ELG3150: Chapter 5

E5.12

The system is a type 0.
The error constants are: $K_p = 0.2$ and $K_v = 0$.
The steady-state error to a ramp input is ∞.
The steady-state error to a step input is

$$e_{ss} = \frac{1}{1+K_p} = 0.833$$

P5.5

(a) The closed-loop transfer function is

$$T(s) = \frac{K_1 K_2 (s+1)}{s^2 + K_1 K_2 s + K_1 K_2}$$

For P. O. of 5%, $\zeta \geq 0.69$. Let us choose $\zeta = 0.69$.
$2\zeta\omega_n = K_1K_2$;

$$\begin{aligned} &\omega_n^2 = K_1 K_2 \\ &2(0.69)\omega_n = \omega_n^2 \\ &\omega_n = 1.38 \\ &K_1 K_2 = 1.9 \end{aligned}$$

When $K_1K_2 \geq 1.9$, then $\zeta \geq 0.69$.

(b) This system is type 2. Therefore the steady-state error for both step and ramp input is zero.

© For step input, the optimum ITAE characteristic equation is

$$s^2 + 1.4\omega_n s + \omega_n^2 = 0$$

For a ramp input, the optimum ITAE characteristic equation is

$$s^2 + 3.2\omega_n s + \omega_n^2 = 0$$

Therefore

$$\begin{aligned} &K_1 K_2 = \omega_n^2 = 3.2\omega_n \\ &\omega_n == 3.2 \\ &K_1 K_2 = 10.24 \end{aligned}$$

DP5.3

The closed-loop transfer function

$$T(s) = \frac{K\omega_n^2}{s^3 + 2\xi\omega_n s2 + \omega_n^2 s + K\omega_n^2}$$

Where $\zeta = 0.2$. From the second-order approximation

$$T_p = \frac{\pi}{\omega n\sqrt{1-\xi^2}}$$

We should have ω_n large in order to have T_p small. From the problem we have

$$0.1 \langle K / \omega_n \langle 0.3$$

We may select $\omega_n = 20$, so $K = 4$. This gives P. O. = 2% and $T_P = 0.9$ seconds.

DP5.3

The closed-loop transfer function is

$$T(s) = \frac{K}{s^2 + qs + K}$$

From the ITAE specifications, we desire

$$T(s) = \frac{\omega_n^2}{s2 + 1.4\omega_n + \omega_n^2}$$

We have $2\zeta\omega_n = 1.4\ \omega_n$, which implies $\zeta = 0.7$.

Since we want $T_s \le 0.5$, we require $\zeta\omega_n \ge 8$

$\omega_n \ge 8/0.7 = 11.4$; we may select $\omega_n = 12$

$$T(s) = \frac{144}{s^2 + 16.8s + 144}$$

Therefore $K = 144$ and $q = 16.8$ and P.O. = 4.5%.