Example:

Derive the transfer function of the given electrical network.

$$Z_1 = \frac{R_1}{R_1Cs + 1}, \qquad Z_2 = R_2$$

The transfer function between the output $E_o(s)$ and the input $E_i(s)$ is

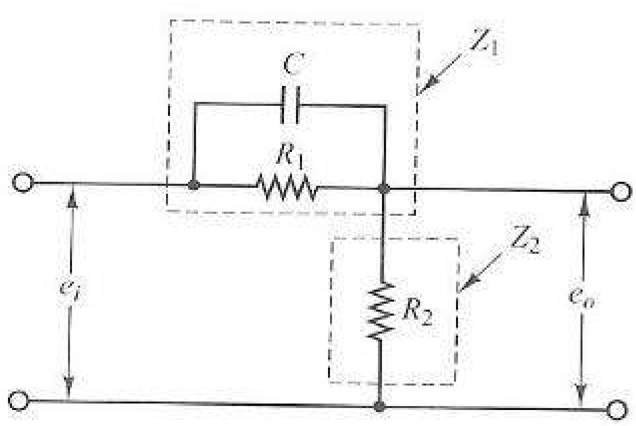
$$\frac{E_o(s)}{E_i(s)} = \frac{Z_2}{Z_1 + Z_2} = \frac{R_2}{R_1 + R_2} \frac{R_1Cs + 1}{\frac{R_1R_2}{R_1 + R_2}Cs + 1}$$

Define

$$R_1C = T, \qquad \frac{R_2}{R_1 + R_2} = \alpha$$

Then the transfer function becomes

$$\frac{E_o(s)}{E_i(s)} = \alpha \frac{Ts + 1}{\alpha Ts + 1} = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}$$



Example:

Derive the transfer function of the given electrical network.

The complex impedances Z_1 and Z_2 are

$$Z_1 = \frac{R_1}{R_1 C_1 s + 1}, \qquad Z_2 = R_2 + \frac{1}{C_2 s}$$

The transfer function between $E_o(s)$ and $E_i(s)$ is

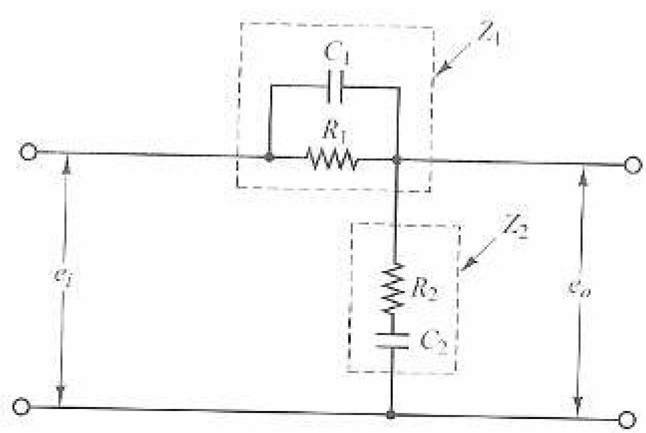
$$\frac{E_o(s)}{E_i(s)} = \frac{Z_2}{Z_1 + Z_2} = \frac{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}{(R_1 C_1 s + 1)(R_2 C_2 s + 1) + R_1 C_2 s}$$

The denominator of this transfer function can be factored into two real terms. Let us define

$$R_1 C_1 = T_1, \qquad R_2 C_2 = T_2, \qquad R_1 C_1 + R_2 C_2 + R_1 C_2 = \frac{T_1}{\beta} + \beta T_2$$

Then $E_o(s)/E_i(s)$ can be simplified to

$$\frac{E_o(s)}{E_i(s)} = \frac{(T_1 s + 1)(T_2 s + 1)}{\left(\frac{T_1}{\beta} s + 1\right)(\beta T_2 s + 1)} = \frac{\left(s + \frac{1}{T_1}\right)\left(s + \frac{1}{T_2}\right)}{\left(s + \frac{\beta}{T_1}\right)\left(s + \frac{1}{\beta T_2}\right)}$$



Example:

For the automobile suspension system shown in the schematic diagram, obtain the transfer function of this system.

$$m\ddot{x}_o + b(\dot{x}_o - \dot{x}_i) + k(x_o - x_i) = 0$$

or

$$m\ddot{x}_o + b\dot{x}_o + kx_o = b\dot{x}_i + kx_i$$

Taking the Laplace transform of this last equation, assuming zero initial conditions, we obtain

$$(ms^2 + bs + k)X_o(s) = (bs + k)X_i(s)$$

Hence the transfer function $X_o(s)/X_i(s)$ is given by

$$\frac{X_o(s)}{X_i(s)} = \frac{bs + k}{ms^2 + bs + k}$$