2009.04.06 RÉMI VAILLANCOURT

#6.49 $y'' - 5y' + 6y = \begin{cases} t & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$ $\quad y(0) = 0 \quad y'(0) = 1$

① $L(y) = Y(s) \qquad = Y(s)$

$L(y') = sY(s) - y(0) \qquad = sY(s)$

$L(y'') = s^2 Y(s) - s\,y(0) - y'(0) \qquad = s^2 Y(s) - 1$

② $f(t) = \begin{cases} t & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$

$$f(t) = t - u(t-1)(t) + u(t-1) \cdot 0$$

$$= t - u(t-1)(t)$$

$$= t - u(t-1)(t) - u(t-1) + u(t-1)$$

$$= t - u(t-1)(t-1) - u(t-1)$$

③ $L(f(t)) = L(t) - L(u(t-1)(t-1)) - L(u(t-1))$

$$= \frac{1}{s^2} - e^{-s} L(t \quad) - \frac{1}{s} e^{-s}$$

$$= \frac{1}{s^2} - e^{-s} \cdot \frac{1}{s^2} - e^{-s} \cdot \frac{1}{s}$$

$$= \frac{1}{s^2} - e^{-s} \left[\frac{1}{s^2} + \frac{1}{s} \right]$$

④ $s^2 Y(s) - 1 - 5(sY(s)) + 6Y(s)$

$= Y(s)(s^2 - 5s + 6) - 1$

⑤ $Y(s)(s^2 - 5s + 6) - 1 = \frac{1}{s^2} - e^{-s}\left[\frac{1}{s^2} + \frac{1}{s}\right]$

$Y(s)(s-2)(s-3) = \frac{1}{s^2} - e^{-s}\left[\frac{1}{s^2} + \frac{1}{s}\right] + 1$

$$Y(s) = \frac{1}{s^2(s-2)(s-3)} - \frac{e^{-s}}{s^2(s-2)(s-3)} - \frac{e^{-s}}{s(s-2)(s-3)} + \frac{1}{(s-2)(s-3)}$$

D 8. R

(6)(a) $\dfrac{1}{(s-2)(s-3)} = \dfrac{A}{(s-2)} + \dfrac{B}{(s-3)} = \dfrac{-1}{(s-2)} + \dfrac{1}{(s-3)}$

$A + B = 0 \Rightarrow A = -B \Rightarrow A = -1$

$-3A - 2B = 1 \Rightarrow 3B - 2B = 1 \quad B = 1$

(b) $\dfrac{1}{s^2(s-2)(s-3)} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{(s-2)} + \dfrac{D}{(s-3)} = \dfrac{5/36}{s} + \dfrac{1/6}{s^2} + \dfrac{-1/4}{(s-2)} + \dfrac{1/9}{(s-3)}$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ -5 & 1 & -3 & -2 & 0 \\ 6 & -5 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 5/36 \\ 0 & 1 & 0 & 0 & 1/6 \\ 0 & 0 & 1 & 0 & -1/4 \\ 0 & 0 & 0 & 1 & 1/9 \end{bmatrix} \quad \begin{matrix} A = 5/36 \\ B = 1/6 \\ C = -1/4 \\ D = 1/9 \end{matrix}$$

(c) $\dfrac{1}{s(s-2)(s-3)} = \dfrac{A}{s} + \dfrac{B}{(s-2)} + \dfrac{C}{(s-3)} = \dfrac{1/6}{s} + \dfrac{-1/2}{(s-2)} + \dfrac{1/3}{(s-3)}$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ -5 & -3 & -2 & 0 \\ 6 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1/6 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 1/3 \end{bmatrix} \quad \begin{matrix} A = 1/6 \\ B = -1/2 \\ C = 1/3 \end{matrix}$$

(7) $Y(s) = \dfrac{-1}{(s-2)} + \dfrac{1}{(s-3)} + \dfrac{5/36}{s} + \dfrac{1/6}{s^2} + \dfrac{-1/4}{(s-2)} + \dfrac{1/9}{(s-3)}$

$\quad - e^{-s}\left[\dfrac{5/36}{s} + \dfrac{1/6}{s^2} + \dfrac{-1/4}{(s-2)} + \dfrac{1/9}{(s-3)} + \dfrac{1/6}{s} + \dfrac{-1/2}{(s-2)} + \dfrac{1/3}{s-3}\right]$

$= \dfrac{-5/4}{(s-2)} + \dfrac{10/9}{(s-3)} + \dfrac{5/36}{s} + \dfrac{1/6}{s^2} - e^{-s}\left[\dfrac{11/36}{s} + \dfrac{1/6}{s^2} + \dfrac{-3/4}{(s-2)} + \dfrac{4/9}{(s-3)}\right]$

$y(t) = -\frac{5}{4}e^{2t} + \frac{10}{9}e^{3t} + \frac{5}{36} + \frac{1}{6}t - \frac{11}{36}u(t-1) - \frac{1}{6}u(t-1)(t-1)$

$\quad + \frac{3}{4}u(t-1)e^{2(t-1)} - \frac{4}{9}u(t-1)e^{3(t-1)}$

$= -\frac{5}{4}e^{2t} + \frac{10}{9}e^{3t} + \frac{1}{6}t + \frac{5}{36} - u(t-1)\left[\frac{11}{36} + \frac{1}{6}(t-1) - \frac{3}{4}e^{2t-2} + \frac{4}{9}e^{3t-3}\right]$

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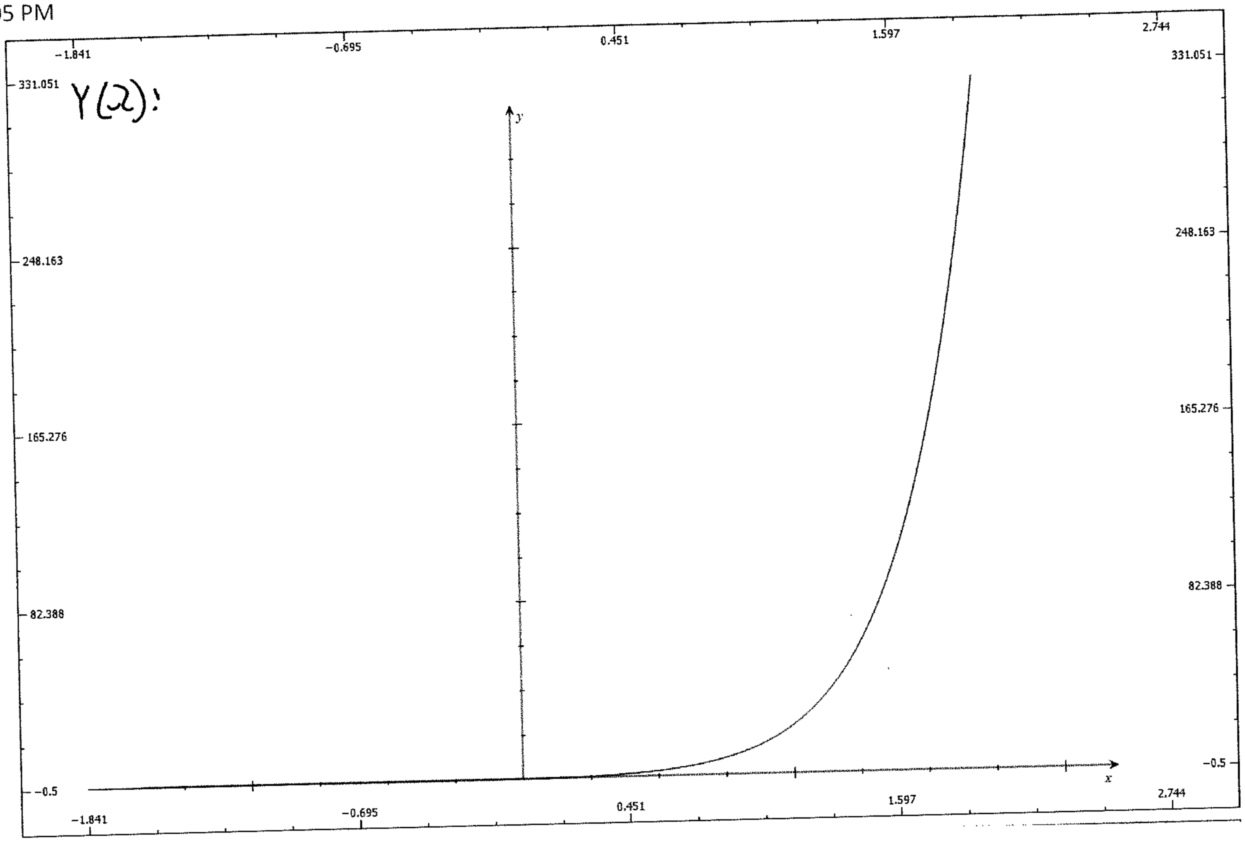
D 8.3

Graphique: Y(x):

6.49



D 8.4

6.52 $y'' + 3y' + 2y = 1 - u(t-1)$ $\quad y(0) = 0 \quad y'(0) = 1$

porons $\mathcal{L}(y) = Y(s)$

$$\mathcal{L}(y'' + 3y' + 2y) = \mathcal{L}(1 - u(t-1)) = s^2 Y(s) - \cancel{s y(0)} - y'(0) + 3(sY(s) - y(0)) + 2Y(s) = \frac{1 - e^{-s}}{s}$$

$$\Rightarrow Y(s)(s^2 + 3s + 2) - 1 = \frac{1 + e^{-s}}{s} \Rightarrow Y(s) = \frac{1 - e^{-s} + s}{s(s+2)(s+1)}$$

$$\frac{1}{s(s+2)(s+1)} = \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s+1)} = \frac{(s^2 + 3s + 2)A + s(s+1)B + s(s+2)C}{s(s+2)(s+1)}$$

$$\Rightarrow \begin{cases} s^2(A + B + C) = 0 \Rightarrow B = -C - 1/2 \Rightarrow 1/2 \\ s(3A + B + 2C) = 0 \Rightarrow 3/2 - C - 1/2 + 2C = 0 \Rightarrow C = -1 \\ 2A = 1 \Rightarrow A = 1/2 \end{cases}$$

$$\frac{1}{s(s+2)(s+1)} = \frac{1}{2s} + \frac{1}{2(s+2)} - \frac{1}{(s+1)}$$

$$\frac{s}{s(s+2)(s+1)} = \frac{1}{(s+2)(s+1)} = \frac{A}{(s+2)} + \frac{B}{(s+1)} = \frac{(s+1)A + (s+2)B}{(s+1)(s+2)}$$

$$\Rightarrow \begin{cases} s(A+B) = 0 \Rightarrow A = -B \\ A + 2B = 1 \Rightarrow -B + 2B = 1 \Rightarrow B = 1 \Rightarrow A = -1 \end{cases}$$

$$\frac{s}{s(s+2)(s+1)} = \frac{-1}{(s+2)} + \frac{1}{(s+1)}$$

$$\Rightarrow Y(s) = \left[\frac{1}{2s} + \frac{1}{2(s+2)} - \cancel{\frac{1}{s+1}}\right] + \left[\frac{-1}{s+2} + \cancel{\frac{1}{s+1}}\right] - e^{-s}\left[\frac{1}{2s} + \frac{1}{2(s+2)} - \frac{1}{s+1}\right]$$

$$= \frac{1}{2s} - \frac{1}{2(s+2)} - e^{-s}\left[\frac{1}{2s} + \frac{1}{2(s+2)} - \frac{1}{s+1}\right]$$

D 8.5

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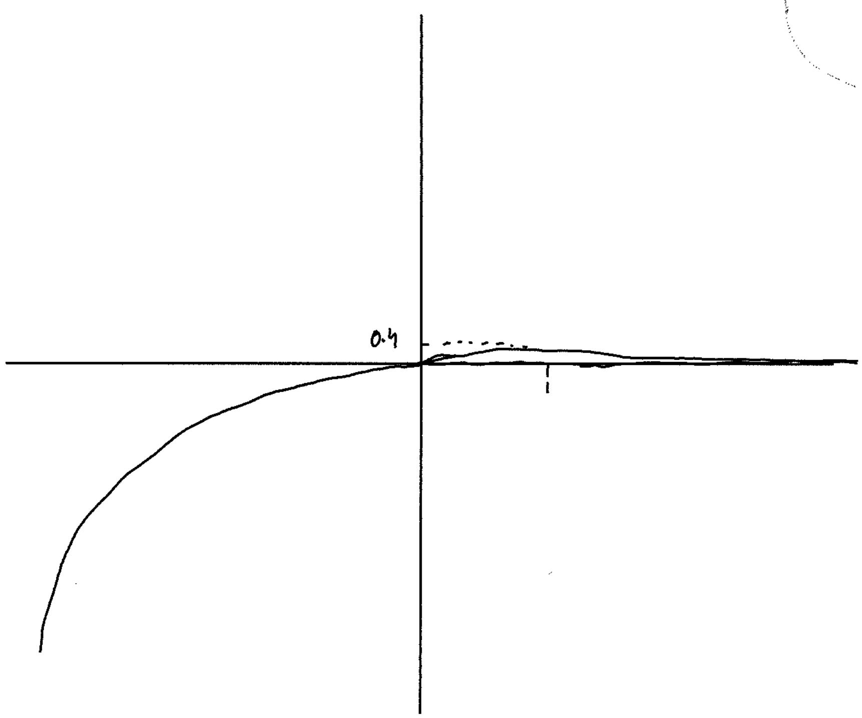
$$\mathcal{L}^{-1}\left(Y(s)\right) = y(t) = \frac{1}{2} - \frac{e^{-2t}}{2} - \frac{1}{2}u(t-1) - \frac{1}{2}u(t-1)e^{-2(t-1)} + u(t-1)e^{-(t-1)}$$

$$\boxed{y(t) = \frac{1}{2} - \frac{e^{-2t}}{2} - u(t-1)\left[\frac{1}{2} + \frac{e^{-2(t-1)}}{2} - e^{-(t-1)}\right]}$$



6.54 $y'' + 5y' + 6y = u(t-1) + \delta(t-2) \quad y(0) = 0 \quad y'(0) = 1$

P 8.6

① $L(y) = Y(s) \qquad\qquad = Y(s)$

$L(y') = sY(s) + y(0) \qquad = sY(s)$

$L(y'') = s^2 Y(s) - s\,y(0) - y'(0) = s^2 Y(s) - 1$

② $y(t) = u(t-1) + \delta(t-2)$

$L(y(t)) = L(u(t-1)) + L(\delta(t-2))$

$= \frac{1}{s} e^{-s} + e^{-2s}$

③ $s^2 Y(s) - 1 + 5(sY(s)) + 6(Y(s))$

$= Y(s)(s^2 + 5s + 6) - 1$

④ $Y(s)(s+2)(s+3) = \frac{1}{s} e^{-s} + e^{-2s} + 1$

$Y(s) = \frac{1}{s(s+2)(s+3} e^{-s} + \frac{1}{(s+2)(s+3)} e^{-2s} + \frac{1}{(s+2)(s+3)}$

⑤ ⓐ $\frac{1}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)} = \frac{1}{(s+2)} + \frac{-1}{(s+3)}$

$A + B = 0 \Rightarrow A = -B \qquad \Rightarrow A = 1$

$3A + 2B = 1 \Rightarrow -3B + 2B = 1 \Rightarrow -B = 1 \Rightarrow B = -1$

ⓑ $\frac{1}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s+3)} = \frac{1/6}{s} + \frac{-1/2}{(s+2)} + \frac{1/3}{(s+3)}$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 5 & 3 & 2 & 0 \\ 6 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1/6 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 1/3 \end{bmatrix} \quad \begin{array}{l} A = 1/6 \\ B = -1/2 \\ C = 1/3 \end{array}$$

⑥ $Y(s) = \left[\frac{1/6}{s} + \frac{-1/2}{(s+2)} + \frac{1/3}{(s+3)}\right] e^{-s} + \left[\frac{1}{(s+2)} + \frac{-1}{(s+3)}\right] e^{-2s} + \frac{1}{(s+2)} + \frac{-1}{(s+3)}$

$Y(t) = u(t-1)\left[\frac{1}{6} - \frac{1}{2} e^{-2(t-1)} + \frac{1}{3} e^{-3(t-1)}\right] + e^{-2t} - e^{-3t}$

$+ u(t-2)\left[e^{-2(t-2)} - e^{-3(t-2)}\right]$

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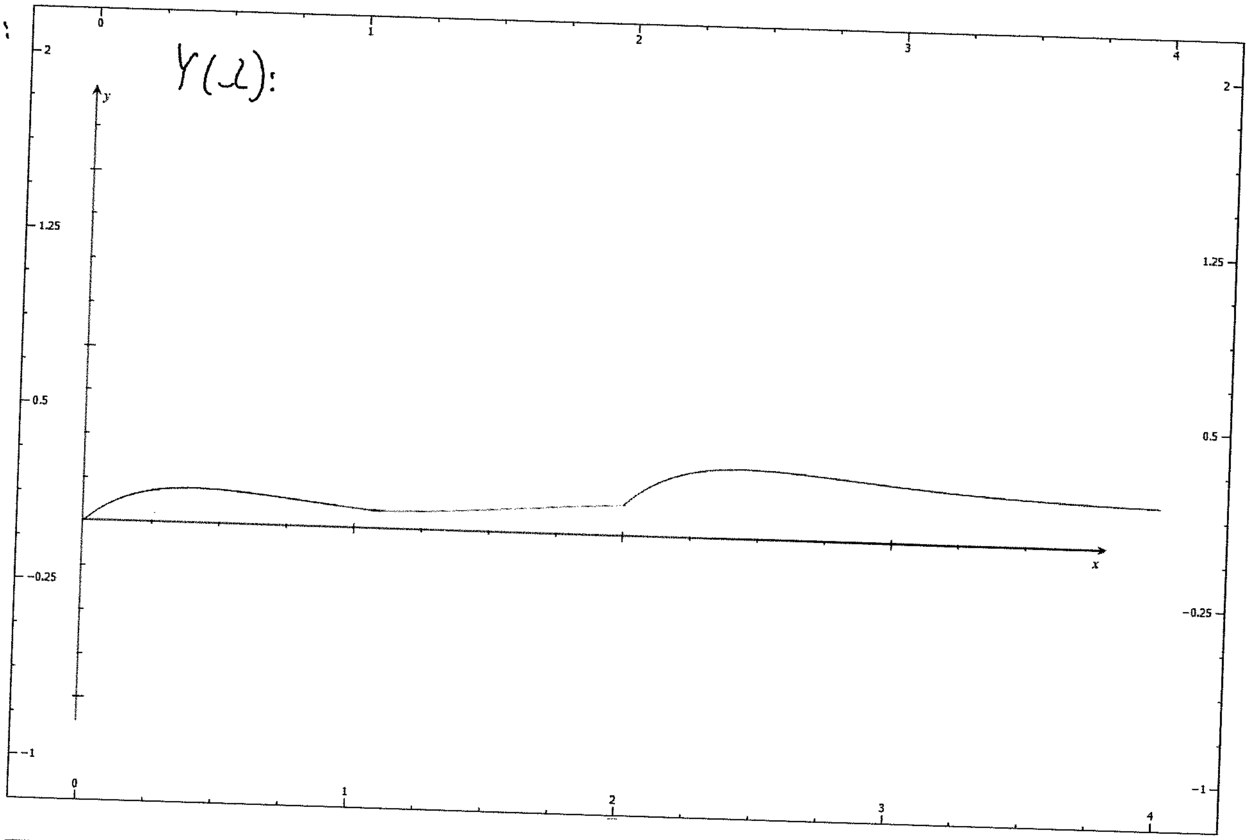
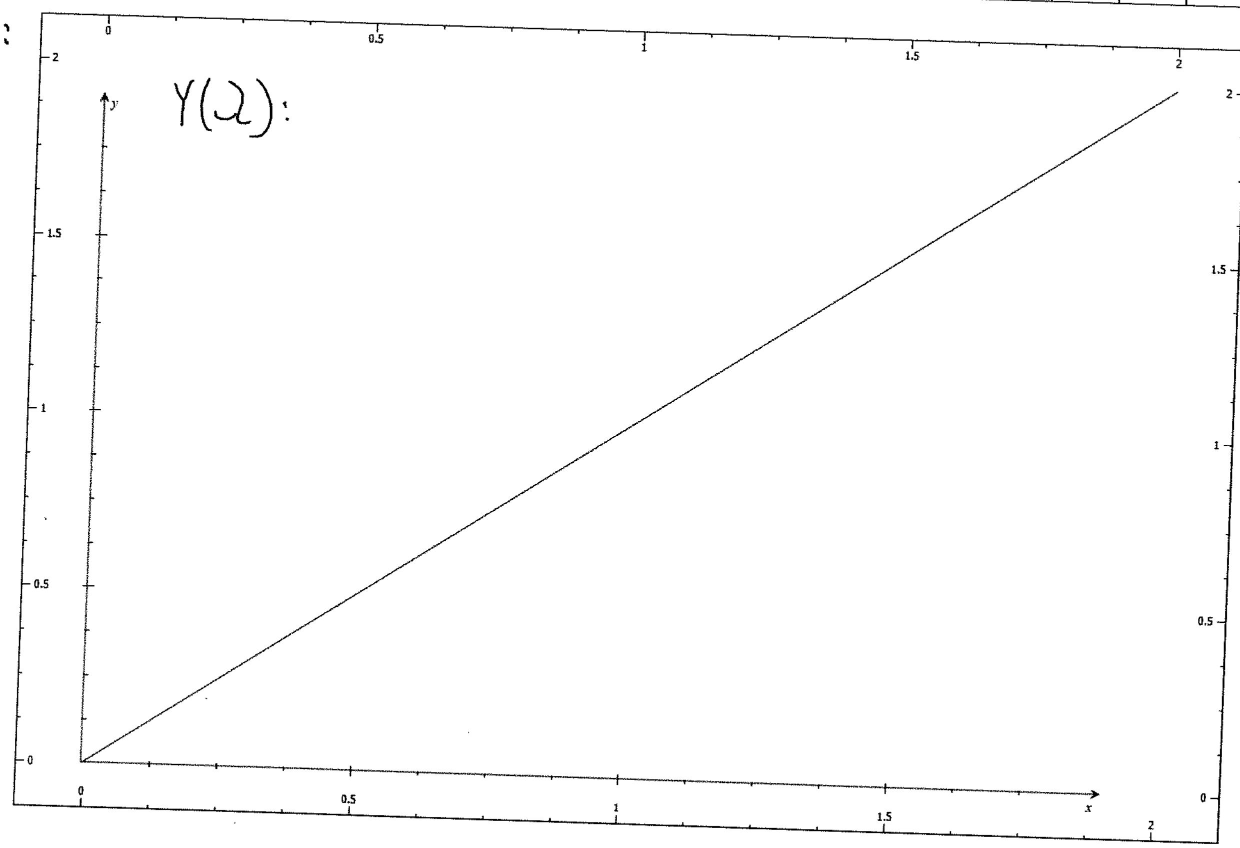
08.

raphique:
#6.54



raphique:
#6.56



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D P. 8

\# 6.56 $y(t) = \sin(t) + \int_0^t y(\tau)\sin(t-\tau)\,d\tau$

① $y(t) = \sin(t) + y * \sin(t)$

② $Y(s) = \frac{1}{s^2+1} + Y(s)\frac{1}{s^2+1}$

$Y(s) - Y(s)\frac{1}{s^2+1} = \frac{1}{s^2+1}$

$Y(s)\left(1 - \frac{1}{s^2+1}\right) = \frac{1}{s^2+1}$

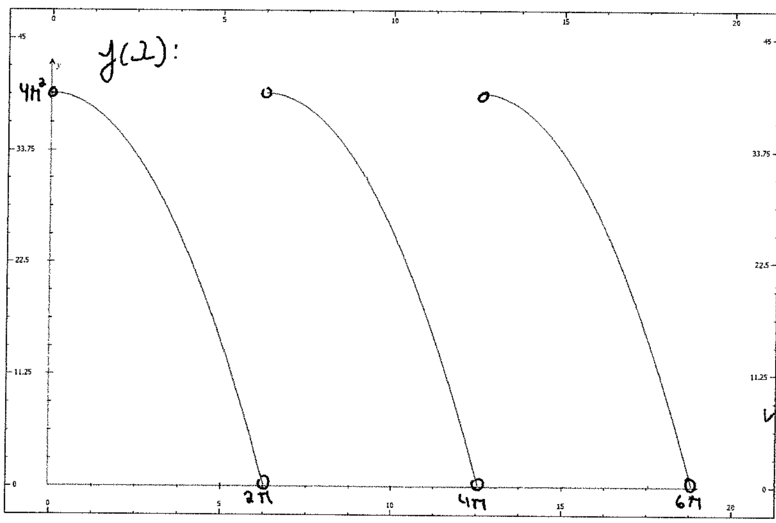
$Y(s)\left(\frac{s^2}{s^2+1}\right) = \frac{1}{s^2+1}$

$Y(s) = \frac{(s^2+1)}{s^2(s^2+1)}$

$Y(s) = \frac{1}{s^2}$

③ $Y(t) = t$

\# 6.61 $f(t) = 4\pi^2 - t^2 \quad 0 < t < 2\pi$



$L(f(t)) = \frac{1}{1-e^{-2\pi s}} \int_0^{2\pi} e^{-st}(4\pi^2 - t^2)\,dt$

$= \frac{1}{1-e^{-2\pi s}} \left[4\pi^2 \int_0^{2\pi} e^{-st}\,dt - \int_0^{2\pi} t^2 e^{-st}\,dt\right]$ 2x intégrations par partie

$= \frac{1}{1-e^{-2\pi s}} \left[4\pi^2\left[\frac{-e^{-st}}{s}\right]_0^{2\pi} + \left[\frac{e^{-st}(s^2t^2+2st+2)}{s^3}\right]_0^{2\pi}\right]$

$= \frac{1}{1-e^{-2\pi s}} \left[\frac{-4\pi^2}{s}\left[e^{-2\pi s} - 1\right] + \left[\frac{4\pi^2 e^{-2\pi s}}{s} + \frac{4\pi e^{-2\pi s}}{s^2} + \frac{2e^{-2\pi s}}{s^3} - \frac{2}{s^3}\right]\right]$

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D8.9

\# 12.12 $y' = x + \sin(-y)$ $y(0) = 0$ $0 \leq x \leq 1$ $h = 0,1$ Ordre 4

① $f(x, y) = x + \sin(-y)$ $x_n = 0 + 0,1\, n$, $n = 0, 1, 2$

$$y_{n+1} = y_n + 1/6\,(K_1 + 2K_2 + 2K_3 + K_4)$$

$$k_1 = 0,1\,(0,1n + \sin(y_n))$$

$$k_2 = 0,1\,(0,1n + 0,05 + \sin(y_n + \tfrac{1}{2}k_1))$$

$$k_3 = 0,1\,(0,1n + 0,05 + \sin(y_n + \tfrac{1}{2}k_2))$$

$$k_4 = 0,1\,(0,1n + 0,1 + \sin(y_n + k_3))_{\#}$$

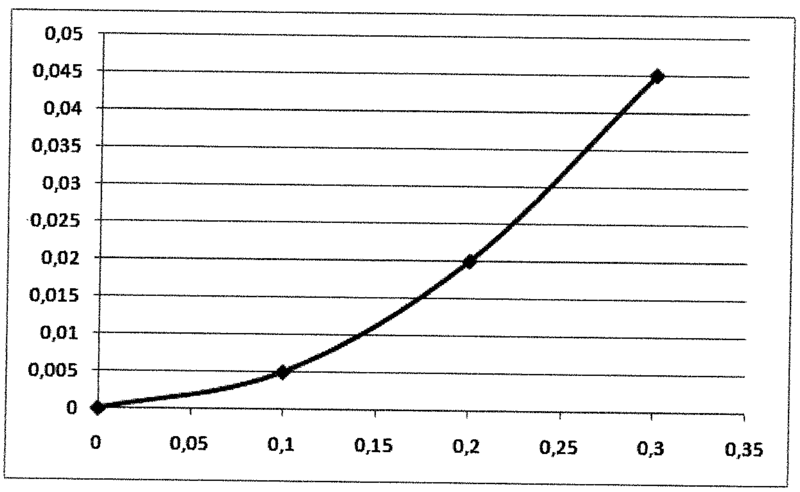
③ $y_0 = 0$

$y_1 = 0,0051708328$

$y_2 = 0,0214025223$

$y_3 = 0,0498576053$

④ Graphique:



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D 8.10

#12.16 $y' = x^2 + 2y^2 \qquad y(0) = 1 \qquad h = 0{,}1$

① $x_n = 0 + 0{,}1\,n$

① $k_1 = 0{,}1\left((0{,}1n)^2 + 2(y_n)^2\right)$

$k_2 = 0{,}1\left((0{,}1n + 0{,}05)^2 + 2(y_n + 0.5k_1)^2\right)$

$k_3 = 0{,}1\left((0{,}1n + 0{,}075)^2 + 2(y_n + 0.75k_2)^2\right)$

$k_4 = 0{,}1\left((0{,}1n + 0{,}1)^2 + 2(y_n + 2/9\,k_1 + 1/3\,k_2 + 4/9\,k_3)^2\right)$

③ $y_{n+1} = y_n + 2/9\,k_1 + 1/3\,k_2 + 4/9\,k_3$

	y_n	E
$y_0 =$	1	0
$y_1 =$	1,249567586	−0,0017685822
$y_2 =$	1,666094382	−0,004925542

O.K

D 8.11

#12.24 $y' = x + \sin(y)$ $\quad y(0) = 0 \quad h = 0,1 \quad f_n^c = x_n + \sin(y_n^c) \quad f_n^p = x_n + \sin(y_n^p)$

$$y^p_{n+1} = y_n^c + \frac{h}{24}\left(55 f_n^c - 59 f_{n-1}^c + 37 f_{n-2}^c - 9 f_{n-3}^c\right)$$

$$y^c_{n+1} = y_n^c + \frac{h}{24}\left(9 f_{n+1}^p + 19 f_n^c - 5 f_{n-1}^c + f_{n-2}^c\right)$$

①

x_n	y_n^c initiales	y_n^p	y_n^c
0	0		
0,1	0,0051708328		
0,2	0,0214025223		
0,3	0,0498576053		
0,4		0,0918146882516	0,0918169392657
0,5		0,1486825834892	0,1486815487269
0,6		0,2219697057807	0,2219620181515

② estimation de l'erreur en $x = 0.5$

$$E \approx -\frac{19}{270}\left[0,1486815487269 - 0,1486825834892\right]$$

$$\approx 7,28166063 \times 10^{-8}$$

~~Arrondir à 6 décimales près!~~

Graphique: