POWER FACTOR: The cosine of angle between voltage and current in an a.c. circuit is known as power factor.

The circuit current I can be resolved into two perpendicular components, namely;

(a) I cos φ in phase with V

(b) I sin φ 90 out of phase with V

The component I cos φ is known as active or wattful component, whereas component I sin φ is called the reactive or wattless component. The reactive component is a measure of the power factor. If the reactive component is small, the phase angle φ is small and hence power factor cos φ will be high. Therefore, a circuit having small reactive current (i.e., I sin φ) will have high power factor and vice-versa.

Power Triangle: If each side of the current triangle oab of Fig. 6.1 is multiplied by voltage V, then we get the power triangle OAB shown below:
$OA = VI \cos \phi$ and represents the active power in watts or kW

$AB = VI \sin \phi$ and represents the reactive power in VAR or kVAR

$OB = VI$ and represents the apparent power in VA or kVA

The following points may be noted from the power triangle:

(i) The apparent power in an a.c. circuit has two components viz., active and reactive power at right angles to each other.

$OB^2 = OA^2 + AB^2$

or $(\text{apparent power})^2 = (\text{active power})^2 + (\text{reactive power})^2$

or $(\text{kVA})^2 = (\text{kW})^2 + (\text{kVAR})^2$

(ii) Power factor, $\cos \phi = \frac{OA}{OB} = \frac{\text{active power}}{\text{apparent power}} = \frac{\text{kW}}{\text{kVA}}$

Thus, the power factor of a circuit may also be defined as the ratio of active power to the apparent power. This is a perfectly general definition and can be applied to all cases, whatever be the waveform.

(iii) The lagging reactive power is responsible for the low power factor. It is clear from the power triangle that smaller the reactive power component, the higher is the power factor of the circuit.

$k\text{VAR} = kVA \sin \phi = (kW/\cos \phi) \sin \phi$

$\therefore k\text{VAR} = kW \tan \phi$. 
THE POWER FACTOR OF A CIRCUIT CAN BE DEFINED IN ONE OF THE FOLLOWING THREE WAYS:

(A) POWER FACTOR = \( \cos \phi \) = COSINE OF ANGLE BETWEEN \( V \) AND \( I \)

(B) POWER FACTOR = \( \frac{R}{Z} \) = RESISTANCE/IMPEDANCE

(C) POWER FACTOR = \( \frac{VI}{\cos \phi V} \) = ACTIVE POWER/APPARENT POWER.
**Significance of reactive power:**

**Active power:** The active power is consumed by the active elements from a circuit ( Resistances), its unit measure being the watt (W).

**Reactive power:** The reactive power is neither consumed in the circuit nor does any useful work. It merely flows back and forth in both directions in the circuit. A wattmeter does not measure reactive power. The reactive power is “consumed” by the circuit’s reactive elements (coils, capacitors and magnetic coupled coils), its unit measure being volt-ampere reactive (VAr).

**Apparent power:** The apparent power—S is by definition the product between the rms values of voltage and current. The apparent power is an indicator upon the circuit’s functioning measured in (KVA), a utility supplies the apparent power, but a consumer is only charged for active power.

The following are the causes of low power factor:

(i) Most of the a.c. motors are of induction type (1φ and 3φ induction motors) which have low lagging power factor. These motors work at a power factor which is extremely small on light load (0·2 to 0·3) and rises to 0·8 or 0·9 at full load.

(ii) Arc lamps, electric discharge lamps and industrial heating furnaces operate at low lagging power factor.
(iii) The load on the power system is varying; being high during morning and evening and low at other times. During low load period, supply voltage is increased which increases the magnetization current. This results in the decreased power factor.

**Power factor correction:** To improve the power factor, some device taking leading power should be connected in parallel with the load. One of such devices can be a capacitor. The capacitor draws a leading current and partly or completely neutralizes the lagging reactive component of load current. This raises the power factor of the load.

![Diagram](image)

**Voltage regulation:** When a transmission line is carrying current, there is a voltage drop in the line due to resistance and inductance of the line. The result is that receiving end voltage \(V_r\) of the line is generally less than the sending end voltage \(V_s\). This voltage drops \((V_s - V_r)\) in the line is expressed as a percentage of receiving end voltage \(V_r\) and is called voltage regulation.

The difference in voltage at the receiving end of a transmission line *between conditions of no load and full load is called* voltage regulation and is expressed as a percentage of the receiving end voltage.

Mathematically, \% age Voltage regulation =

\[
\frac{V_s - V_r}{V_r} \times 100
\]

Obviously, *it is desirable that the voltage regulation of a transmission line should be low i.e., the increase in load current should make very little difference in the receiving end voltage.*
Transmission efficiency: The power obtained at the receiving end of a transmission line is generally less than the sending end power due to losses in the line resistance. The ratio of receiving end power to the sending end power of a transmission line is known as the transmission efficiency of the line i.e.

\[ \eta_T = \frac{\text{Receiving end power}}{\text{Sending end power}} \times 100 \]
\[ = \frac{V_R I_R \cos \phi_R}{V_S I_S \cos \phi_S} \times 100 \]
Examples:

1) Suppose a circuit draws a current of 10 A at a voltage of 200 V and its p.f. is 0.8 lagging. Then, calculate apparent power, active power and reactive power.

- Apparent power = VI = 200 × 10 = 2000 VA
- Active power = VI cos φ = 200 × 10 × 0.8 = 1600 W
- Reactive power = VI sin φ = 200 × 10 × 0.6 = 1200 VAR

The circuit receives an apparent power of 2000 VA and is able to convert only 1600 watts into active power. The reactive power is 1200 VAR and does no useful work. It merely flows into and out of the circuit periodically. In fact, reactive power is a liability on the source because the source must supply the additional current (i.e., I sin φ).

2) An alternator is supplying a load of 300 kW at a p.f. of 0.6 lagging. If the power factor is raised to unity, how many more kilowatts can alternator supply for the same kVA loading?

\[
\text{kVA} = \frac{\text{kW}}{\cos \phi} = \frac{300}{0.6} = 500 \text{ kVA}
\]

kW at 0.6 p.f. = 300 kW

kW at 1 p.f. = 500 × 1 = 500 kW

∴ Increased power supplied by the alternator

= 500 − 300 = 200 kW

Note the importance of power factor improvement. When the p.f. of the alternator is unity, the 500 kVA are also 500 kW and the engine driving the alternator has to be capable of developing this power together with the losses in the alternator. But when the power factor of the load is 0.6, the power is only 300 kW. Therefore, the engine is developing only 300 kW, though the alternator is supplying its rated output of 500 kVA.
3)

(i) Lighting load of 20 kW at unity power factor.
(ii) Induction motor load of 100 kW at p.f. 0.707 lagging.
(iii) Synchronous motor load of 50 kW at p.f. 0.9 leading.

**Calculate the total kW and kVA delivered by the generator and the power factor at which it works.**

**Solution:** Using the suffixes 1, 2 and 3 to indicate the different loads, we have,

\[
\text{kVA}_1 = \frac{\text{kW}_1}{\cos \phi_1} = \frac{20}{1} = 20 \text{ kVA}
\]

\[
\text{kVA}_2 = \frac{\text{kW}_2}{\cos \phi_2} = \frac{100}{0.707} = 141.4 \text{ kVA}
\]

\[
\text{kVA}_3 = \frac{\text{kW}_3}{\cos \phi_3} = \frac{50}{0.9} = 55.6 \text{ kVA}
\]

These loads are represented in Fig. 6.10. The three kVAs are not in phase. In order to find the total kVA, we resolve each kVA into rectangular components – kW and kVAR as shown in Fig. 6.10. The total kW and kVAR may then be combined to obtain total kVA.

\[
\text{kVAR}_1 = \text{kVA}_1 \sin \phi_1 = 20 \times 0 = 0
\]

\[
\text{kVAR}_2 = \text{kVA}_2 \sin \phi_2 = -141.4 \times 0.707 = -100 \text{ kVAR}
\]

\[
\text{kVAR}_3 = \text{kVA}_3 \sin \phi_3 = +55.6 \times 0.436 = +24.3 \text{ kVAR}
\]

Note that kVAR$_2$ and kVAR$_3$ are in opposite directions: kVAR$_2$ being a lagging while kVAR$_3$ being a leading kVAR.

Total kW = 20 + 100 + 50 = **170 kW**

Total kVAR = 0 - 100 + 24.3 = **-75.7 kVAR**

Total kVA = \sqrt{(\text{kW})^2 + (\text{kVAR})^2} = \sqrt{(170)^2 + (75.7)^2} = **186 kVA**

Power factor = \frac{\text{Total kW}}{\text{Total kVA}} = \frac{170}{186} = **0.914 lagging**

The power factor must be lagging since the resultant kVAR is lagging.
4) An overhead 3-phase transmission line delivers 5000 kW at 22 kV at 0·8 p.f. lagging. The resistance and reactance of each conductor is 4 Ω and 6 Ω respectively. Determine: (i) sending end voltage (ii) percentage regulation (iii) transmission efficiency

Load power factor, \( \cos \phi_R = 0.8 \) lagging

Receiving end voltage/phase, \( V_R = \frac{22,000}{\sqrt{3}} = 12,700 \) V

Impedance/phase, \( \vec{Z} = 4 + j \times 6 \)

Line current, \( I = \frac{5000 \times 10^3}{3 \times 12700 \times 0.8} = 164 \) A

As \( \cos \phi_R = 0.8 \) \( \therefore \sin \phi_R = 0.6 \)

Taking \( \vec{V}_R \) as the reference phasor (see Fig. 10.6),

\[
\vec{V}_R = V_R + j 0 = 12700 \text{ V}
\]

\[
\vec{I} = I (\cos \phi_R - j \sin \phi_R) = 164 (0.8 - j 0.6) = 131.2 - j 98.4
\]

(i) Sending end voltage per phase is

\[
\vec{V}_S = \vec{V}_R + \vec{I} \cdot \vec{Z} = 12700 + (131.2 - j 98.4) (4 + j 6)
\]

\[
= 12700 + 524.8 + j 787.2 - j 393.6 + 590.4
\]

\[
= 13815.2 + j 393.6
\]

Magnitude of \( V_S \) \( = \sqrt{(13815.2)^2 + (393.6)^2} = 13820.8 \) V

Line value of \( V_S \) \( = \sqrt{3} \times 13820.8 = 23938 \) V = 23.938 kV

(ii) Percentage Regulation \( = \frac{V_S - V_R}{V_R} \times 100 = \frac{13820.8 - 12700}{12700} \times 100 = 8.825\% \)

(iii) Line losses \( = 3^2 \times R = 3 \times (164)^2 \times 4 = 322,752 \) W = 322.752 kW

\( \therefore \) Transmission efficiency \( = \frac{5000}{5000 + 322.752} \times 100 = 93.94\% \)