Tutorial No. 1, ELG2336, winter 2008

Problem 3.5

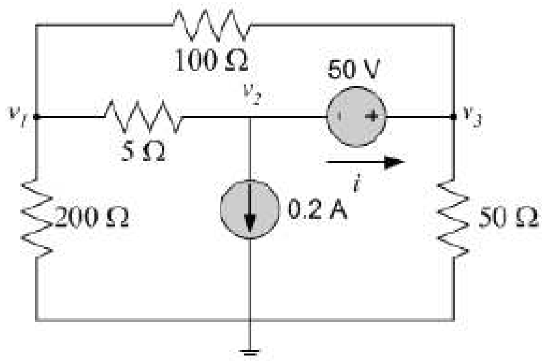


Figure 1: Problem 3.5

Known quantities:

Circuit shown in Figure 1 with resistance values, current and voltage source values.

Find:

The current, i, through the voltage source using node voltage analysis.

Analysis:

At node 1:

$$\frac{v_1}{200} + \frac{v_1 - v_2}{5} + \frac{v_1 - v_3}{100} = 0 \qquad (1)$$

At node 2:

$$\frac{v_2 - v_1}{5} + i + 0.2 = 0 \qquad (2)$$

At node 3:

$$-i + \frac{v_3 - v_1}{100} + \frac{v_3}{50} = 0 \qquad (3)$$

And for the voltage source we have:

$$v_3 - v_2 = 50 \qquad (4)$$

We have four equations with for unknowns, i.e. v_1, v_2, v_3, i. Solving the system for i, we have the followings:

$(4) \rightarrow v_2 = v_3 - 50$

$(3) \rightarrow 0.03v_3 = i + 0.01v_1 \rightarrow v_1 = 3v_3 - 100i$

$(2) \rightarrow 0.2v_2 + i + 0.2 = 0.2v_1 \xrightarrow{(4),(3)} 0.2(v_3 - 50) + i + 0.2 = 0.2(3v_3 - 100i)$

$\rightarrow 0.2v_3 - 10 + i + 0.2 = 0.6v_3 - 20i \rightarrow 0.4v_3 = 21i - 9.8 \qquad (\star)$

$(1) \rightarrow 0.215v_1 - 0.2v_2 - 0.01v_3 = 0 \xrightarrow{(4),(3)} 0.215(3v_3 - 100i) - 0.2(v_3 - 50) - 0.01v_3 = 0$

$\rightarrow 0.435v_3 = 21.5i - 10 \xrightarrow{(\star)} 0.435(21i - 9.8) = 8.6i - 4$

$\rightarrow 0.535i = 0.263 \rightarrow i = 0.491\ A$

Problem 3.6

Known quantities:
Circuit shown in Figure 2 with resistance values, current and voltage source values.

Find:
The three node voltages shown in Figure 2 using node voltage analysis.

Analysis:

At node 1:

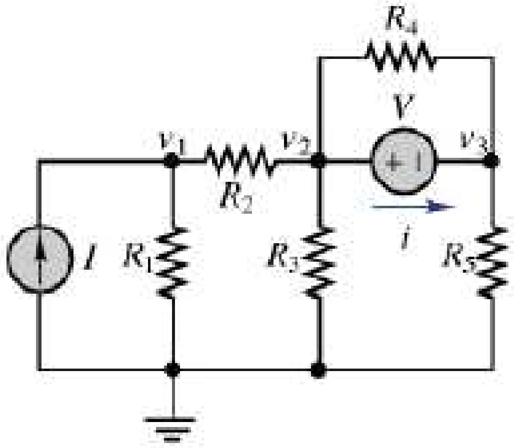


Figure 2: Problem 3.6

$$\frac{v_1}{200} + \frac{v_1 - v_2}{75} - 0.2 = 0 \qquad (1)$$

At node 2:

$$\frac{v_2 - v_1}{75} + \frac{v_2}{25} + \frac{v_2 - v_3}{50} + i = 0 \qquad (2)$$

At node 3:

$$-i + \frac{v_3 - v_2}{50} + \frac{v_3}{100} = 0 \qquad (3)$$

For the voltage source we have: $v_3 + 10 = v_2 \quad (4)$. We have again four equations with for unknowns, i.e. v_1, v_2, v_3, i. Solving the system, we have the followings:

$$(3) \rightarrow -i + 0.03v_3 = 0.02v_2 \xrightarrow{(3,4)} -i + 0.03v_3 = 0.02(v_3 + 10)$$

$$\rightarrow 0.01v_3 = i + 0.2 \rightarrow v_3 = 100i + 20 \qquad (\star)$$

$$(2) \rightarrow 2v_2 - 2v_1 + 6v_2 + 3v_2 - 3v_3 + 150i = 0 \rightarrow 11v_2 - 3v_3 - 2v_1 + 150i = 0$$

$$\xrightarrow{(4)} 8v_3 + 110 - 2v_1 + 150i = 0 \rightarrow v_1 = 75i + 4v_3 + 55 \qquad (\sharp)$$

$$(1) \rightarrow 3v_1 + 8v_1 - 8v_2 - 120 = 0 \rightarrow 11v_1 - 8v_2 = 120$$

$$\xrightarrow{(\sharp),(4),(\star)} 11(75i + 4(100i + 20) + 55) - 8(100i + 20 + 10) = 120$$

$$\rightarrow 11(475i + 135) - 8(100i + 30) = 120 \rightarrow 4425i = -1125 \rightarrow i = -0.254\ A$$

$$\Rightarrow \begin{cases} v_3 = -5.42\ V, \\ v_2 = 4.58\ V, \\ v_1 = 14.24\ V. \end{cases}$$

Problem 3.14

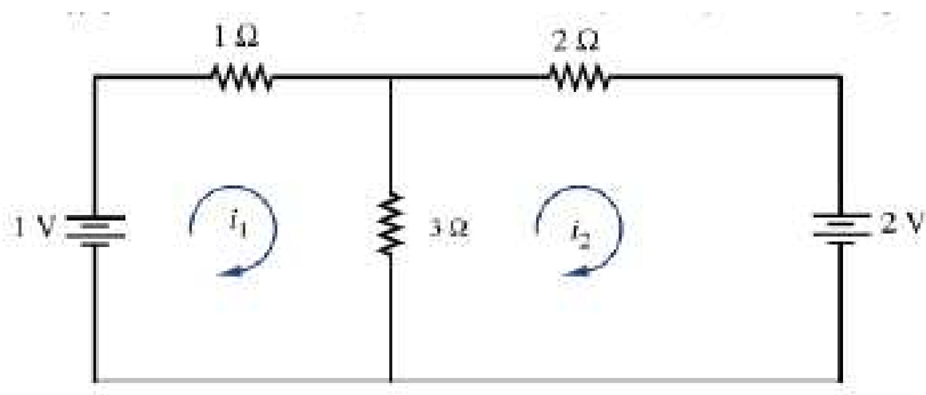


Figure 3: Problem 3.14

Known quantities:
Circuit shown in Figure 3 with resistance values and voltage source values.

Find:
Current i_1 and i_2

Analysis:

For mesh of i_1:

$$i_1 + 3(i_1 - i_2) - 1 = 0 \qquad (1)$$

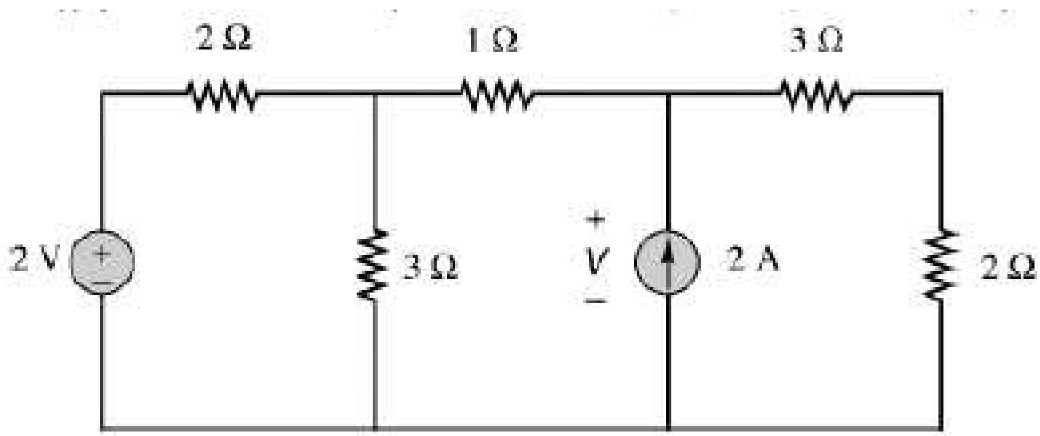
For mesh of i_2:

$$2i_2 + 3(i_2 - i_1) + 2 = 0 \qquad (2)$$

Solving for i_1 and i_2, we have:

$$(2) \rightarrow 5i_2 = 3i_1 - 2$$
$$(1) 4i_1 - 3i_2 - 1 = 0 \xrightarrow{(2)} 20i_1 - 3(3i_1 - 2) - 5 = 0$$
$$\rightarrow 11i_1 = -1 \rightarrow i_1 = -0.091A$$
$$\rightarrow i_2 = -0.455A$$

Problem 3.14



Known quantities:
Circuit shown in Figure with resistance values, current and voltage source values.

Find:
Voltage across the current source.

Analysis:

We observe that the number of meshes are less than the number of nodes, therefore we use mesh analysis to find v.

For mesh of i_1:

$$i_1(2+3) + i_2(-3) + i_3(0) - 2 = 0 \qquad (1)$$

We take a mesh of i_2 and i_3 to avoid the unknown voltage drop on the current source:

$$i_1(-3) + i_2(1+3) + i_3(3+2) = 0 \qquad (2)$$

And for the current source:

$$i_1(0) + i_2(1) + i_3(-1) + 2 = 0 \qquad (3)$$

Solving the three equations with three variables, we have:

$$\begin{aligned}
&(2) \rightarrow 3i_1 = 4i_2 + 5i_3 \xrightarrow{(3)} 3i_1 = 4(i_3 - 2) + 5i_3 = 9i_3 - 8 \quad (\star) \\
&(1) \rightarrow 5i_1 - 3i_2 = 2 \xrightarrow{(\star),(3)} 5(9i_3 - 8) - 9(i_3 - 2) = 6 \\
&\quad \rightarrow 36i_3 = 28 \rightarrow i_3 = 0.778\ A \\
&\quad \Rightarrow v = i_3(3+2) = 3.89\ V
\end{aligned}$$

Problem 3.25

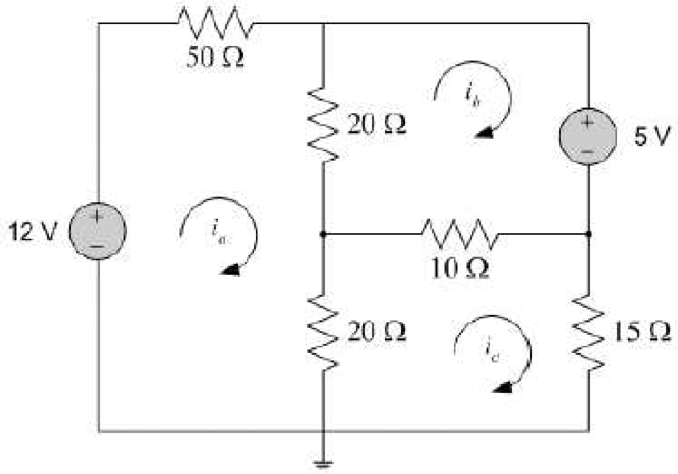


Figure 4: Problem 3.25

Known quantities:
Circuit shown in Figure 4 with resistance values and voltage source values.

Find:
Voltage across the 10Ω resistance in the circuit of Figure 4.

Analysis:

We observe that the number of meshes are less than the number of nodes, therefore we prefer mesh analysis to find v.

For mesh of i_a:

$$i_a(50 + 20 + 20) + i_b(-20) + i_c(-20) - 12 = 0 \qquad (1)$$

For mesh of i_b:

$$i_a(-20) + i_b(20 + 10) + i_c(-10) + 5 = 0 \qquad (2)$$

For mesh of i_c:

$$i_a(-20) + i_b(-10) + i_c(20 + 10 + 15) = 0 \qquad (3)$$

We can put the above equations in a matrix format:

$$\begin{bmatrix} 90 & -20 & -20 \\ -20 & 30 & -10 \\ -20 & -10 & 45 \end{bmatrix} \times \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 12 \\ -5 \\ 0 \end{bmatrix}$$

You have two options now, either find the inverse matrix or use techniques from Calculus I with row operations on both sides of the equation to find a triangular matrix. Since finding the inverse matrix is very standard, we use the latter technique as follows:

We can re-write the equation as follows:

$$\begin{bmatrix} 9 & -2 & -2 \\ -2 & 3 & -1 \\ -2 & -1 & 4.5 \end{bmatrix} \times \mathbf{i} = \begin{bmatrix} 1.2 \\ -0.5 \\ 0 \end{bmatrix}$$

We would like to find a lower triangular matrix by row operations on both sides of the above equation. So, we proceed as follows:

1. Multiply 2nd row by 9/2 and add 1st row to it.

2. Also, multiply 3nd row by 9/2 and add 1st row to it. After these operations, the result would be:

$$\begin{bmatrix} 9 & -2 & -2 \\ 0 & 11.5 & -6.5 \\ 0 & -6.5 & 18.25 \end{bmatrix} \times \mathbf{i} = \begin{bmatrix} 1.2 \\ -1.05 \\ 1.2 \end{bmatrix}$$

3. Then, multiply 3rd row by 11.5/6.5 and add the new 2nd row to it. The result will look like:

$$\begin{bmatrix} 9 & -2 & -2 \\ 0 & 11.5 & -6.5 \\ 0 & 0 & 25.79 \end{bmatrix} \times \mathbf{i} = \begin{bmatrix} 1.2 \\ -1.05 \\ 1.07 \end{bmatrix}$$

4. From the 3rd row, we can easily find i_c, since:

$$25.79i_c = 1.07 \rightarrow i_c = 41.6\ mA$$

5. By knowing $i_c = 41.6\ mA$, find i_b from the 2nd row, because:

$$11.5i_b - 6.5i_c = -1.05 \rightarrow i_b = -67.8\ mA$$

6. Finally, having known i_b and i_c, calculate i_a from the first row:

$$9i_a - 2i_b - 2i_c = 1.2 \rightarrow i_a = 127.5\ mA$$

$$\Rightarrow \begin{cases} i_a = 127.5\ mA, \\ i_b = -67.8\ mA, \\ i_c = 41.6\ mA, \\ v = 10(i_b - i_c) = 10(-0.109) = -1.09\ V. \end{cases}$$

Problem 3.26

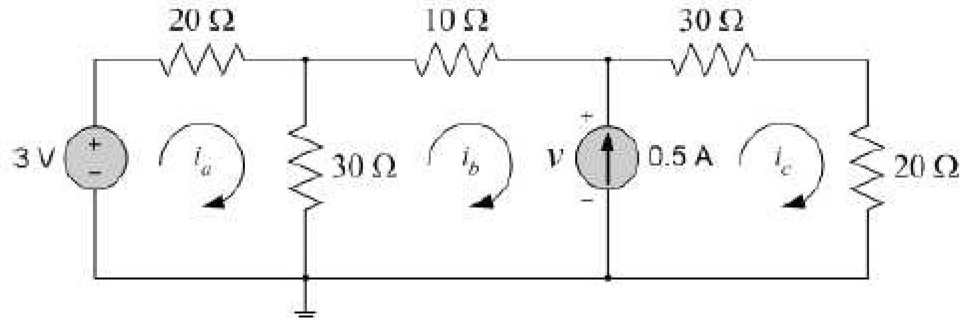


Figure 5: Problem 3.26

Known quantities:
Circuit shown in Figure 5 with resistance values, current and voltage source values.

Find:
Voltage across the current source in the circuit of Figure 4.

Analysis:

We use mesh analysis to find v.

For mesh of i_a:

$$i_a(20+30) + i_b(-30) + i_c(0) - 3 = 0 \qquad (1)$$

For mesh of i_b and i_c:

$$i_a(-30) + i_b(10+30) + i_c(30+20) = 0 \qquad (2)$$

For the current source:

$$i_b - i_c + 0.5 = 0 \qquad (3)$$

Putting these three equations in a matrix format, we get:

$$\begin{bmatrix} 50 & -30 & 0 \\ -30 & 40 & 50 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -0.5 \end{bmatrix}$$

We can re-write the equation as follows:

$$A \times \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 0.3 \\ 0 \\ -0.5 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 5 & -3 & 0 \\ -3 & 4 & 5 \\ 0 & 1 & -1 \end{bmatrix}$$

This time we find the inverse of A; the reader can verify that

$$A^{-1} = \begin{bmatrix} 0.2500 & 0.0833 & 0.4167 \\ 0.0833 & 0.1389 & 0.6944 \\ 0.0833 & 0.1389 & -0.3056 \end{bmatrix}$$

By using the inverse matrix, we have:

$$A^{-1}A \times \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = A^{-1} \begin{bmatrix} 0.3 \\ 0 \\ -0.5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} -0.133 \\ -0.322 \\ 0.178 \end{bmatrix}$$

Therefore, $v = i_c(30 + 20) = 8.89\ V$.