Examples: -

1) A 60-Hz short transmission line, having $R = 0.62$ ohms per phase and $L = 93.24$ millihenrys per phase, supplies a three-phase, wye-connected 100 MW load of 0.9 lagging power factor at 215 kV line-to-line voltage. Calculate the sending-end voltage per phase.

The line current $I (= I_s = I_R)$ is

$$I = \frac{100 \times 10^6}{\sqrt{3} \times 215 \times 10^3 \times 0.9} = 298.37 \text{ A}$$

and the per-phase voltage at the receiving end is

$$V_R = \frac{215 \times 10^3}{\sqrt{3}} = 124.13 \text{ kV}$$

The phasor diagram illustrating the operating conditions is that of Fig. 4-11, with $R = 0.62 \Omega$ and $X = \omega L = 377 \times 93.24 \times 10^{-3} = 35.15 \Omega$. Hence,

$$V_s = V_R + I(R + jX)$$

$$= 124.13 \times 10^3 / 0^\circ + (298.37/ -25.8^\circ)(0.62 + j35.15)$$

$$\approx 124.13 \times 10^3 / 0^\circ + (298.37/ -25.8^\circ)(35.15/90^\circ)$$

$$= (128.69 + j9.44) \text{ kV} = 129.04/4.2^\circ \text{ kV}$$
2) Determine the voltage regulation and efficiency of the above problem

\[
\text{Percent voltage regulation} = \frac{|V_s| - |V_R|}{|V_R|} \times 100 = \frac{129.04 - 124.13}{124.13} \times 100 = 3.955 \text{ percent}
\]

To calculate the efficiency, we first determine the loss in the line, which is

\[
\text{Line loss} = 3I^2R = 3 \times 298.37^2 \times 0.62 = 0.166 \text{ MW}
\]

The power received at the load is given to be 100 MW, so the power sent is 100 + 0.166 = 100.166 MW.

\[
\text{Efficiency} = \frac{100}{100.166} = 99.83 \text{ percent}
\]

3)

A 10-km-long, single-phase short transmission line has \(0.5/60^\circ \Omega/\text{km}\) impedance. The line supplies a 316.8-kW load at 0.8 power factor lagging. What is the voltage regulation if the receiving-end voltage is 3.3 kV?

\[
\cos^{-1} 0.8 = 36.87^\circ
\]

and

\[
Z = (0.5/60^\circ)(10) = 5/60^\circ \Omega
\]

Then

\[
I = \frac{316.8 \times 10^3}{3.3 \times 10^3 \times 0.8/36.87^\circ} = 120/36.87^\circ \text{ A}
\]

and

\[
IZ = (5/60^\circ)(120/36.87^\circ) = (551.77 + j235.69) \text{ V}
\]

Now

\[
V_s = (3300 + j0) + (551.77 + j235.69) = (3851.77 + j235.69) \text{ V}
\]

so

\[
|V_s| = 3858.97 \text{ V}
\]

Percent voltage regulation = \(\frac{3858.97 - 3300}{3300} \times 100 = 16.94 \text{ percent}\)
The per-phase impedance of a short transmission line is \( (0.3 + j0.4) \Omega \). The sending-end line-to-line voltage is 3300 V, and the load at the receiving end is 300 kilowatts per phase at 0.8 power factor lagging. Calculate (a) the receiving-end voltage and (b) the line current.

(a) On a per-phase basis,

\[
V_S = \frac{3300}{\sqrt{3}} = 1905.25 \text{ V} \tag{1}
\]

and

\[
I = \frac{300 \times 10^3}{(0.8)V_R} = \frac{3.75 \times 10^5}{V_R} \text{ A} \tag{2}
\]

\[
V_S^2 = (V_R \cos \phi_R + RI)^2 + (V_R \sin \phi_R + XI)^2 \tag{3}
\]

Substituting (1), (2), and other known values into (3) yields

\[
1905.25^2 = \left(0.8V_R + \frac{0.3 \times 3.75 \times 10^5}{V_R}\right)^2 + \left(0.6V_R + \frac{0.4 \times 3.75 \times 10^5}{V_R}\right)^2
\]

from which we find that \( V_R = 1805 \text{ V} \).

(b) From (2), we have

\[
I = \frac{3.75 \times 10^5}{1905} = 207.75 \text{ A}
\]
5) Determine a) Sending end power factor b) power loss from the sending end power and c) compare the result with the earlier used formula from the previous example

\[ V_R \cos \phi_R + RI = (1805)(0.8) + (0.3)(207.75) = 1506.33 \text{ V} \]

and \( V_s = 1905.26 \text{ V}. \) Hence

\[ \cos \phi_s = \frac{1506.33}{1905.25} = 0.79 \]

and the sending end has 0.79 power factor lagging.

(b) The sending-end power is

\[ P_s = V_s I \cos \phi_s = 1905.25 \times 207.75 \times 0.79 = 312.94 \text{ kW} \]

The power loss per phase is then

\[ P_L = 312.94 - 300 = 12.94 \text{ kW} \]

(c) By direct calculation, the per-phase power loss in the line is

\[ I^2R = (207.75)^2(0.3) = 12.948 \text{ kW} \]

6) A short 3-Φ transmission line with an impedance of \((6 + j8) \Omega\) per phase has sending and receiving end voltages of 120 kV and 110 kV respectively for some receiving end load at a p.f. of 0.9 lagging. Determine (i) power output and (ii) sending end power factor.

Resistance of each conductor, \( R = 6 \Omega \)
Reactance of each conductor, \( X_L = 8 \Omega \)
Load power factor, \( \cos \phi_R = 0.9 \) lagging
Receiving end voltage/phase, \( V_R = 110 \times 10^3/\sqrt{3} = 63508 \text{ V} \)
Sending end voltage/phase, \( V_s = 120 \times 10^3/\sqrt{3} = 69282 \text{ V} \)

Let \( I \) be the load current. Using approximate expression for \( V_s \), we get,

\[ V_s = V_R + IR \cos \phi_R + I X_L \sin \phi_R \]

or

\[ 69282 = 63508 + I \times 6 \times 0.9 + I \times 8 \times 0.435 \]

or

\[ 8.88 I = 5774 \]

or

\[ I = 5774/8.88 = 650.2 \text{ A} \]

(i) Power output

\[ \text{Power output} = \frac{3 V_R I \cos \phi_R}{1000} \text{ kW} = \frac{3 \times 63508 \times 650.2 \times 0.9}{1000} = 1,114.90 \text{ kW} \]

(ii) Sending end p.f., \( \cos \phi_s = \frac{V_R \cos \phi_R + IR}{V_s} = \frac{63508 \times 0.9 + 650.2 \times 6}{69282} = 0.88 \text{ lag} \]
7) A 3-phase load of 2000 kVA, 0.8 p.f. is supplied at 6.6 kV, 50 Hz by means of a 33 kV transmission line 20 km long and 33/6.6 kV step-down transformer. The resistance and reactance of each conductor are 0.4 Ω and 0.5 Ω per km respectively. The resistance and reactance of transformer primary are 7.5 Ω and 13.2 Ω, while those of secondary are 0.35 Ω and 0.65 Ω respectively. Find the voltage necessary at the sending end of transmission line when 6.6 kV is maintained at the receiving end. Determine also the sending end power factor and transmission efficiency.

**Solution.** Fig. 10.7 shows the single diagram of the transmission system. Here, the voltage drop will be due to the impedance of transmission line and also due to the impedance of transformer.

Resistance of each conductor = 20 × 0.4 = 8 Ω
Reactance of each conductor = 20 × 0.5 = 10 Ω

Let us transfer the impedance of transformer secondary to high tension side i.e., 33 kV side.

Equivalent resistance of transformer referred to 33 kV side

\[ = \text{Primary resistance} + 0.35 \times (33/6.6)^2 \]
\[ = 7.5 + 8.75 = 16.25 \Omega \]

Equivalent reactance of transformer referred to 33 kV side

\[ = \text{Primary reactance} + 0.65 \times (33/6.6)^2 \]
\[ = 13.2 + 16.25 = 29.45 \Omega \]

Total resistance of line and transformer is

\[ R = 8 + 16.25 = 24.25 \Omega \]

![Diagram](image)

**Fig. 10.7**

Total reactance of line and transformer is

\[ X_L = 10 + 29.45 = 39.45 \Omega \]

Receiving end voltage per phase is

\[ V_R = \frac{33,000}{\sqrt{3}} = 19052 \text{ V} \]

Line current,

\[ I = \frac{2000 \times 10^3}{\sqrt{3} \times 33000} = 35 \text{ A} \]

Using the approximate expression for sending end voltage \( V_S \) per phase,

\[ V_S = V_R + IR \cos \phi_R + IX_L \sin \phi_R \]
\[ = 19052 + 35 \times 24.25 \times 0.8 + 35 \times 39.45 \times 0.6 \]
\[ = 19052 + 679 + 828 = 20559 \text{ V} = 20.559 \text{ kV} \]

Sending end line voltage

\[ = \sqrt{3} \times 20.559 \text{ kV} = 35.6 \text{ kV} \]

Sending end p.f.

\[ \cos \phi_S = \frac{V_R \cos \phi_R + IR}{V_S} = \frac{19052 \times 0.8 + 35 \times 24.25}{20559} = 0.7826 \text{ lag} \]

Line losses

\[ = \frac{3I^2R}{1000} \text{ kW} = \frac{3 \times (35)^2 \times 24.25}{1000} = 89.12 \text{ kW} \]

Output power

\[ = 2000 \text{ kVA} \times 0.8 = 1600 \text{ kW} \]

Transmission efficiency

\[ = \frac{1600}{1600 + 89.12} \times 100 = 94.72\% \]