MAT2784 - Devoir 9 - MAT 2784 B D9.1

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#5.6 $\sum_{n=0}^{\infty} \frac{(-1)^n}{k^n} x^{2n}$

① on pose $w = x^2$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{k^n} w^n$$

ⓐ $\frac{1}{R_1} = \lim_{n\to\infty} \left| \frac{(-1)^n(-1)}{k^n\, k} \cdot \frac{k^n}{(-1)^n} \right| = \lim_{n\to\infty} \left| \frac{-1}{k} \right|$

$R_1 = |k|$

ⓑ $|x^2| = |w| < |k|$

$|x| < |k|^{1/2} = R$

$$\boxed{R = |k|^{1/2} \text{ pour } x < |k|^{1/2}}$$

② $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2n}{k^n} x^{2n-1}$

$$\frac{1}{R'} = \overline{\lim_{n\to\infty}} \left| \frac{(-1)^n 2n}{k^n} \right|^{1/(2n-1)}$$

$$= \overline{\lim_{n\to\infty}} \left| (-1)^n \cdot 2n \right|^{1/(2n-1)} \cdot \overline{\lim_{n\to\infty}} \left| \frac{1}{k^n} \right|^{(1/n)(1/(2n-1))}$$

$$= \overline{\lim_{n\to\infty}} \left(\frac{1}{|k|} \right)^{1/(2-1/n)}$$

$$= \frac{1}{|k|^{1/2}}$$

$R' = |k|^{1/2}$

$$\boxed{R' = |k|^{1/2} \text{ pour } x < |k|^{1/2}}$$

#5.9 $\quad y'' - 3y' + 2y = 0$

① on pose $y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$

$y'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \dots$

$y''(x) = 2a_2 + 6a_3 x + 12 a_4 x^2 + 20 a_5 x^3 + \dots$

②
$$\left.\begin{array}{l} y'' \\ -3y' \\ 2y \end{array}\right\} = \left\{\begin{array}{l} 2a_2 + 6a_3 x + 12 a_4 x^2 + 20 a_5 x^3 + \dots \\ -3a_1 - 6a_2 x - 9a_3 x^2 - 12 a_4 x^3 + \dots \\ 2a_0 + 2a_1 x + 2a_2 x^2 + 2a_3 x^3 + \dots \end{array}\right.$$

③
$x^0: \quad 2a_2 - 3a_1 + 2a_0 = 0$

$x^1: \quad 6a_3 - 6a_2 + 2a_1 = 0$

$x^2: \quad 12a_4 - 9a_3 + 2a_2 = 0$

$x^3: \quad 20a_5 - 12a_4 + 2a_3 = 0$

④ ⓐ $a_2 = \dfrac{3a_1 - 2a_0}{2}$ ⓑ $6a_3 - 6\left(\dfrac{3a_1 - 2a_0}{2}\right) + 2a_1 = 0$

$6a_3 = 9a_1 - 6a_0 - 2a_1$

$a_3 = \dfrac{7a_1 - 6a_0}{6}$

ⓒ $12a_4 - 9\left(\dfrac{7a_1 - 6a_0}{6}\right) + 2\left(\dfrac{3a_1 - 2a_0}{2}\right) = 0$

$12a_4 - \dfrac{21}{2}a_1 + 9a_0 + 3a_1 - 2a_0 = 0$

$12a_4 = \dfrac{15}{2}a_1 - 7a_0 \qquad a_4 = \dfrac{5}{8}a_1 - \dfrac{7}{12}a_0$

ⓓ $20a_5 - 12\left(\dfrac{5}{8}a_1 - \dfrac{7}{12}a_0\right) + 2\left(\dfrac{7a_1 - 6a_0}{6}\right) = 0$

$20a_5 - \dfrac{15}{2}a_1 + 7a_0 + \dfrac{7}{3}a_1 - 2a_0 = 0$

$20a_5 = \dfrac{31}{6}a_1 - 5a_0 \qquad a_5 = \dfrac{31}{120}a_1 - \dfrac{1}{4}a_0$

⑤ $y(x) = a_0 + a_1 x + \dfrac{3a_1 - 2a_0}{2}x^2 + \dfrac{7a_1 - 6a_0}{6}x^3 + \left(\dfrac{5}{8}a_1 - \dfrac{7}{12}a_0\right)x^4$
$+ \left(\dfrac{31}{120}a_1 - \dfrac{1}{4}a_0\right)x^5 + \dots$

#5.29 $f(x) = \begin{cases} 0 & -1 < x < 0 \\ 1 & 0 < x < 1 \end{cases}$

① $f(x) = \sum_{m=0}^{\infty} a_m P_m(x) \qquad -1 < x < 1$

$$a_m = \frac{2m+1}{2} \int_{-1}^{1} f(x) P_m(x)\, dx$$

② $a_0 = \frac{1}{2} \int_0^1 dx = \frac{1}{2} \left[x \right|_0^1 = \frac{1}{2}$

$a_1 = \frac{3}{2} \int_0^1 x\, dx = \frac{3}{2} \left[\frac{x^2}{2} \right|_0^1 = \frac{3}{4}$

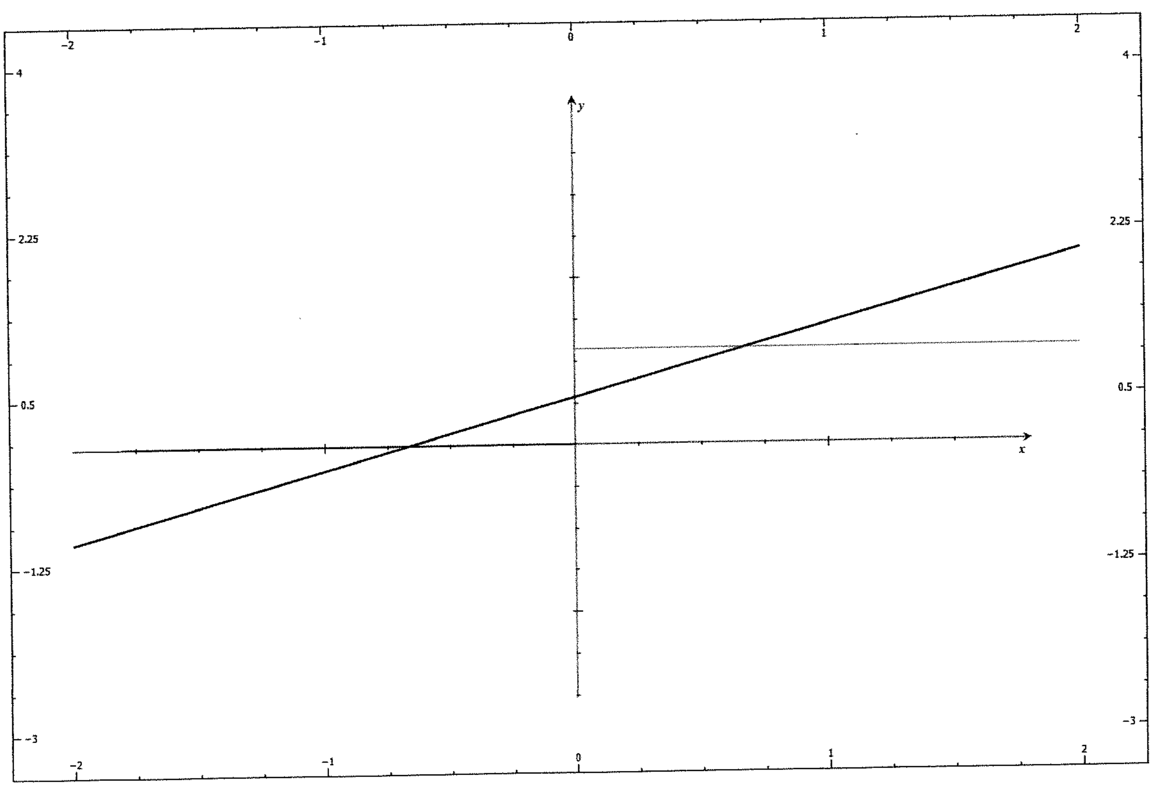
$a_2 = \frac{5}{2} \int_0^1 \frac{1}{2}(3x^2 - 1)\, dx = \frac{5}{4} \left[\int_0^1 3x^2\, dx - \int_0^1 dx \right]$

$= \frac{5}{4} \left[x^3 - x \right]_0^1 = 0$

③ $f(x) \approx \frac{1}{2} P_0(x) + \frac{3}{4} P_1(x) + 0 P_2(x)$

$f(x) \approx \frac{1}{2} + \frac{3}{4} x$

Graphique:



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D9.4

#5.31 $I = \int_{0,2}^{1,5} e^{-x^2} dx$

① $x = \dfrac{(1,5-0,2)t + 1,5 + 0,2}{2}$ $\qquad dx = \dfrac{1,5-0,2}{2} dt$

$= \dfrac{13}{20} t + \dfrac{17}{20}$ $\qquad = \dfrac{13}{20} dt$

② $I = \dfrac{13}{20} \int_{-1}^{1} e^{-\left(\frac{13}{20}t + \frac{17}{20}\right)^2} dt$

③ $I = \dfrac{13}{20}\left[\dfrac{5}{9} f\left(-\sqrt{3/5}\right) + \dfrac{8}{9} f(0) + \dfrac{5}{9} f\left(\sqrt{3/5}\right)\right]$

$= \dfrac{13}{20}\left[\dfrac{5}{9}(0,886857) + \dfrac{8}{9}(0,485536) + \dfrac{5}{9}(0,160104)\right]$

$= 0,6586020857$

#5.34 ① $(n+1) P_{n+1}(x) = (2n+1) x P_n(x) - n P_{n-1}(x)$

Pour $P_4(x)$, on pose $n=3$

$4 P_4(x) = 7x P_3(x) - 3 P_2(x)$

$P_4(x) = \dfrac{7x P_3(x) - 3 P_2(x)}{4}$

② $P_3(x) = \dfrac{1}{2}(5x^3 - 3x)$ $\qquad P_2(x) = \dfrac{1}{2}(3x^2 - 1)$

③ $P_4(x) = \dfrac{x(7/2)(5x^3 - 3x) - (3/2)(3x^2 - 1)}{4}$

$P_4(x) = \dfrac{35}{8} x^3 - \dfrac{21}{8} x^2 - \dfrac{9}{8} x^2 + \dfrac{3}{8}$

$= \dfrac{35}{8} x^3 - \dfrac{30}{8} x^2 + \dfrac{3}{8}$

$= \dfrac{1}{8}(35x^3 - 30x^2 + 3)$