ELG 4151

Linear Systems

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Agenda for this tutorial session

- 1) I will talk about the Linear System Elements (Devices): Elements Store Energy (Inductive Elements & Capacitive Elements), Elements Dissipate Energy, Elements Transfer Energy: Mass, Inertia, Inductor, Pipe, Spring, Rotary Spring, Capacitor, Tank, Friction, Rotational Friction, Damper, Resistor, Liquid Resistance, Gears, Transformer, Lever, Motor.

  (25 min)
- 2) I will present two examples on how to drive the mathematical model and state space representation of two mechanical systems.

  (20 min)
- 3) I will present two examples on how to drive the mathematical model and state space representation of two electrical systems.

  (20 min)
- 4) I will present one example on how to drive the mathematical model and state space representation of an electromechanical system.

  (10 min)

Elements of Linear Systems

- Elements Store Energy
  - Inductive Elements
    - Mass
    - Inertia
    - Inductor
    - Pipe
  - Capacitive Elements
    - Spring
    - Rotary Spring
    - Capacitor
    - Tank

- Elements Dissipate Energy
  - Friction
  - Rotational Friction
  - Damper
  - Resistor
  - Liquid Resistance
- Elements Transfer Energy
  - Gears
  - Transformer
  - Lever
  - Motor

Mathematical Formulas and State Variables of Energy Storage Elements

- **Mass**

$$\sum F = M\dot{v} \xrightarrow{s.v.} v$$

- **Inertia**

$$\sum T = J\dot{\omega} \xrightarrow{s.v.} \omega$$

- **Inductor**

$$e = L\,\dot{i} \qquad \xrightarrow{s.v.} \; i$$

- **Pipe**

$$\tilde{P} = I_w \dot{Q} \qquad \xrightarrow{s.v.} \; Q$$

- **Spring**

$$F_s = kx \xrightarrow{s.v.} x$$

- **Rotary Spring**

$$T = k\theta \xrightarrow{s.v.} \theta$$

- **Capacitor**

$$e = \frac{1}{C} q \xrightarrow{s.v.} q$$

- **Tank**

$$P = \rho g h \xrightarrow{s.v.} h$$

Mathematical Formulas of Energy Dissipating Elements

- **Friction**

$$F = b\,v$$

- **Rotational Friction**

$$T = b\omega$$

- **Damper**

$$F = b\,v$$

- **Resistor**

$$e = Ri$$

- **Liquid Resistance**

$$\tilde{P} = R_w Q$$

Mathematical Formulas of Energy Transferring Elements

$$P_{in} = P_{out}$$

$$P_{elct} = vi \qquad\qquad P_{mech} = T\omega$$

- **Gears**

$$T_1\omega_1 = T_2\omega_2$$

$$\omega_2 = \frac{r_1}{r_2}\omega_1 \qquad\qquad T_2 = \frac{r_2}{r_1}T_1$$

- **Transformer**

$$v_1 i_1 = v_2 i_2$$

$$v_2 = \frac{n_2}{n_1} v_1 \qquad i_2 = \frac{n_1}{n_2} i_1$$

- **Lever**

$$F_1 l_1 = F_2 l_2$$

- **Motor**

$$e_b i = T\omega$$

$$e_b = k_t \omega \qquad T = K_t i$$

Your Questions