ELG3150: Chapter 4

E4.1

(a) The system sensitivity to τ is

$$
\begin{aligned}
G(s) &= \frac{100}{\tau s+1} \\
S_\tau^T &= S_G^T S_\tau^G \\
S_G^T &= \frac{\partial T}{\partial G}\times\frac{G}{T} = \frac{1}{1+GH(s)} = \frac{1}{1+\dfrac{100}{3s+1}} = \frac{3s+1}{3s+101} \\
S_\tau^G &= \frac{-\tau s}{\tau s+1} = \frac{-3s}{3s+1} \\
S_\tau^T &= \frac{\tau s+1}{3s+101}\times\frac{-\tau s}{\tau s+1} = \frac{-3s}{3s+101}[\text{when } \tau=3]
\end{aligned}
$$

(b) The closed-loop transfer function

$$
T(s) = \frac{G(s)}{1+GH(s)} = \frac{100}{3s+101} = \frac{100/101}{\dfrac{3}{101}s+1} = \frac{0.99}{\tau_c s+1}
$$

The time constant $\tau_c = 3/101 = 0.0297$.

P4.2

$$
G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}
$$

K_a = Actuator constant
K_1 = Roll Sensor constant

(a) The open-loop transfer function is

$$
T(s) = K_a G(s)
$$

Therefore

$$
S_{K_a}^T = 1
$$

The closed-loop transfer function

$$
\begin{aligned}
T(s) &= \frac{K_a G(s)}{1+K_a K_1 G(s)} \\
S_{K_1}^T &= \frac{\partial T}{\partial K_1}.\frac{K_1}{T} = \frac{-K_a K_1 G(s)}{1+K_a K_1 G(s)} \\
S_{K_a}^T &= \frac{1}{1+K_1 K_a G(s)}
\end{aligned}
$$

(b) The tracking error, $E(s) = \theta_d(s) - \theta(s) = -\theta(s)$ since $\theta_d(s) = 0$. The transfer function from the wave disturbance to the output $\theta(s)$ is

$$
\theta(s) = \frac{G(s)}{1+K_1 K_a G(s)} T_d(s)
$$

Consider a step disturbance input for the open- and closed-loop systems. For the open-loop system

$$e_{ss} = -\lim_{s\to 0} sG(s)\frac{A}{s} = -A$$

For the closed-loop system

$$e_{ss} = \lim_{s\to 0} s\left(\frac{-G(s)}{1+K_1K_aG(s)}\right)\frac{A}{s} = \frac{-A\omega_n^2}{1+K_1K_a\omega_n^2}$$

P4.12

(a) The two transfer functions

$$T_1(s) = \frac{Y(s)}{R(s)} = \frac{K_1K_2}{1+0.0099K_1K_2}$$

$$T_2(s) = \frac{Y(s)}{R(s)} = \frac{K_1K_2}{(1+0.09K_1)(1+0.09K_2)}$$

(b) When $K_1 = K_2 = 100$

$$T_1(s) = \frac{Y(s)}{R(s)} = \frac{100^2}{1+0.0099\times 10000} = 100$$

$$T_1(s) = \frac{Y(s)}{R(s)} = \frac{K_1K_2}{(1+0.09\times 100)(1+0.09\times 100)} = 100$$

© The sensitivity

$$S_{K_1}^{T_1} = \frac{\partial T_1}{\partial K_1}.\frac{K_1}{T_1} = \frac{1}{1+0.0099K_1K_2} = 0.01$$

$$S_{K_1}^{T_2} = \frac{\partial T_2}{\partial K_1}.\frac{K_1}{T_2} = \frac{1}{1+0.09K_1K_2} = 0.1$$

For $K_1 = 100$

$$S_{K_1}^{T_1} = \frac{S_{K_1}^{T_2}}{10}$$

AP4.1

The plant transfer function

$$G_p(s) = \frac{R}{RCs+1}$$

The closed-loop transfer function

$$H(s) = \frac{1}{1+GG_p(s)}Q_3(s) + \frac{GG_p(s)}{1+GG_p(s)}H_d(s)$$

With $E(s) = H_d(s)\text{-}H(s)$

$$E(s) = \frac{-1}{1+GG_p(s)}Q_3(s)$$

(a) When $G(s) = K$

$$e_{ss} = \lim_{s\to 0} sE(s) = \frac{-1}{1+KR}$$

(b) When $G(s) = K/s$

$$e_{ss} = \lim_{s \to 0} sE(s) = 0$$