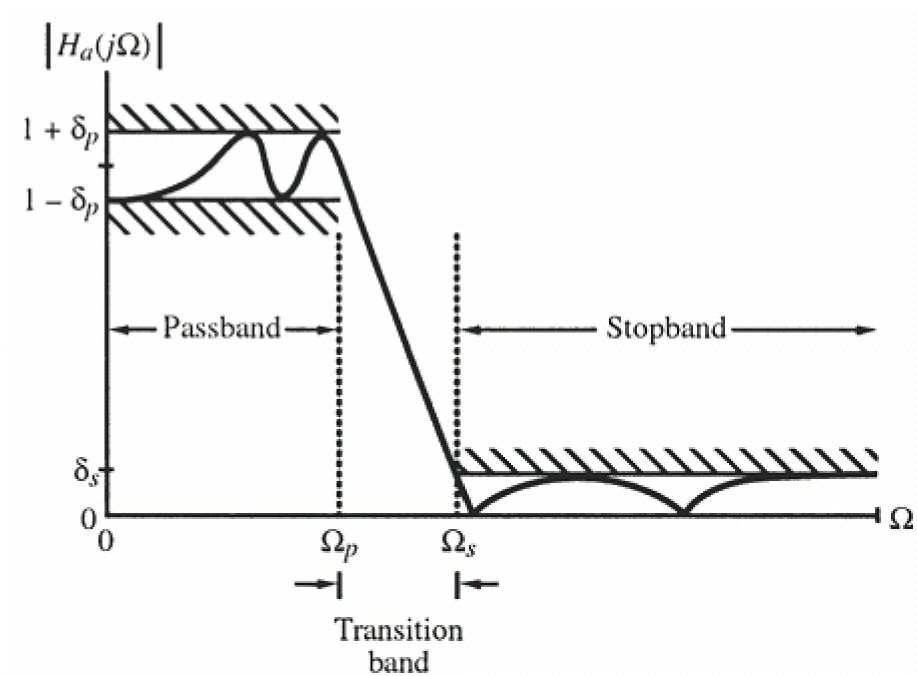
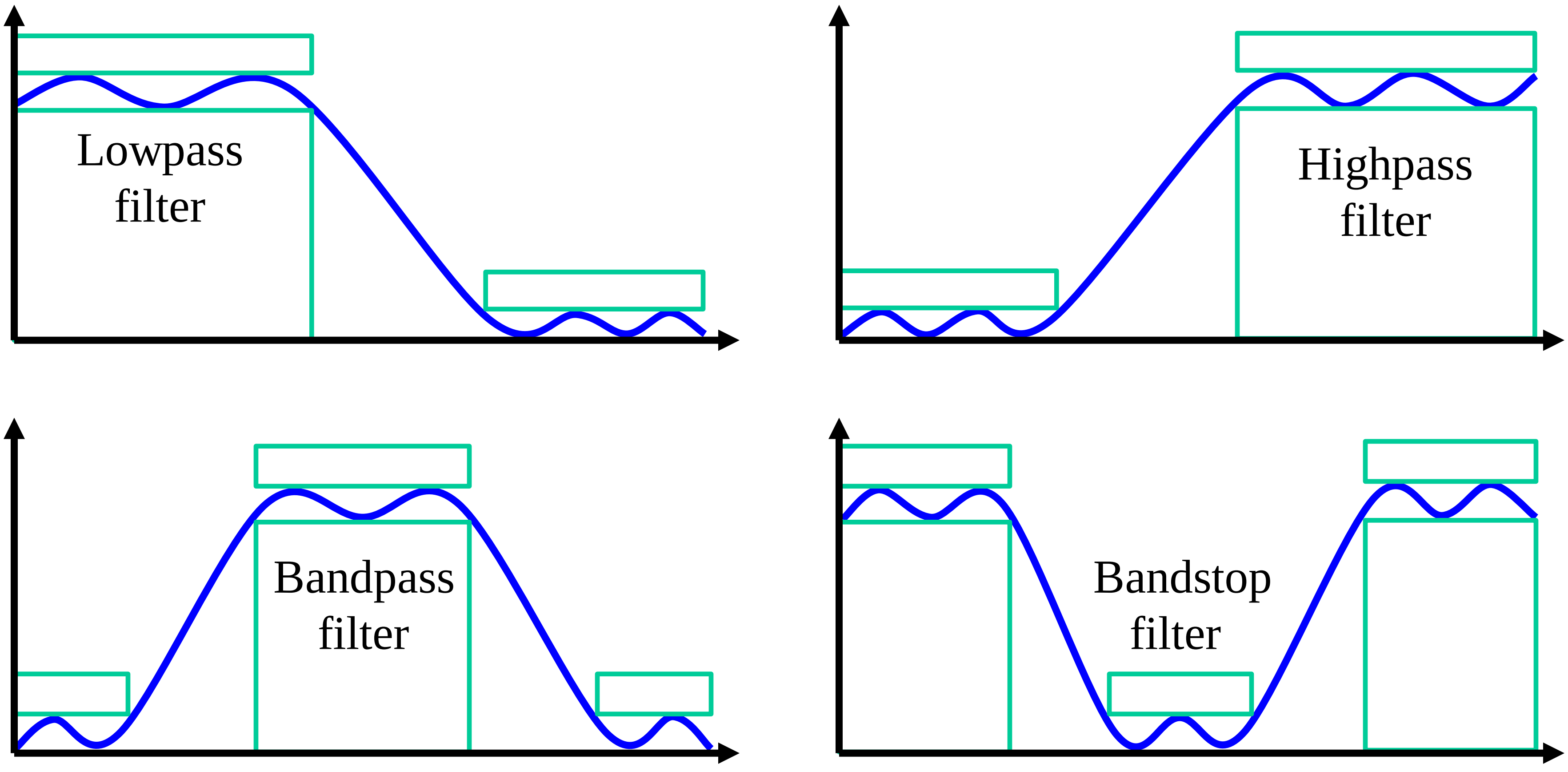
ELG4139: Active Filter Design

This simple filter is a kind of integrator, in which the capacitor integrates the charge provided through the resistor.

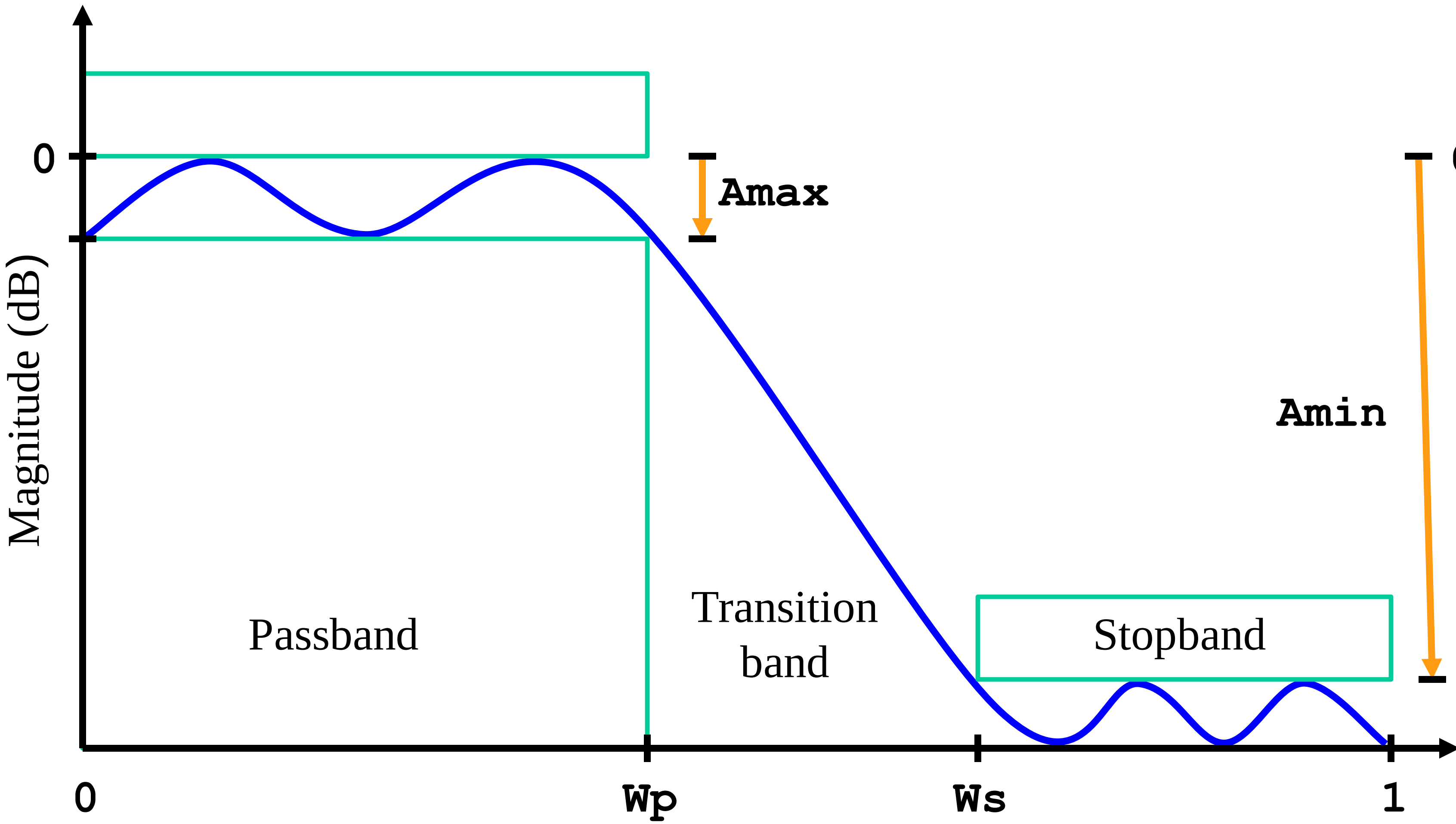


Filter Configurations

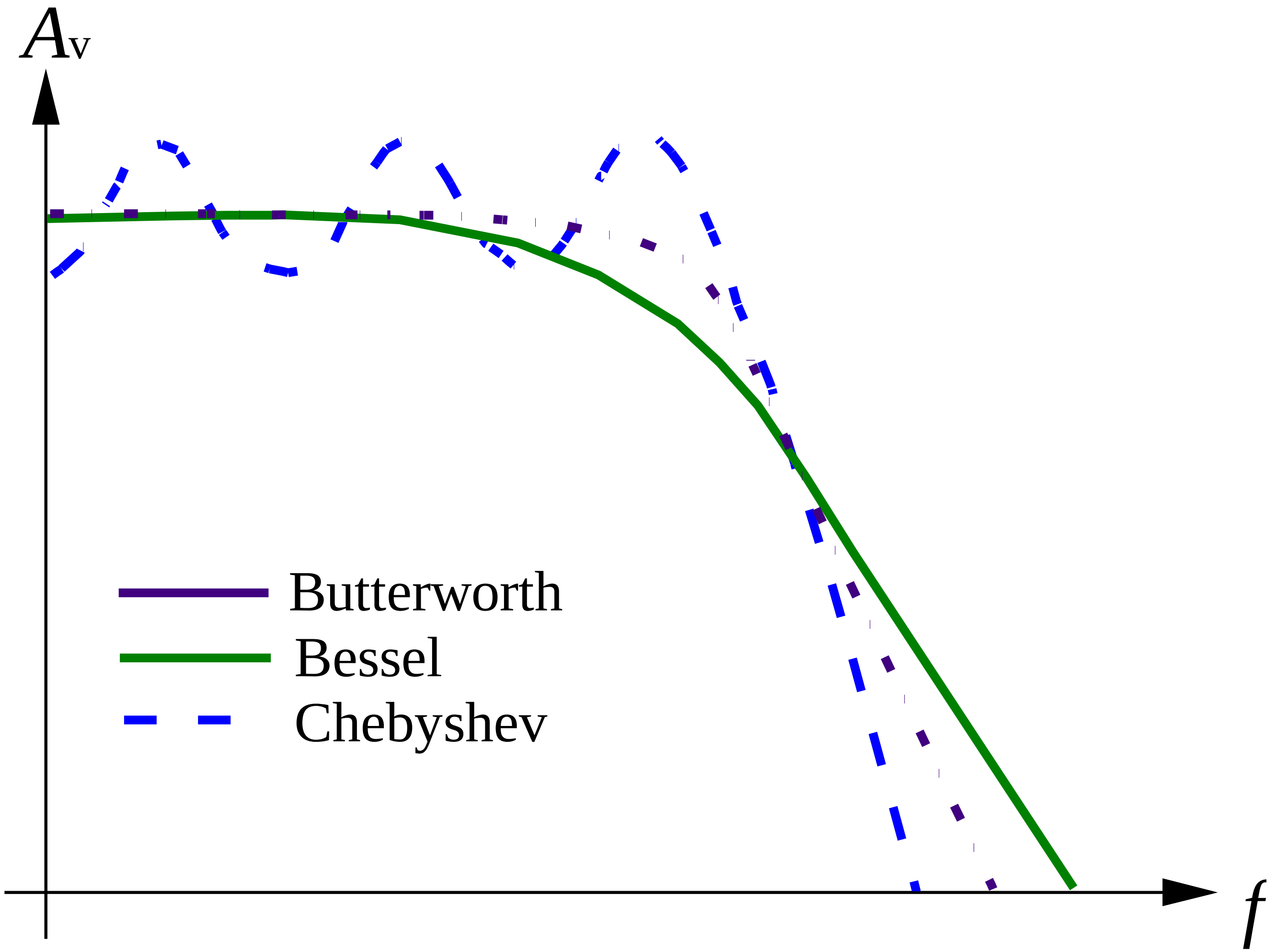


Normalization convention: $0 \leq f \leq 1$ = Nyquist frequency

Filter Specifications



Filter Response Characteristics



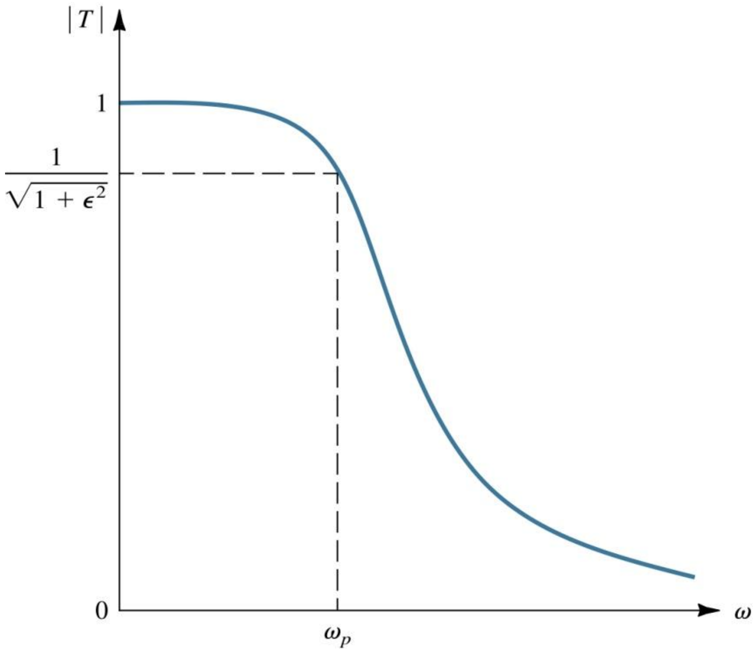
Butterworth Characteristic

- Very flat amplitude, $A_{v(dB)}$ response in the pass-band.
- Role-off rate is 20dB/decade/pole.
- Phase response is not linear.
- Used when all frequencies in the pass-band must have the same gain.
- Often referred to as a maximally flat response.

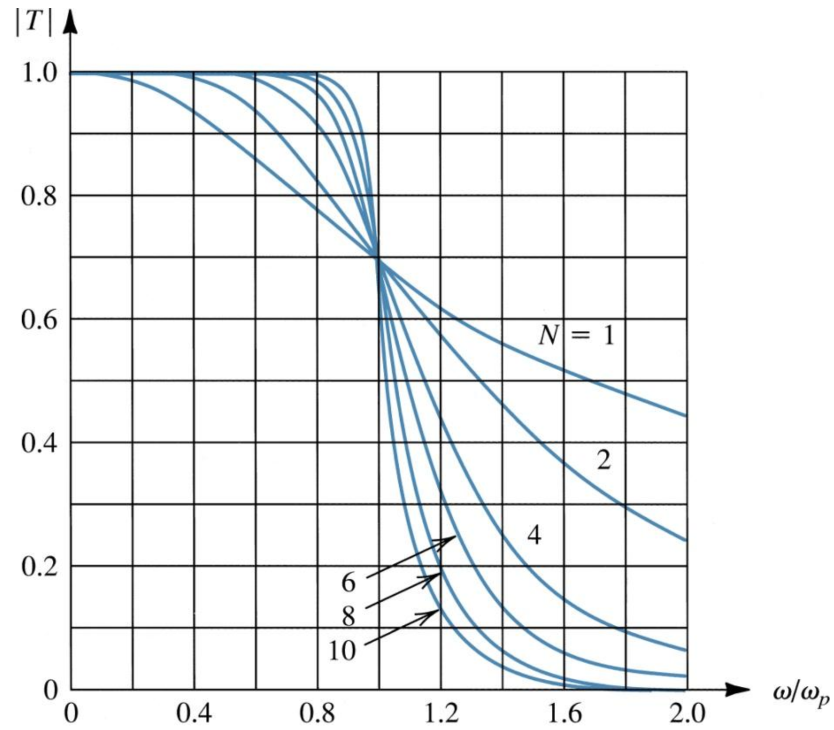
Design Criteria

- Requirements of a Customer:
  - $A_{\max}$
  - $A_{\min}$;
  - Selectivity $= \frac{w_s}{w_p}$
- Find Number of Stages N
- Select the Configuration (for example Butterworth)
- Draw the Circuit and Find its Transfer Function
- Write the Standard Transfer Function
- Equate both Transfer Functions and Compare the Appropriate Sections to Find Values of Parameters Basing on the Quantities Given: Q and w.

Butterworth Filters



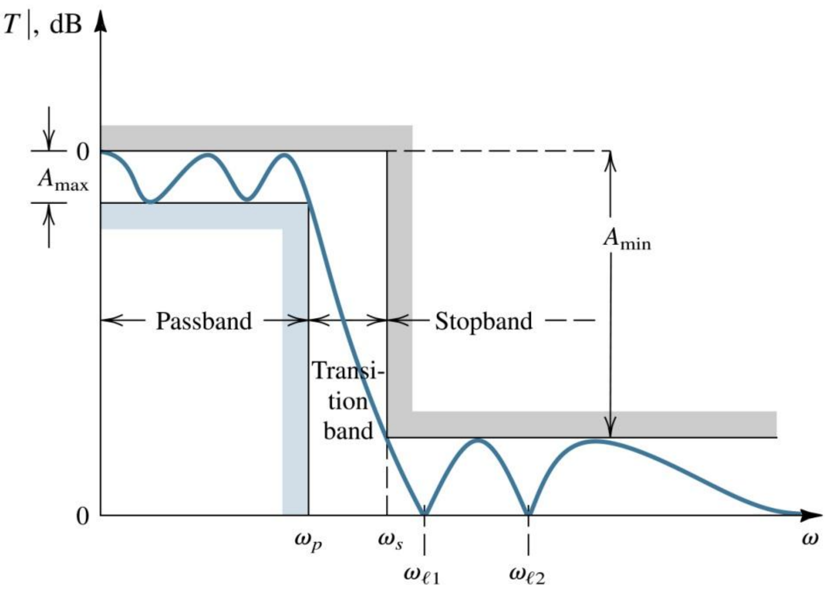
The magnitude response of a Butterworth filter.



Magnitude response for Butterworth filters of various order with ε = 1. Note that as the order increases, the response approaches the ideal brickwall type transmission.

Finding Order of the Filter N

$$A(\omega_s) = 10\log[1+\varepsilon^2(\frac{\omega_s}{\omega_p})^{2N}] = A_{\min}$$



$$1+\varepsilon^2(\frac{\omega_s}{\omega_p})^{2N} = 10^{\frac{A_{\min}}{10}}$$

$$(\frac{\omega_s}{\omega_p})^{2N} = \frac{10^{\frac{A_{\min}}{10}} - 1}{\varepsilon^2}$$

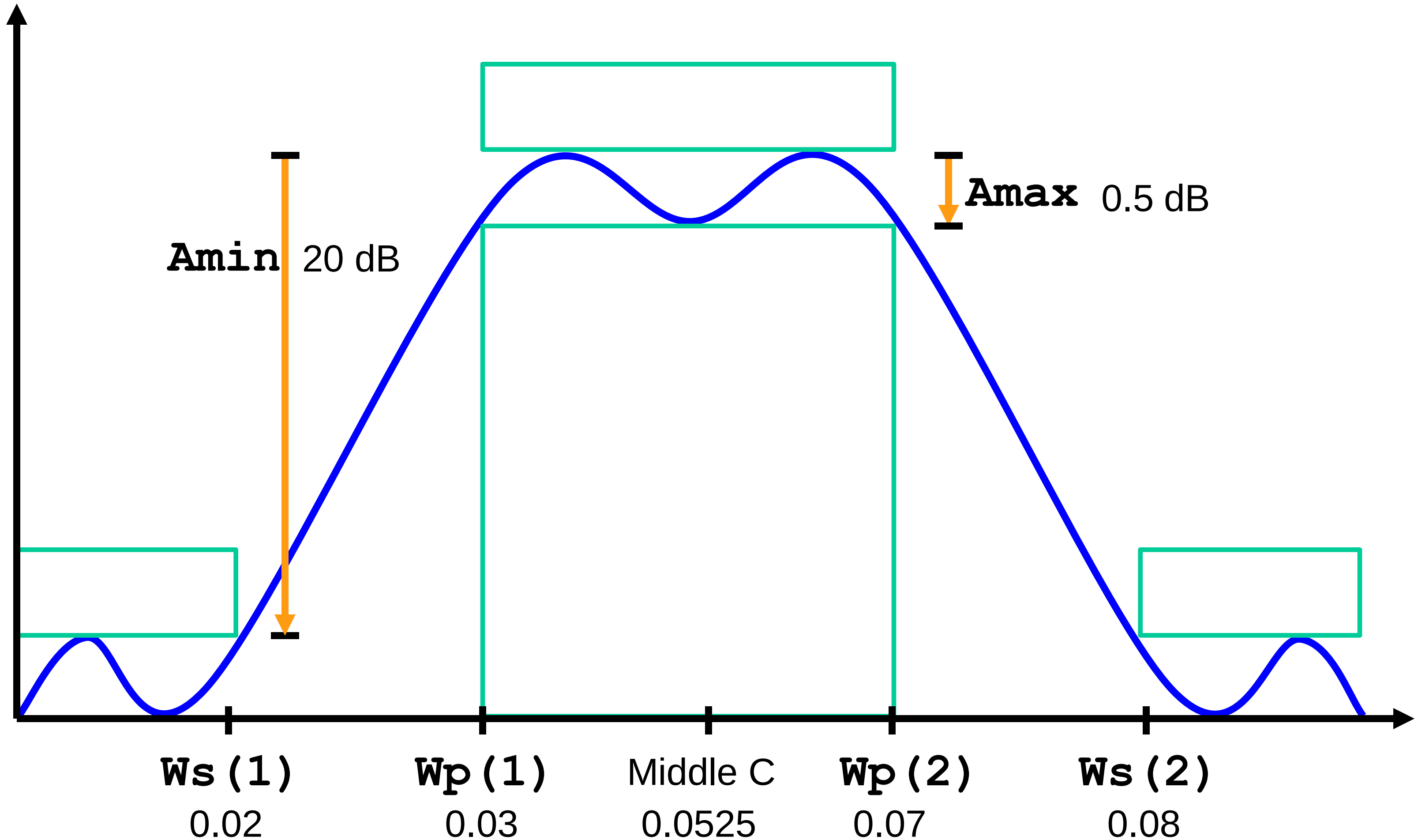
$$\log[(\frac{\omega_s}{\omega_p})^{2N}] = \log[\frac{10^{\frac{A_{\min}}{10}} - 1}{\varepsilon^2}]$$

$$N = \frac{\log[\frac{10^{\frac{A_{\min}}{10}} - 1}{\varepsilon^2}]}{\log[(\frac{\omega_s}{\omega_p})^2]}$$

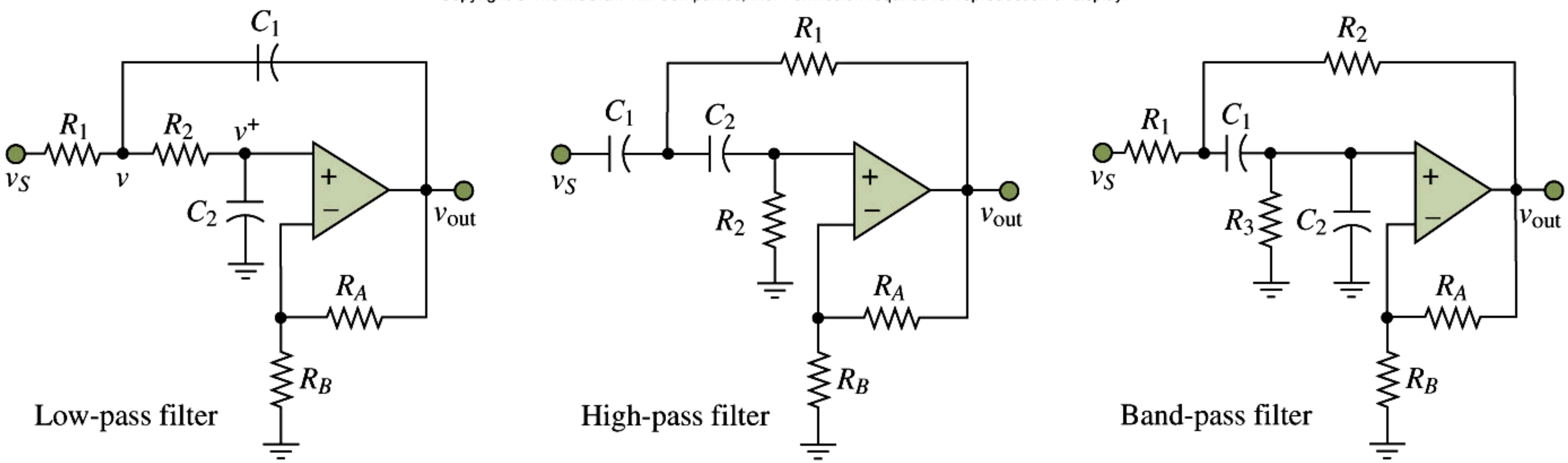
$$\varepsilon = \sqrt{10^{\frac{A_{\max}}{10}} - 1}$$

Example: Bandpass Filter

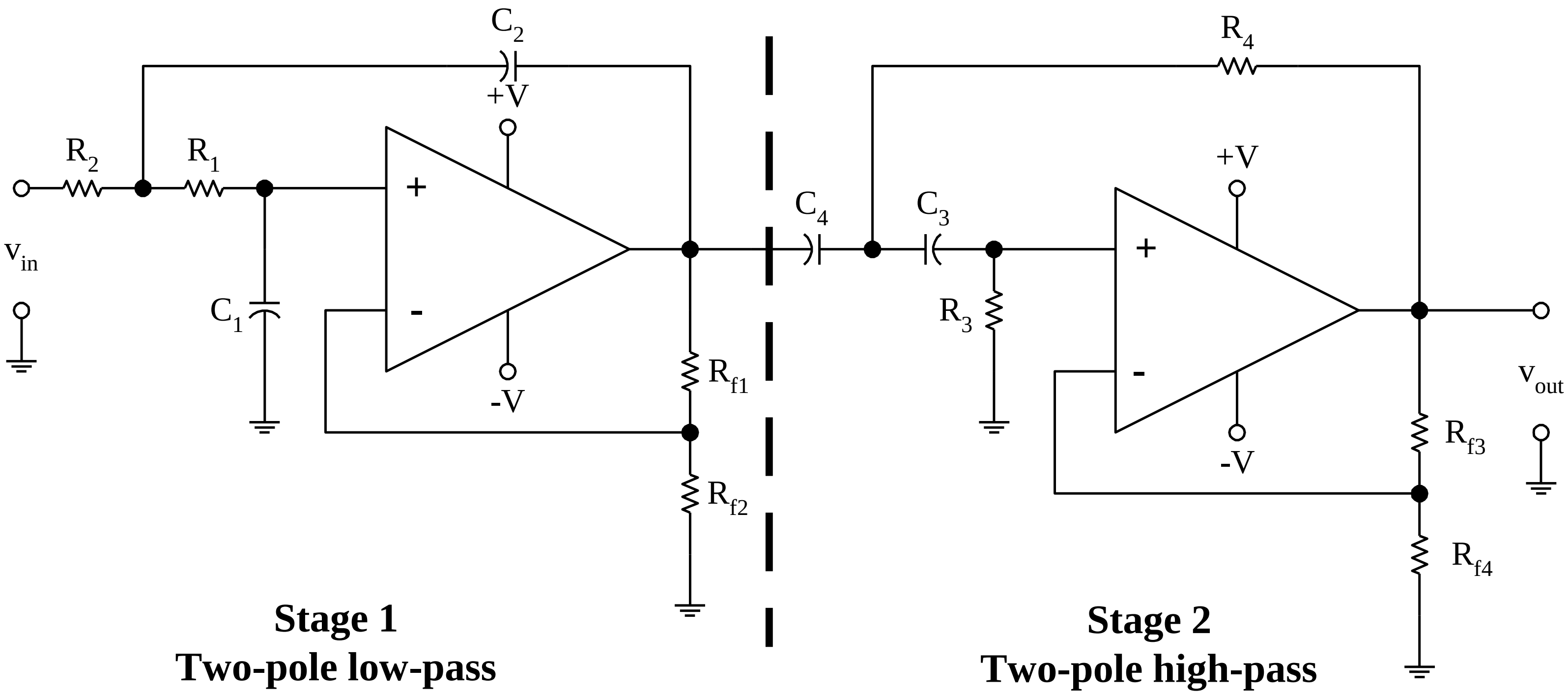
Order $N \le 10$, sampling frequency fs = 10,000 Hz

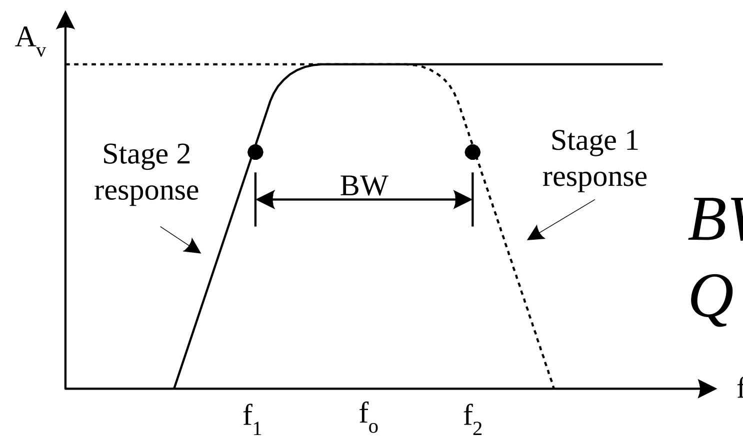


Sallen and Key Active Filters



Two-Stage Band-Pass Filter: N = 4

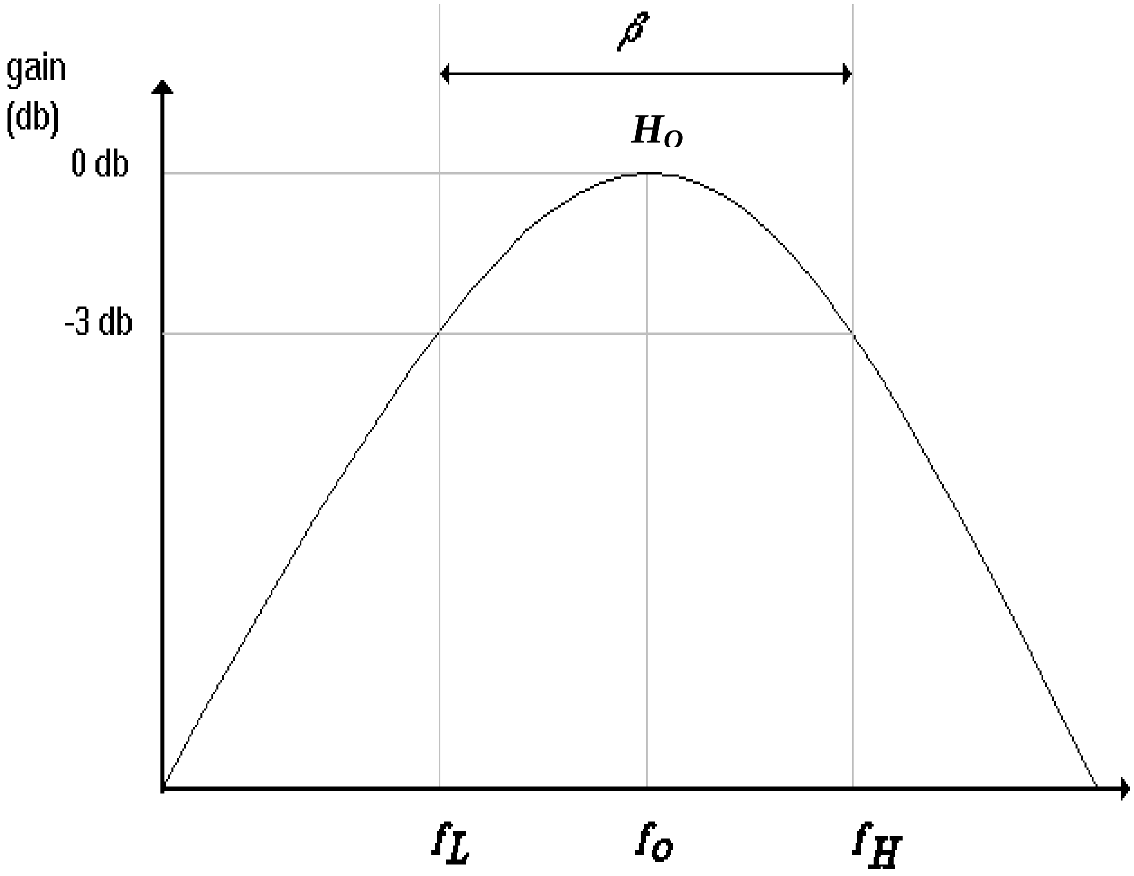




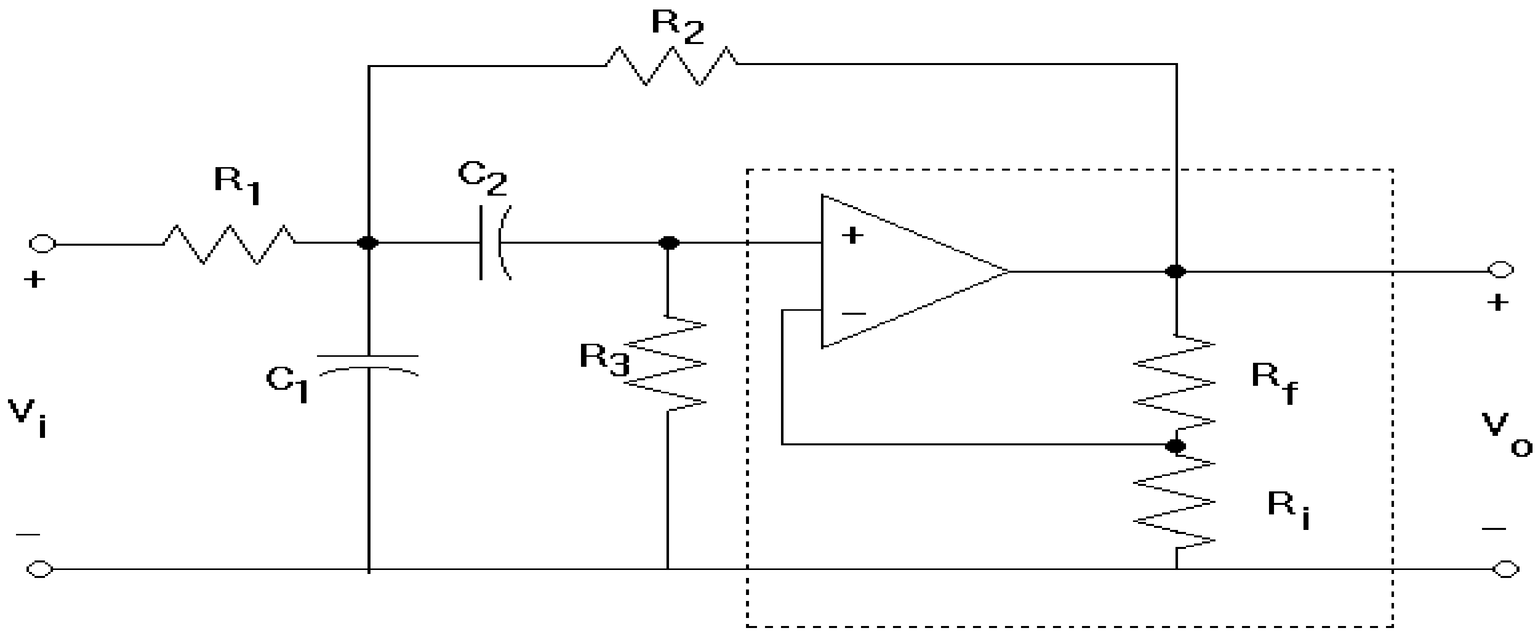
$$BW = f_2 - f_1$$
$$Q = f_0 / BW$$

Band-Pass Filter Performance

- Quality Factor $Q = f_o / \beta$
  - Dimensionless figure of merit used to measure the selectivity of a filter expressed as ratio of center frequency to bandwidth



Design of Second Order Sallen-Key Band-Pass Filter



$$T(s)=\frac{V_o}{V_i}=\frac{\dfrac{Ks}{R_1C_1}}{s^2+\left(\dfrac{1}{R_1C_1}+\dfrac{1}{R_3C_1}+\dfrac{1}{R_3C_2}+\dfrac{1-K}{R_2C_1}\right)s+\dfrac{R_1+R_2}{R_1R_2R_3C_1C_2}} \qquad (2)$$

$$K=\frac{V_o}{V_x}=1+\frac{R_f}{R_i} \qquad (1)$$

Equal Component Sallen-Key Filter

$$R_1 = R_2 = \frac{R_3}{2} = R \text{ and } C_1 = C_2 = C \qquad (3)$$

$$T(s) = \frac{\frac{Ks}{RC}}{s^2 + \left(\frac{3-K}{RC}\right)s + \left(\frac{1}{RC}\right)^2} \qquad (4)$$

$$s_{1,2} = \frac{-\left(\frac{3-K}{RC}\right) \pm \sqrt{\left(\frac{3-K}{RC}\right)^2 - 4\left(\frac{1}{RC}\right)^2}}{2} \qquad (9)$$

In order to ensure stability of the filter, we must ensure that the poles of the transfer function lie in the left-half of the complex s-plane, or $\Re(s_{1,2}) < 0$. Thus, we must ensure that the gain of the op-amp is less than 3, i.e., $K < 3$.

Standard Second Order Filter
Compare (4) with (5)

$$T(s) = \frac{H_o\left(\frac{\omega_o}{Q}\right)s}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2} \qquad (5)$$

$$\omega_o = \frac{1}{RC}, \quad Q = \frac{1}{3-K}, \quad H_o = \frac{K}{3-K}$$

Note: For your design, let

$$R_1 = R_2 = \frac{R_3}{2} = R_i = 2k\Omega$$

Frequency Scaling

Frequency scaling is a method of changing a filter's frequency of operation

This method is extremely useful once one has designed a filter with a satisfactory response (i.e., ω_o, Q, and H_o) and then merely wants to change, for example, the center frequency

To increase the center frequency of a filter without affecting any of its other characteristics (i.e., Q and H_o), we can simply divide all frequency determining capacitors *or* divide all frequency determining resistors by the desired scaling factor

As an example, to triple the center frequency, divide all capacitor values by 3 *or* divide all resistor values by 3

Design of Fifth Order Butterworth Low Pass Filter

Design a fifth order Butterworth low pass filter with a gain (μ) of 1 and a cutoff frequency 10 kHz.

The normalized Butterworth low pass filter equation is:

$$\frac{V_2(s)}{V_1(s)} = H(s) = \frac{\mu}{R_1R_2C_1C_2s^2 + \left(R_1C_1 + R_1C_2 + R_2C_2 - \mu R_1C_1\right)s + 1}$$

For a gain (μ) of 1

$$\frac{V_2(s)}{V_1(s)} = H(s) = \frac{1}{R_1R_2C_1C_2s^2 + \left(\cancel{R_1C_1} + R_1C_2 + R_2C_2 - \cancel{(1)*R_1C_1}\right)s + 1}$$

For frequency scaling: Replace s with s/w_c

$$\frac{V_2(s)}{V_1(s)} = H(s) = \frac{1}{R_1 R_2 C_1 C_2 s^2 + \left(R_1 C_2 + R_2 C_2\right)s + 1}$$

The normalized denominator for the fifth order Butterworth low pass filter is:

$$H(s) = \frac{1}{(s+1)(s^2+0.618s+1)(s^2+1.618s+1)}$$

Stage 1: First order filter

Stage 2: Second order filter

Stage 3: Second order filter

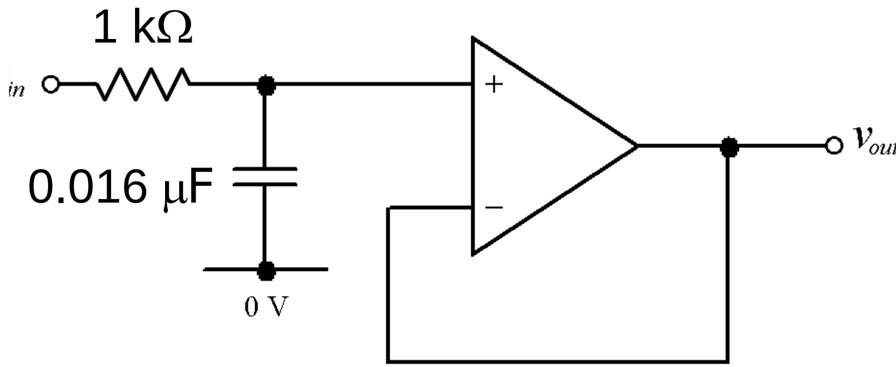
Stage 1: First Order Filter

Assume the resistor in the design as 1 kΩ.

$$\frac{v_{out}}{v_{in}} = \frac{1/RC}{s + 1/RC}$$

v_i

$$\frac{v_{out}}{v_{in}} = \frac{1/RC}{s/20000\pi + 1/RC}$$



$$1/RC = \omega_c = 20000\pi$$

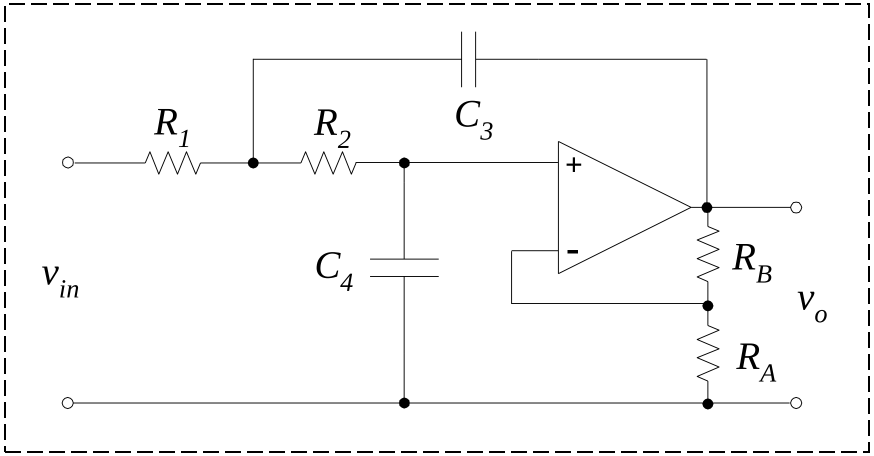
$$(1000 \times C) = \frac{1}{20000\pi}$$

$$C = \frac{1}{(1000 \times 20000)\pi} = 1.59 \times 10^{-8} \approx 0.016\mu F$$

Stage 2 and 3: Second Order Filter

$$H(s) = \frac{1}{(s^2 + 0.618s + 1)}$$

Defines one of the second order filters



$$H(s) = \frac{1}{\left(\frac{s^2}{(20000\pi)^2} + 0.618\frac{s}{20000\pi} + 1\right)}$$

$$H(s) = \frac{1}{R_1R_2C_1C_2s^2 + (R_1C_2 + R_2C_2)s + 1}$$

$$H(s) = \frac{1}{R_1R_2C_1C_2s^2 + (R_1C_2 + R_2C_2)s + 1}$$

$$H(s) = \frac{1}{(1000)^2C_1C_2s^2 + 1000(C_2 + C_2)s + 1}$$

$$\frac{1}{(1000)^2 C_1 C_2 s^2 + 1000(2C_2)s + 1}$$

$$= \frac{1}{\dfrac{s^2}{(20000\pi)^2} + 0.618\dfrac{s}{20000\pi} + 1}$$

$$1000(2C_2) = \frac{0.618}{20000\pi} \qquad C_2 = \frac{0.618}{40000000\pi} = 0.0049\,\mu F$$

$$(1000)^2 C_1 C_2 = \frac{1}{(20000\pi)^2}$$

$$C_1 = \frac{1}{(0.0049\times10^{-6})(1.0\times10^{6})(20000\pi)^2}$$

$$C_1 = \frac{1}{0.0049(20000\pi)^2} = 5.17\times10^{-8} = 0.05\,\mu F$$

$$H(s)=\frac{1}{(s^2+1.618s+1)}$$

Defines the other second order filters

$$H(s)=\frac{1}{\left(\frac{s^2}{(20000\pi)^2}+1.618\frac{s}{20000\pi}+1\right)}$$

$$H(s)=\frac{1}{R_1R_2C_1C_2s^2+\left(R_1C_2+R_2C_2\right)s+1}$$

$$H(s)=\frac{1}{(1000\times 1000)C_1C_2s^2+\left(1000C_2+1000C_2\right)s+1}$$

$$H(s)=\frac{1}{(1000)^2C_1C_2s^2+1000\left(C_2+C_2\right)s+1}$$

$$\frac{1}{(1000)^2 C_1 C_2 s^2 + 1000(2C_2)s + 1}$$

$$= \frac{1}{\frac{s^2}{(20000\pi)^2} + 1.618\frac{s}{20000\pi} + 1}$$

$$1000(2C_2) = \frac{1.618}{20000\pi} \qquad C_2 = \frac{1.618}{40000000\pi} = 1.29\times 10^{-8} = 0.013\mu F$$

$$(1000)^2 C_1 C_2 = \frac{1}{(20000\pi)^2} \qquad C_1 = \frac{1}{(0.013\times 10^{-6})(1.0\times 10^{6})(20000\pi)^2}$$

$$C_1 = \frac{1}{0.013\times(20000\pi)^2} = 1.95\times 10^{-8} = 0.0195\,\mu F$$