ELG3311: Solutions for Assignment 1

Problem 2-6:

A 15-kVA 8000/230-V distribution transformer has an impedance referred to the primary of 80 + *j*300Ω. The components of the excitation branch referred to the primary side are R_C = 350 kΩ and X_M = 70 kΩ.

(a) If the primary voltage is 7967 V and the load impedance is Z_L = 3.0 +*j*1.5 Ω, what is the secondary voltage of the transformer? What is the voltage regulation of the transformer?

(b) If the load is disconnected and a capacitor of –*j*4.0 Ω is connected in its place, what is the secondary voltage of the transformer? What is its voltage regulation under these conditions?

Solution:

(a) The turns ratio is

$$a = 8000/230 = 34.78$$

The load impedance referred to the primary side is

$$Z_L^P = a^2 Z_L = (34.78)^2(3.0 + j1.5) = 3629 + j1815\,\Omega$$

The referred secondary current is

$$I_S^P = \frac{V_P}{Z_i^P + Z_L^P} = \frac{7967\angle 0°}{(80 + j300) + (3629 + j1815)} = 1.87\angle -29.7° \text{ A}$$

The referred secondary voltage is

$$V_S^P = I_S^P Z_L^P = (1.87\angle -29.7°)(3629 + j1815) = 7588\angle -3.1° \text{ V}$$

The actual secondary voltage is

$$V_S = \frac{V_S^P}{a} = \frac{7588\angle -3.1°}{34.78} = 218.2\angle -3.1° \text{ V}$$

The voltage regulation is

$$VR = \frac{V_P - V_S^P}{V_S^P} = \frac{7967 - 7588}{7588} \times 100\% = 4.99\%$$

(b) The turns ratio is

$$a = 8000/230 = 34.78$$

The load impedance referred to the primary side is

$$Z_L^P = a^2 Z_L = (34.78)^2(-j4.0) = -j4839\,\Omega$$

The referred secondary current is

$$I_S^P = \frac{V_P}{Z_i^P + Z_L^P} = \frac{7967\angle 0°}{(80 + j300) + (-j4839)} = 1.75\angle 89.0°\text{ A}$$

The referred secondary voltage is

$$V_S^P = I_S^P Z_L^P = (1.75\angle 89.0°)(-j4839) = 8468\angle -1.0°\text{ V}$$

The actual secondary voltage is

$$V_S = \frac{V_S^P}{a} = \frac{8468\angle -1.0°}{34.78} = 243.2\angle -1.0°\text{ V}$$

The voltage regulation is

$$VR = \frac{V_P - V_S^P}{V_S^P} = \frac{7967 - 8468}{8468} \times 100\% = -5.92\%$$

Problem 2-7:

A 5000-kVA 230/13.8-kV single-phase power transformer has a per-unit resistance of 1 percent and a per-unit reactance of 5 percent (data taken from the transformer's nameplate). The open-circuit test performed on the low-voltage side of the transformer yielded the following data: V_{OC} = 13.8 kV, I_{OC} = 15.1 A, P_{OC} = 44.9 kW

(a) Find the equivalent circuit referred to the low-voltage side of this transformer.

(b) If the voltage on the secondary side is 13.8 kV and the power supplied is 4000 kW at 0.8 PF lagging, find the voltage regulation of the transformer. Find its efficiency.

Solution:

(a) The components of the excitation branch relative to the low-voltage side (secondary side) are

$$|Y_{EX}| = |G_C - jB_M| = \frac{I_{OC}}{V_{OC}} = \frac{15.1}{13.8k} = 0.0010942$$

$$\theta = -\cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}} = -\cos^{-1}\frac{44.9k}{(13.8k)(15.1)} = -77.56°$$

$$Y_{EX} = |Y_{EX}|\angle\theta = G_C - jB_M = 0.0010942\angle -77.56° = 0.0002358 - j0.0010685$$

$$R_C = \frac{1}{G_C} = \frac{1}{0.0002358} = 4240\,\Omega$$

$$X_M = \frac{1}{B_M} = \frac{1}{0.0010685} = 936\,\Omega$$

The base impedance of this transformer referred to the low-voltage side (secondary side) is

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(13.8k)^2}{5000k} = 38.09\,\Omega$$

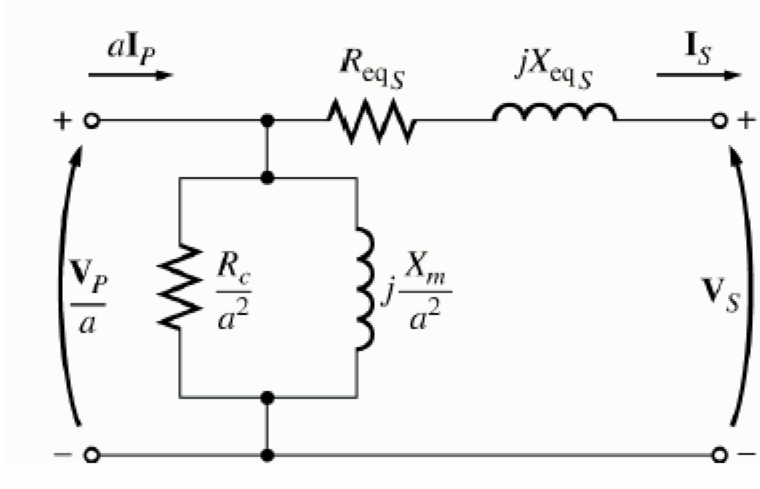
So that

$$R_{EQ} = 1\% Z_{base} = (0.01)(38.09) = 0.38\,\Omega$$

$$X_{EQ} = 5\% Z_{base} = (0.05)(38.09) = 1.90\,\Omega$$

The resulting equivalent circuit is shown below

$$R_{EQ,S} = 0.38\,\Omega \qquad X_{EQ,S} = 1.9\,\Omega$$
$$R_{C,S} = 4240\,\Omega \qquad X_{M,S} = 936\,\Omega$$



(b) The secondary current is

$$I_S = \frac{P_{out}}{V_S PF} = \frac{4000\text{ kW}}{(13.8\text{ kV})(0.8)} = 362.3\text{ A}$$
$$\theta = -\cos^{-1} PF = -\cos^{-1}(0.8) = -36.87°$$
$$I_S = 362.3\angle -36.87°\text{ A}$$

The voltage on the primary side of the transformer (referred to the secondary side) is

$$V_P^S = V_S + I_S Z_{EQ,S} = 13800\angle 0° + (362.3\angle -36.87°)(0.38 + j1.9) = 14330\angle 1.9°\text{ V}$$

The voltage regulation of the transformer is

$$VR = \frac{V_P^S - V_S}{V_S} \times 100\% = \frac{14330 - 13800}{13800} \times 100\% = 3.84\%$$

The transformer copper losses and core losses are

$$P_{copper} = I_S^2 R_{EQ,S} = (362.3)^2(0.38) = 49.9\text{ kW}$$
$$P_{core} = \frac{(V_P^S)^2}{R_C} = \frac{(14330)^2}{4240} = 48.4\text{ kW}$$

Therefore the efficiency of this transformer at these conditions is

$$\eta = \frac{P_{out}}{P_{out} + P_{copper} + P_{core}} \times 100\% = \frac{4000\,\text{kW}}{4000\,\text{kW} + 49.9\,\text{kW} + 48.4\,\text{kW}} \times 100\% = 97.6\%$$

Problem 2-8:

A 200-MVA, 15/200-kV single-phase power transformer has a per-unit resistance of 1.2 percent and a per-unit reactance of 5 percent (data taken from the transformer's nameplate). The magnetizing impedance is j80 per unit.

(a) Find the equivalent circuit referred to the low-voltage side of this transformer.

(b) Calculate the voltage regulation of this transformer for a full-load current at power factor of 0.8 lagging.

(c) Assume that the primary voltage of this transformer is a constant 15 kV, and plot the secondary voltage as a function of load current for currents from no load to full load. Repeat this process for power factors of 0.8 lagging, 1.0, and 0.8 leading.

Solution:

(a) The base impedance of this transformer referred to the low-voltage side (primary) is

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(15k)^2}{200M} = 1.125\,\Omega$$

So that

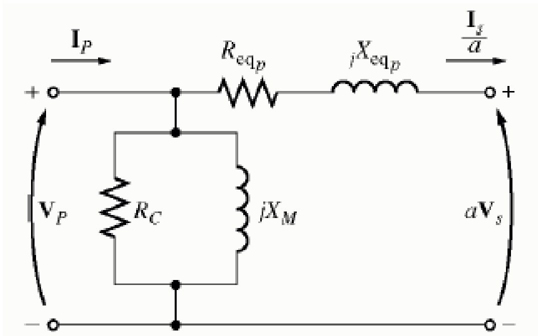
$$R_{EQ} = 1.2\% Z_{base} = (0.012)(1.125) = 0.0135\,\Omega$$
$$X_{EQ} = 5\% Z_{base} = (0.05)(1.125) = 0.0563\,\Omega$$

And

$$X_M = (80) Z_{base} = (80)(1.125) = 90\,\Omega$$

The equivalent circuit referred to the low-voltage side of this transformer is shown below

$$R_{EQ,P} = 0.0135\Omega \qquad X_{EQ,P} = 0.0563\Omega$$
$$R_{C,P} = not - spcified \qquad X_{M,P} = 90\Omega$$



(b) The current on the secondary side of the transformer (referred to the primary side) is

$$I_S^P = \frac{P_{out}}{V_S PF} = \frac{200\text{ MW}}{(15\text{ kW})(0.8)} = 16667 A$$

$$\theta = -\cos^{-1} PF = -\cos^{-1}(0.8) = -36.87°$$

$$I_S^P = 16667\angle -36.87° \text{A}$$

The voltage on the primary side of the transformer is

$$V_P = V_S^P + I_S^P Z_{EQ,P} = 15000\angle 0° + (16667\angle -36.87°)(0.0135 + j0.0563) = 15755\angle 2.24° V$$

Therefore the voltage regulation of the transformer is

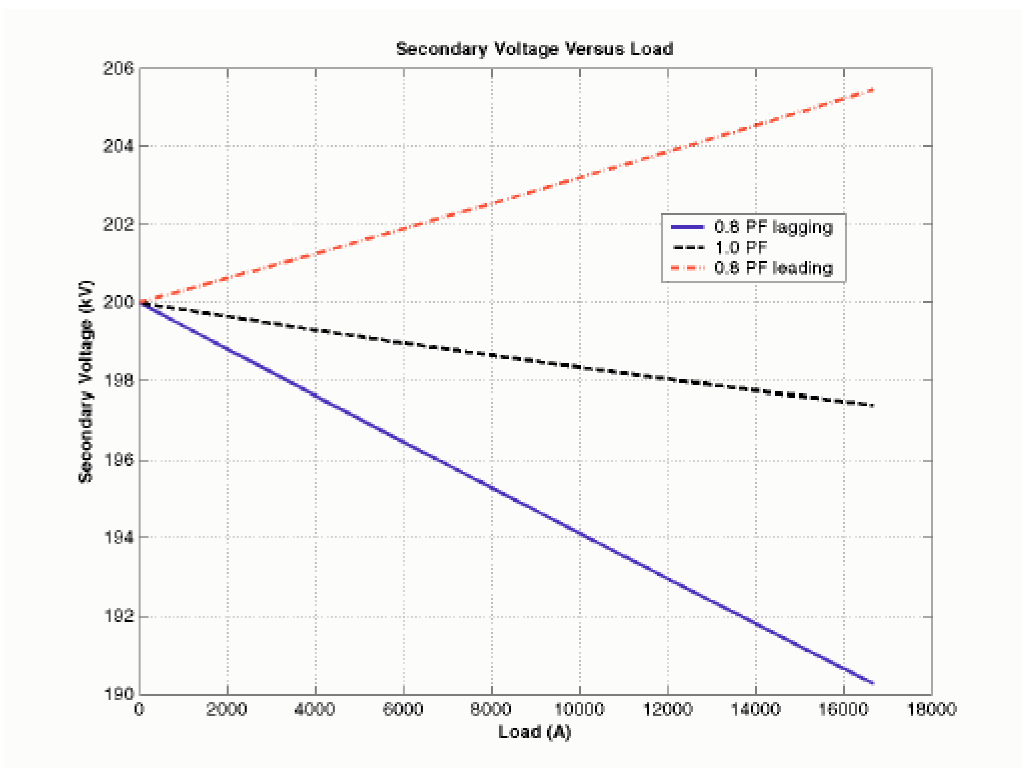
$$VR = \frac{V_P - V_S^P}{V_S^P} \times 100\% = \frac{15755 - 15000}{15000} \times 100\% = 5.03\%$$

(c) This problem is repetitive in nature, and is ideally suited for MATLAB. A program to calculate the secondary voltage of the transformer as a function of load is shown below:

```
% M-file: prob2_8.m
% M-file to calculate and plot the secondary voltage
% of a transformer as a function of load for power
% factors of 0.8 lagging, 1.0, and 0.8 leading.
% These calculations are done using an equivalent
% circuit referred to the primary side.

% Define values for this transformer
VP = 15000;            % Primary voltage (V)
amps = 0:166.67:16667; % Current values (A)
Req = 0.0135;          % Equivalent R (ohms)
Xeq = 0.0563;          % Equivalent X (ohms)

% Calculate the current values for the three
% power factors. The first row of I contains
% the lagging currents, the second row contains
% the unity currents, and the third row contains
% the leading currents.
I(1,:) = amps .* ( 0.8 - j*0.6); % Lagging
I(2,:) = amps .* ( 1.0 ); % Unity
```

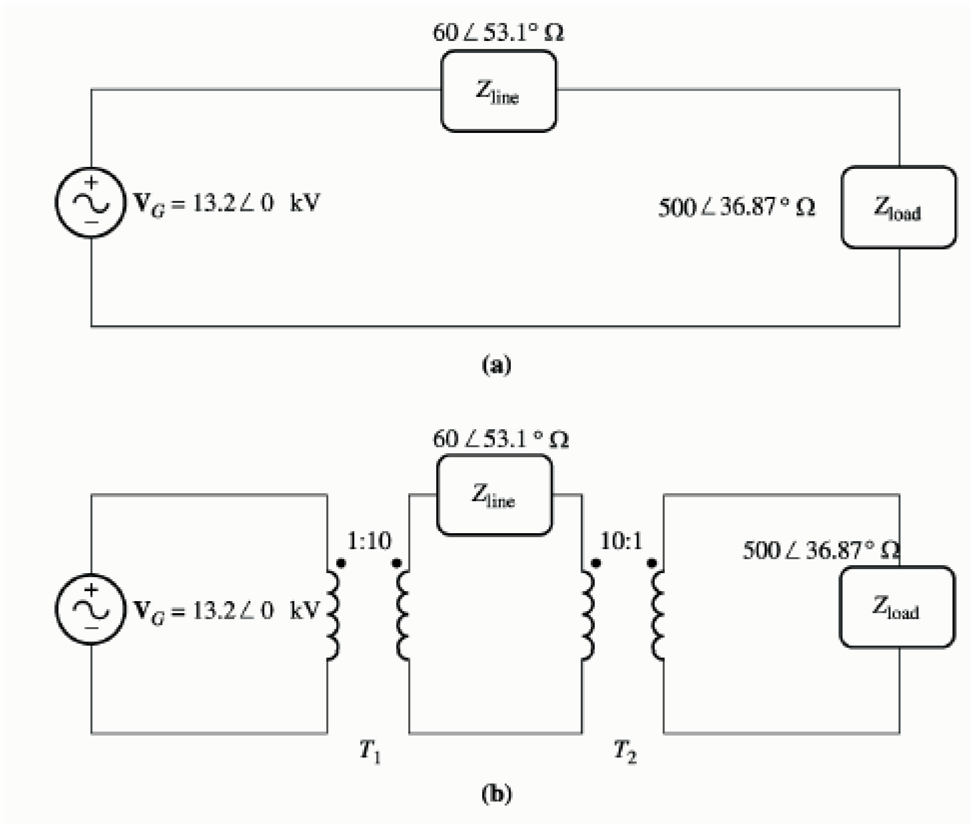
```
I(3,:) = amps .* ( 0.8 + j*0.6); % Leading

% Calculate VS referred to the primary side
% for each current and power factor.
aVS = VP - (Req.*I + j.*Xeq.*I);

% Refer the secondary voltages back to the
% secondary side using the turns ratio.
VS = aVS * (200/15);

% Plot the secondary voltage (in kV!) versus load
plot(amps,abs(VS(1,:)/1000),'b-','LineWidth',2.0);
hold on;
plot(amps,abs(VS(2,:)/1000),'k--','LineWidth',2.0);
plot(amps,abs(VS(3,:)/1000),'r-.','LineWidth',2.0);
title ('\bfSecondary Voltage Versus Load');
xlabel ('\bfLoad (A)');
ylabel ('\bfSecondary Voltage (kV)');
legend('0.8 PF lagging','1.0 PF','0.8 PF leading');
grid on;
hold off;
```

The resulting plot of secondary voltage versus load is shown below:



Problem 2-14:

A 13.2-kV single-phase generator supplies power to a load through a transmission line. The load's impedance is $Z_{load} = 500\ \angle 36.87°\Omega$, and the transmission line's impedance is $Z_{line} = 60\ \angle 53.1°\Omega$.



(a) If the generator is directly connected to the load, what is the ratio of the load voltage to the generated voltage? What are the transmission losses of the system?
(b) If a 1:10 step-up transformer is placed at the output of the generator and a 10:1 transformer is placed at the load end of the transmission line, what is the new ratio of the load voltage to the generated voltage? What are the transmission losses of the system now? (Note: The transformers may be assumed to be ideal.)

Solution:

(a) In the case of the directly-connected load, the line current is

$$I_{line} = I_{load} = \frac{V_G}{Z_{line} + Z_{load}} = \frac{13.2\angle 0°\text{kV}}{60\angle 53.1° + 500\angle 36.87°} = 23.66\angle -38.6°\text{A}$$

The load voltage is

$$V_{load} = I_{load} Z_{load} = (23.66\angle -38.6°)(500\angle 36.87°) = 11.83\angle -1.73° kV$$

The ratio of the load voltage to the generated voltage is

$$ratio = \frac{V_{load}}{V_G} = \frac{11.83\ \text{kV}}{13.2\ \text{kV}} = 0.896$$

The resistance in the transmission line is

$$R_{line} = |Z_{line}| \cos\theta = (60)(\cos 53.1°) = 36\Omega$$

The transmission losses in the system are

$$P_{loss} = I_{line}^2 R_{line} = (23.66)^2 (36) = 20.1\ \text{kW}$$

(b) In this case, a 1:10 step-up transformer precedes the transmission line a 10:1 step-down transformer follows the transmission line. If the transformers are removed by referring the transmission line to the voltage levels found on either end, then the impedance of the transmission line becomes

$$Z'_{line} = a^2 Z_{line} = (1/10)^2 (60\angle 53.1°) = 0.60\angle 53.1° \Omega$$

The current in the referred transmission line and in the load becomes

$$I'_{line} = I_{load} = \frac{V_G}{Z'_{line} + Z_{load}} = \frac{13.2\angle 0° kV}{0.60\angle 53.1° + 500\angle 36.87°} = 26.37\angle -36.89° A$$

The load voltage is

$$V_{load} = I_{load} Z_{load} = (26.37\angle -36.89°)(500\angle 36.87°) = 13.185\angle -0.02° kV$$

The ratio of the load voltage to the generated voltage is

$$ratio = \frac{V_{load}}{V_G} = \frac{13.185k}{13.2k} = 0.9989$$

The current in the transmission line is

$$I_{line} = a I_{load} = (1/10)(26.37) = 2.637 A$$

The losses in the transmission line are

$$P_{loss} = I_{line}^2 R_{line} = (2.637)^2 (36) = 250 W$$

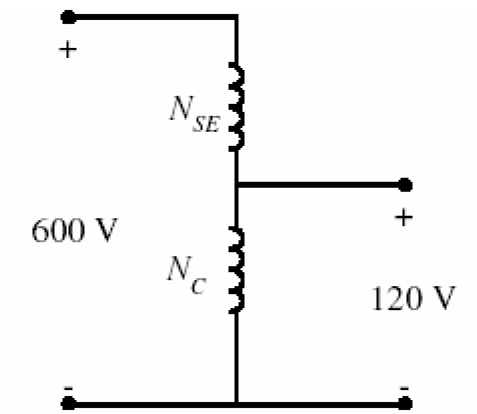
Problem 2-15:

A 5000-VA, 480/120-V conventional transformer is to be used to supply power from a 600-V source to a 120-V load. Consider the transformer to be ideal, and assume that all insulation can handle 600 V.

(a) Sketch the transformer connection that will do the required job.
(b) Find the kilovoltampere rating of the transformer in the configuration.
(c) Find the maximum primary and secondary currents under these conditions.

Solution:

(a) The common winding must be the *smaller* of the two windings, and $N_{SE} = 4\ N_C$. The transformer connection is shown below:



(b) The kVA rating of the transformer is

$$S_{IO} = \frac{N_{SE} + N_C}{N_C} S_W = \frac{4N_C + N_C}{N_C}(5000) = 6250 VA$$

(c) The maximum primary current for this configuration is

$$I_P = \frac{S_{IO}}{V_P} = \frac{6250}{600} = 10.4 A$$

The maximum secondary current is

$$I_S = \frac{S_{IO}}{V_S} = \frac{6250}{120} = 52.1 A$$

Problem 2-19:

A 20-kVA, 20,000/480-V, 60-Hz distribution transformer is tested with the following results:

Open-circuit test (measured from secondary side)	Short-circuit test (measured from primary side)
V_{OC} = 480 V	V_{SC} = 1130 V
I_{OC} = 1.60 A	I_{SC} = 1.00 A
P_{OC} = 305 W	P_{SC} = 260 W

(a) Find the per-unit equivalent circuit for this transformer at 60 Hz.
(b) What would be the rating of this transformer be if it were operated on a 50-Hz power system?
(c) Sketch the per-unit equivalent circuit of this transformer referred to the primary side *if it is operating at 50 Hz*.

Solution:

(a) The base impedance of this transformer referred to the primary side is

$$Z_{base,P} = \frac{V_P^2}{S} = \frac{(20000)^2}{20k} = 20\ \text{k}\Omega$$

The base impedance of this transformer referred to the secondary side is

$$Z_{base,S} = \frac{V_S^2}{S} = \frac{(480)^2}{20k} = 11.52\ \Omega$$

The open-circuit test yields the values for the excitation branch (referred to the secondary side)

$$\left|Y_{EX}\right| = \frac{I_{OC}}{V_{OC}} = \frac{1.60}{480} = 0.00333$$

$$\theta = -\cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}} = -\cos^{-1}\frac{305}{(480)(1.60)} = -66.6°$$

$$Y_{EX} = \left|Y_{EX}\right|\angle\theta = G_C - jB_M = 0.00333\angle -66.6° = 0.00132 - j0.00306$$

$$R_C = \frac{1}{G_C} = \frac{1}{0.00132} = 757\Omega$$

$$X_M = \frac{1}{B_M} = \frac{1}{0.00306} = 327\Omega$$

The excitation branch elements can be expressed in per-unit as

$$R_{C,pu} = \frac{R_C}{Z_{base,S}} = \frac{757}{11.52} = 65.7\,pu \qquad X_{M,pu} = \frac{X_M}{Z_{base,S}} = \frac{327}{11.52} = 28.4\,pu$$

The short-circuit test yields the values for the series impedances (referred to the primary side)

$$\left|Z_{EQ}\right| = \frac{V_{SC}}{I_{SC}} = \frac{1130}{1.00} = 1130$$

$$\theta = \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}} = \cos^{-1}\frac{260}{(1130)(1.00)} = 76.7^\circ$$

$$Z_{EQ} = \left|Z_{EQ}\right|\angle\theta = R_{EQ} + jX_{EQ} = 1130\angle 76.7^\circ = 260 + j1100$$

$$R_{EQ} = 260\,\Omega$$

$$X_{EQ} = 1100\,\Omega$$

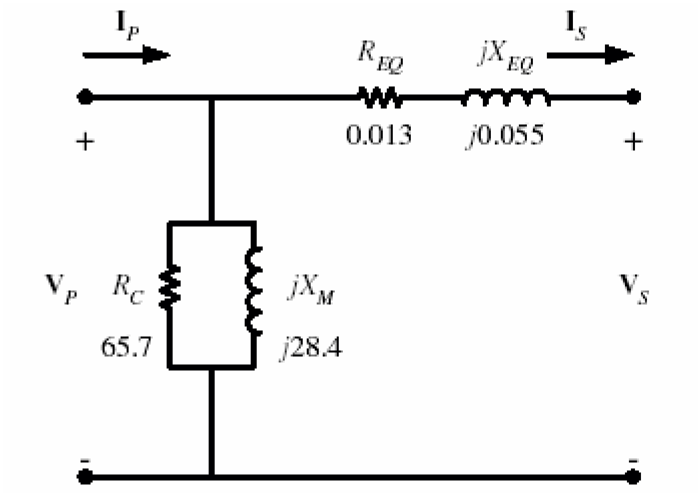
The resulting per-unit impedances are

$$R_{EQ,pu} = \frac{R_{EQ}}{Z_{base,P}} = \frac{260}{20k} = 0.013\,pu \qquad X_{EQ,pu} = \frac{X_{EQ}}{Z_{base,P}} = \frac{1100}{20k} = 0.055\,pu$$

Therefore the per-unit equivalent circuit is show below

$$R_{EQ,pu} = 0.013\,pu \qquad X_{EQ,pu} = 0.055\,pu$$

$$R_{C,pu} = 65.7\,pu \qquad X_{M,pu} = 28.4\,pu$$



(b) If this transformer were operated at 50 Hz, both the voltage and apparent power would have to be derated by a factor of 50/60, so its ratings would be

$$S_{50} = (50/60)S_{60} = (50/60)(20k) = 16.67 \text{ kVA}$$
$$V_{P,50} = (50/60)V_{P,60} = (50/60)(20000) = 16667V$$
$$V_{S,50} = (50/60)V_{S,60} = (50/60)(480) = 400V$$

(c) The transformer parameters referred to the primary side at 60 Hz are

$$R_{EQ} = Z_{base}R_{EQ,pu} = (20k)(0.013) = 260\Omega$$
$$X_{EQ} = Z_{base}X_{EQ,pu} = (20k)(0.055) = 1100\Omega$$
$$R_C = Z_{base}R_{C,pu} = (20k)(65.7) = 1.31M\Omega$$
$$X_M = Z_{base}X_{M,pu} = (20k)(28.4) = 568k\Omega$$

At 50 Hz, the resistances will be unaffected but the reactances are reduced in direct proportion to the decrease in frequency.

$$R_{EQ,50} = R_{EQ,60} = 260\Omega$$
$$X_{EQ,50} = (50/60)X_{EQ,60} = (50/60)(1100) = 917\Omega$$
$$R_{C,50} = R_{C,60} = 1.31M\Omega$$
$$X_{M,50} = (50/60)X_{M,60} = (50/60)(568) = 473k\Omega$$

The base impedance of the transformer operating at 50 Hz referred to the primary side is

$$Z_{base,P,50} = \frac{V_P^2}{S} = \frac{(16667)^2}{16.67k} = 16.67k\Omega$$

The resulting per-unit equivalent circuit referred to the primary at 50 Hz is shown below

$$R_{EQ,pu,50} = \frac{R_{EQ,50}}{Z_{base,P,50}} = \frac{260}{16.67k} = 0.016\,pu \qquad X_{EQ,pu,50} = \frac{X_{EQ,50}}{Z_{base,P,50}} = \frac{917}{16.67k} = 0.055\,pu$$

$$R_{C,pu,50} = \frac{R_{C,50}}{Z_{base,P,50}} = \frac{1.31M}{16.67k} = 78.5\,pu \qquad X_{M,pu,50} = \frac{X_{M,50}}{Z_{base,P,50}} = \frac{473k}{16.67k} = 28.4\,pu$$