Test mi-session 2
Durée: 80 min
Place: MRT219
17 mars 2006
10:00–11:20

MAT 3380

Mid-term 2
Time: 80 min
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17 March 2006
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Prof.: Rémi Vaillancourt

Instructions:

(a) *Test à livre fermé. Tout genre de calculatrices autorisé.*
Closed book mid-term. All types of calculators are allowed.

(b) *Répondre sur le questionnaire.*
Answer on the question sheets.

(c) *Les 6 questions ont toutes la même valeur.*
All six questions have the same value.

(d) *Une réponse sans calcul à l'appui ne sera pas corrigée.*
Show all computation. Bare answers will not be graded.

(e) *Tous les angles sont en RADIANS.*
Prière de tester et d'ajuster votre calculatrice.
All angles are in RADIAN measures.
Test and adjust your calculators.

$$\sin 1.123456789 = 0.90160112364453$$

"The purpose of computing is insight, not numbers", Hamming

Qu. 1. Solve the linear system $Ax = b$ by forward and backward substitutions given that $A = LU$ and

$$L = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad U = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 5 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 13 \\ 1 \end{bmatrix},$$

Qu. 2. Find the Cholesky decomposition and compute the determinant of the matrix :

$$A = \begin{bmatrix} 4 & -2 & -2 \\ -2 & 10 & 1 \\ -2 & 1 & 21 \end{bmatrix}.$$

Give the detail of your computation.

Qu. 3. Permute the equations of the system :

$$\begin{array}{rcrcrcr} 6x_1 & + & x_2 & - & x_3 & = & 3 \\ -x_1 & + & x_2 & + & 7x_3 & = & -17 \\ x_1 & + & 5x_2 & + & x_3 & = & 0 \end{array} \quad \text{with} \quad \begin{array}{l} x_1^{(0)} = 1 \\ x_2^{(0)} = 1 \\ x_3^{(0)} = 1 \end{array}$$

to insure convergence of the Gauss–Seidel scheme and iterate this scheme once.

Qu. 4. Draw Gershgorin's disk for the matrix :

$$\begin{bmatrix} 1-i & 0.3+0.4i & 0.5i \\ 0.3i & -1+i & 0.3 \\ 0 & 0.6+0.8i & 1+i \end{bmatrix}.$$

Qu. 5. Prove the uniqueness of one of the following three matrix decompositions :

(a) $A = LU$ where A is nonsingular, L lower triangular with $l_{ii} = 1$ and U is upper triangular.
(b) $A = GG^T$ where A is positive definite, G lower triangular with $g_{ii} > 0$.
(c) $A = QR$ wher A is nonsingular, Q orthogonal and R upper triangularwith $r_{ii} > 0$.

One may assume that $A \in \mathbb{R}^{3\times 3}$.

Qu. 6. Consider the Householder reflection :

$$P = I - 2\,\frac{vv^T}{v^T v}.$$

Qu. 6(a). Show that P is symmetric : $P^T = P$.

Qu. 6(b). Show that P is orthogonal : $P^{-1} = P^T$.

Qu. 6(c). Find the eigenvalues of P. (Show your work. Just guessing is insufficient.)

Qu. 6(d). Calculer le déterminant de P.

One may assume that $A \in \mathbb{R}^{3\times 3}$.