The system with feedback is given by

\[ \dot{x} = Ax = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} x , \]

where \( x_1(0) = 1 \), and \( x_2(0) = 0 \). The characteristic equation is

\[ \det[sI - A] = \det \begin{bmatrix} s + 2 & 1 \\ -1 & s \end{bmatrix} = s(s + 2) + 1 = s^2 + 2s + 1 = 0 . \]

The roots are \( s_{1,2} = -1 \). The solution is

\[ x(t) = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} x(0) = \begin{bmatrix} \phi_{11} \\ \phi_{21} \end{bmatrix} \]

since \( x_1(0) = 1 \) and \( x_2(0) = 0 \). We compute the elements of the state transition matrix as follows:

\[ \phi_{21}(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s^2 + 2s + 1} \right\} , \]

therefore

\[ x_2(t) = te^{-t} . \]

Similarly,

\[ \phi_{11}(t) = \mathcal{L}^{-1} \left\{ \frac{s}{s^2 + 2s + 1} \right\} . \]

Therefore,

\[ x_1(t) = (1 - t)e^{-t} . \]

Let

\[ u = -k_1 x_1 - k_2 x_2 + \alpha r \]

where \( r(t) \) is the command input. A state variable representation of the plant is

\[ \dot{x} = \begin{bmatrix} -5 & -2 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u \]

\[ y = \begin{bmatrix} 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} u . \]

The closed-loop transfer function is

\[ T(s) = \frac{\alpha}{s^2 + (k_1/2 + 5)s + 4 + k_2} . \]

To meet the performance specifications we need \( \omega_n = 4.8 \) and \( \zeta = 0.826 \). Therefore, the desired characteristic polynomial is

\[ q(s) = s^2 + 2(0.826)4.8s + 23 = s^2 + 8s + 23 . \]

Equating coefficients and solving for \( k_1 \) and \( k_2 \) yields \( k_2 = 19 \) and \( k_1 = 6 \). Select \( \alpha = 23 \) to obtain zero steady-state error to a step input.
Let \( u = -Kx \).

Then, Ackermann’s formula is

\[ K = [0,0,...,1]P^{-1}_c q(A) \]

where \( q(s) \) is the desired characteristic polynomial, which in this case is

\[ q(s) = s^2 + 4s + 8. \]

A state-space representation of the limb motion dynamics is

\[ \dot{x} = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u. \]

The controllability matrix is

\[ P_c = [B \mid AB] = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \]

and

\[ P_c^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}. \]

Also, we have

\[ q(A) = A^2 + 4A + 8I = \begin{bmatrix} 4 & 0 \\ 2 & 8 \end{bmatrix}. \]

Using Ackermann’s formula, we have

\[ K = [2 \ 8]. \]