P11.10 The system with feedback is given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{x} ,$$

where $x_1(0) = 1$, and $x_2(0) = 0$. The characteristic equation is

$$\det[s\mathbf{I} - \mathbf{A}] = \det \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} = s(s+2) + 1 = s^2 + 2s + 1 = 0 .$$

The roots are $s_{1,2} = -1$. The solution is

$$\mathbf{x}(t) = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \mathbf{x}(0) = \begin{bmatrix} \phi_{11} \\ \phi_{21} \end{bmatrix}$$

since $x_1(0) = 1$ and $x_2(0) = 0$. We compute the elements of the state transition matrix as follows:

$$\phi_{21}(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s^2 + 2s + 1} \right\} ,$$

therefore

$$x_2(t) = te^{-t} .$$

Similarly,

$$\phi_{11}(t) = \mathcal{L}^{-1} \left\{ \frac{s}{s^2 + 2s + 1} \right\} .$$

Therefore,

$$x_1(t) = (1 - t)e^{-t} .$$

P11.11 Let

$$u = -k_1 x_1 - k_2 x_2 + \alpha r$$

where $r(t)$ is the command input. A state variable representation of the plant is

$$\dot{\mathbf{x}} = \begin{bmatrix} -5 & -2 \\ 2 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \end{bmatrix} u .$$

The closed-loop transfer function is

$$T(s) = \frac{\alpha}{s^2 + (k_1/2 + 5)s + 4 + k_2} .$$

To meet the performance specifications we need $\omega_n = 4.8$ and $\zeta = 0.826$. Therefore, the desired characteristic polynomial is

$$q(s) = s^2 + 2(0.826)4.8s + 23 = s^2 + 8s + 23 .$$

Equating coefficients and solving for k_1 and k_2 yields $k_2 = 19$ and $k_1 = 6$. Select $\alpha = 23$ to obtain zero steady-state error to a step input.

P11.16 Let

$$u = -\mathbf{K}\mathbf{x} .$$

Then, Ackermann's formula is

$$\mathbf{K} = [0, 0, ..., 1]\mathbf{P}_c^{-1} q(\mathbf{A})$$

where $q(s)$ is the desired characteristic polynomial, which in this case is

$$q(s) = s^2 + 4s + 8 .$$

A state-space representation of the limb motion dynamics is

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u .$$

The controllability matrix is

$$\mathbf{P}_c = [\mathbf{B} \mid \mathbf{AB}] = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

and

$$\mathbf{P}_c^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} .$$

Also, we have

$$q(\mathbf{A}) = \mathbf{A}^2 + 4\mathbf{A} + 8\mathbf{I} = \begin{bmatrix} 4 & 0 \\ 2 & 8 \end{bmatrix} .$$

Using Ackermann's formula, we have

$$\mathbf{K} = [2 \quad 8] .$$