $T(s) = \frac{k}{s^2 + s + k}$,

3) System stable with uncertain parameters;

use Routh_Hurwitz criterion:

Example: $T(s) = \frac{k}{s^2 + (2+k)s + (6-k)}$

4) PID controllers;