ELG3150
Problems for Chapter 10

P10.5, P10.18, AP10.4, DP10.7

P10.5 A stabilized precision rate table uses a precision tachometer and a dc direct-drive torque motor, as shown in Figure P10.5 in the textbook. It is desired to maintain a high steady-state accuracy for the speed control. To obtain a zero steady-state error for a step command design, select a proportional plus integral compensator as discussed in section 10.6. Select the appropriate gain constants so that the system has an overshoot of approximately 10% and a settling time (2% criterion) in the range of 0.4 to 0.6 second.

Solution

We desire P.O. < 10% and $0.4 < T_s < 0.6$ sec. The compensator is a PI-type, given by

$$G_c(s) = K_2 + \frac{K_3}{s} = \frac{K_2 s + K_3}{s} = \frac{K_2(s+a)}{s}$$

where $a = K_3 / K_2$. So, $e_{ss} = 0$ for a step input and

$$G(s) = \frac{2.5K_a}{(s+0.1)(0.1s+1)} = \frac{25K_a}{(s+0.1)(s+10)}$$

The open-loop transfer function is

$$G_c G(s) = \frac{25K_a K_2(s+a)}{s(s+0.1)(s+10)}$$

Using root locus methods, we select a = 0.03 (after several iterations) and determine $K_a K_2$ to yields ζ = 0.65. This results in

$$K_a K_2 = 2.3$$

The root locus is shown in Figure P10.5. The design specifications are met, but not without difficulty. The settling time specification is most difficult since we cannot place dominant poles with $\zeta\omega_n$ > 10 (see the root locus, Figure P10.5).

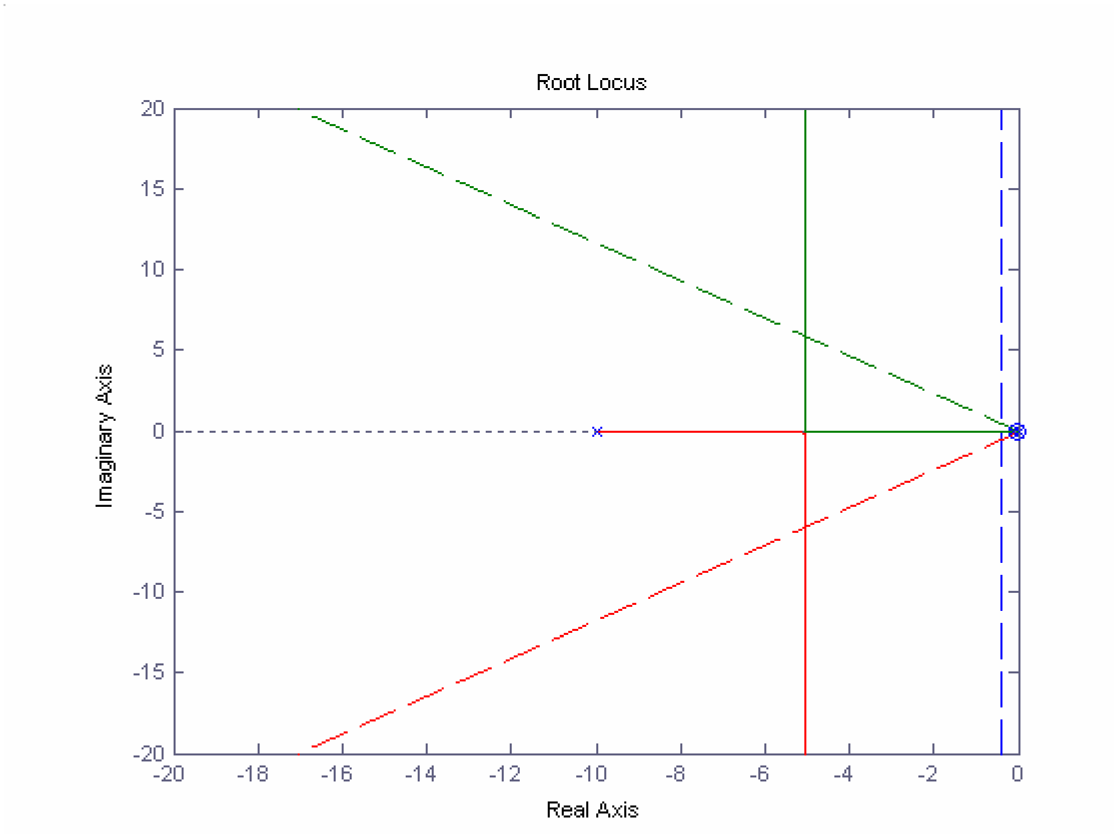

Figure P10.5

P10.18 A unity feedback control system for a robot submarine has a plant with a third-order transfer function

$$G(s) = \frac{K}{s(s+10)(s+50)}$$

It is desired that the overshoot be approximately 7.5% for a step input and the settling time (2% criterion) of the system be 400ms. Find a suitable phase-lead compensator by using root locus methods. Let the zero of the compensator be located at s=-15, and determine the compensator pole. Determine the resulting system K_v.

Solution

The plant transfer function is

$$G(s) = \frac{K}{s(s+10)(s+50)}$$

We desire $\zeta\omega_n > 10$ to meet $T_s < 0.4$ sec and $\zeta = 0.65$ to meet $P.O. < 7.5\%$. Try a pole at s = -120. The root locus is shown in Figure P10.18. The gain K = 6000 for $\zeta = 0.65$. Thus,

$$GG_c(s) = \frac{6000(s/15+1)}{s(s+10)(s+50)(s.120+1)} \qquad \text{and} \qquad K_v = \frac{6000}{500} = 12.$$

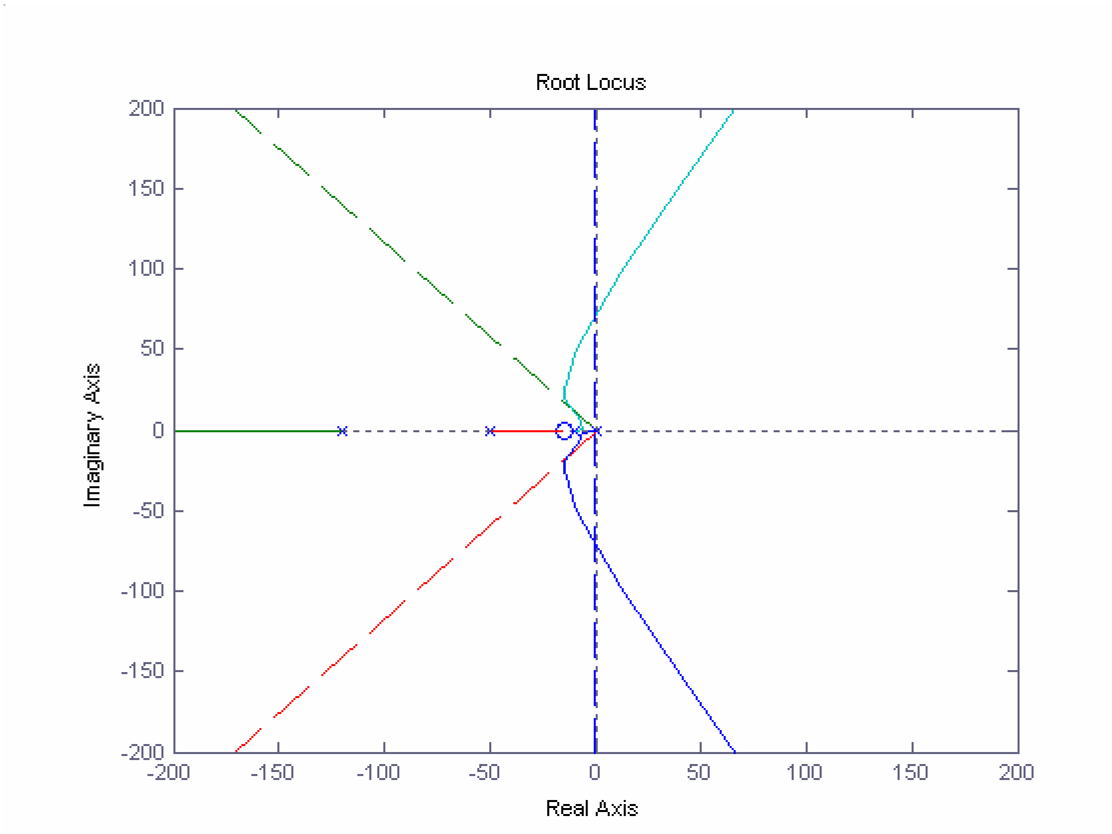

Figure P10.18

AP10.4 A dc motor control system with unity feedback has the form shown in Figure AP10.4 in the textbook. Select K_1 and K_2 so that the system response has a settling time (2% criterion) less than 0.6 second and an overshoot less than 5% for a step input.

Solution
The closed-loop transfer functions

$$T(s) = \frac{64K_1}{s^2 + 64(1+K_1K_2)s + 64K_1}$$

From the performance specifications, we determine that the natural frequency and damping of the dominant poles should be ω_n = 11.3 and ζ = 0.707. So,

$$s^2 + 64(1+K_1K_2)s + 64K_1 = s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 16s + 128$$

Solving for the gains yields $K_1 = 2$ and $K_2 = -0.375$. The closed-loop transfer function is

$$T(s) = \frac{128}{s^2 + 16s + 128}$$

The performance results are $P.O. = 4.3\%$ and $T_s = 0.53$ seconds.

DP10.7 The linear model for the system is a feedback system with unity feedback and

$$\frac{Y(s)}{E(s)} = G(s) = \frac{K(s+4000)}{s(s+1000)(s+3000)(s+p_1)(s+p_1^*)},$$

where $p_1 = 2000 + j2000$, and Y(s) is position.

The specifications for the system are (1) settling time of less than 12ms, (2) an overshoot to a step position command of less than 10%, and (3) a steady-state velocity error of less than .5%. Determine a compensator scheme to achieve these stringent specifications.

Solution

The stringent design specifications are $K_v > 200$; $T_s < 12$ ms and percent overshoot $P.O. < 10\%$. A suitable compensator is

$$G_c(s) = K\frac{s+403}{s+2336}$$

where K = 1.9476e+13. Then,

$P.O. = 9.5\%$ $\qquad$ $T_s = 10$ ms $\qquad$ $K_v = 560$.

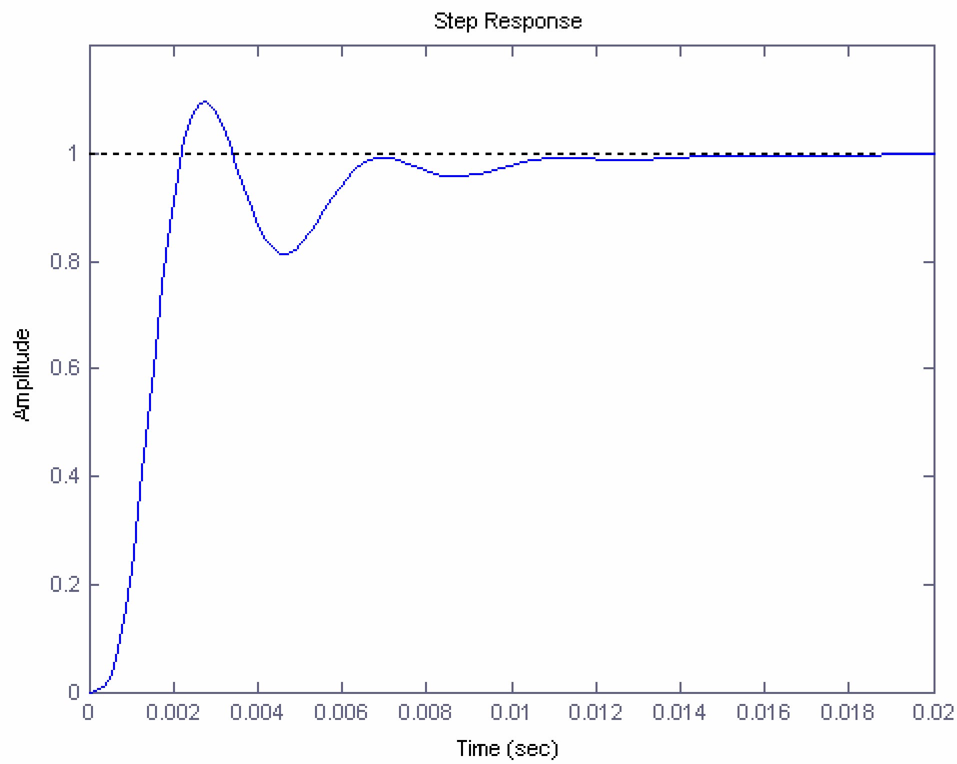

Figure DP10.7 Step response for the tape transport system.