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D6.1

MAT 2734 B

15 MARS 2009

DEV. no 6

RÉMI VAILLANCOURT

#4.2 $y' = Ay \qquad A = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

① $\det \begin{bmatrix} 2-\lambda & 0 & 4 \\ 0 & 2-\lambda & 0 \\ -1 & 0 & 2-\lambda \end{bmatrix}$

$$= (2-\lambda)\begin{vmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ -1 & 2-\lambda \end{vmatrix} + 4\begin{vmatrix} 0 & 2-\lambda \\ -1 & 0 \end{vmatrix}$$

$$= (2-\lambda)(2-\lambda)^2 + 4(2-\lambda) \qquad \frac{4 \pm \sqrt{16-4\cdot1\cdot8}}{2} = 2 \pm 2i$$

$$= (2-\lambda)\left((2-\lambda)^2+4\right)$$

$$= (2-\lambda)(\lambda^2 - 4\lambda + 8) \qquad \lambda_1 = 2 \quad \lambda_2 = 2+2i \quad \lambda_3 = 2-2i$$

②ⓐ $(A - 2I)u = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = 0 \qquad 4u_3 = 0 \qquad 0u_2 = 0$

$$-u_1 = 0$$

$$u = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

ⓑ $(A - (2-2i))v = \begin{bmatrix} 2i & 0 & 4 \\ 0 & 2i & 0 \\ -1 & 0 & 2i \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$

$$\left[\begin{array}{ccc|c} 2i & 0 & 4 & 0 \\ 0 & 2i & 0 & 0 \\ -1 & 0 & 2i & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -2i & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 2i & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -2i & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$v_1 - 2i\,v_3 = 0 \qquad v_1 - 2i\,2 = 0 \qquad v_1 = 2it$

$v_2 = 0$

$v_3 = t$

$$v = t\begin{bmatrix} 2i \\ 0 \\ 1 \end{bmatrix}$$

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D 6.2

② $(A-(2+2i))w = \begin{bmatrix} -2i & 0 & 4 \\ 0 & -2i & 0 \\ -1 & 0 & -2i \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = 0$

$$\left[\begin{array}{ccc|c} -2i & 0 & 4 & 0 \\ 0 & -2i & 0 & 0 \\ -1 & 0 & -2i & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 2i & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & -2i & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 2i & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$w_1 + 2i w_3 = 0 \qquad w_1 = -2it$

$w_2 = 0$

$w_3 = t$

$$w = 2 \cdot \begin{bmatrix} -2i \\ 0 \\ 1 \end{bmatrix}$$

③ $\vec{V} = \begin{bmatrix} 2 \\ 0 \\ i \end{bmatrix} e^{(2+2i)x} = \begin{bmatrix} 2 \\ 0 \\ i \end{bmatrix} e^{2x} e^{2ix} = e^{2x} \begin{bmatrix} 2 \\ 0 \\ i \end{bmatrix} (\cos(2x) + i\sin(2x))$

$$= e^{2x} \begin{bmatrix} 2\cos(2x) + 2i\sin(2x) \\ 0 \\ i\cos(2x) - \sin(2x) \end{bmatrix} = e^{2x} \begin{bmatrix} 2\cos(2x) \\ 0 \\ -\sin(2x) \end{bmatrix} + i e^{2x} \begin{bmatrix} 2\sin(2x) \\ 0 \\ \cos(2x) \end{bmatrix}$$

④ $y(x) = c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 2\cos(2x) \\ 0 \\ -\sin(2x) \end{bmatrix} e^{2x} + c_3 \begin{bmatrix} 2\sin(2x) \\ 0 \\ \cos(2x) \end{bmatrix} e^{2x}$

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D6.3

4.7 $\quad y' = Ay + f(x) \qquad A = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix} \quad f(x) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

① $A = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

$$\det \begin{bmatrix} 1-\lambda & 1 \\ 3 & 1-\lambda \end{bmatrix} = (1-\lambda)^2 - 3 = 0$$

$$(1-\lambda) = \pm\sqrt{3}$$

$$\lambda_1 = 1+\sqrt{3} \quad \lambda_2 = 1-\sqrt{3}$$

② ⓐ $(A - (1+\sqrt{3})I)u = \begin{bmatrix} -\sqrt{3} & 1 \\ 3 & -\sqrt{3} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0 \qquad \begin{matrix} -\sqrt{3}u_1 + u_2 = 0 \\ 3u_1 - \sqrt{3}u_2 = 0 \end{matrix}$

$$-\sqrt{3}u_1 + u_2 = 3u_1 - \sqrt{3}u_2 \qquad u = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$$

$$u_2 + \sqrt{3}u_2 = 3u_1 + \sqrt{3}u_1$$

$$u_2(1+\sqrt{3}) = u_1(3+\sqrt{3})$$

$$u_2 = \sqrt{3}u_1$$

② $(A - (1-\sqrt{3}))v = \begin{bmatrix} \sqrt{3} & 1 \\ 3 & \sqrt{3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \qquad v = \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix}$

$$\sqrt{3}v_1 + v_2 = 3v_1 + \sqrt{3}v_2$$

$$v_1(\sqrt{3}-3) = v_2(\sqrt{3}-1)$$

$$v_2 = -\sqrt{3}v_1$$

③ $y_h(x) = C_1 \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} e^{(1+\sqrt{3})x} + C_2 \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} e^{(1-\sqrt{3})x}$

④ $f(x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} := e_1 + e_2$

$$y_p(x) = \begin{bmatrix} a \\ b \end{bmatrix} \quad y_p'(x) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$y_p(x) = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0 \qquad \begin{matrix} a+b = -1 \rightarrow a = 0 \\ 3a+b = -1 \rightarrow b = -1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$y_p(x) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

⑤ $y(x) = C_1 \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} e^{(1+\sqrt{3})x} + C_2 \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} e^{(1-\sqrt{3})x} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

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D6.4

#49 $y' = Ay + f(x) \qquad A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \qquad f(x) = \begin{bmatrix} e^{x} \\ -e^{x} \end{bmatrix}$

① $\det \begin{bmatrix} 2-\lambda & -1 \\ 3 & -2-\lambda \end{bmatrix} = \lambda^2 - 1 \qquad \lambda_1 = 1 \qquad \lambda_2 = -1$

②ⓐ $(A - I)u = 0 \qquad \begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0 \qquad u_1 = u_2$

$u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

ⓑ $(A + I)v = 0 \qquad \begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \qquad 3v_1 = v_2$

$v = \begin{bmatrix} 1/3 \\ 1 \end{bmatrix}$

ⓒ $y_h(x) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{x} + c_2 \begin{bmatrix} 1/3 \\ 1 \end{bmatrix} e^{-x}$

③ $f(x) = e^{x} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\quad y_p(x) = \vec{a} e^{x}$, $y_p(x)$ apparaît dans $y_h(x)$, donc

$y_p(x) = e^{x}(\vec{a} x + \vec{b})$

ⓐ $y_p'(x) = e^{x}(\vec{a} x + \vec{b}) + e^{x}(\vec{a}) = A e^{x}(\vec{a} x + \vec{b}) + e^{x} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\vec{a} x + \vec{b} + \vec{a} = A(\vec{a} x + \vec{b}) + \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

① $\vec{a} = A \vec{a} x \qquad \vec{a} = \alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha \\ \alpha \end{bmatrix}$

② $\vec{b} + \vec{a} = A\vec{b} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad \left[\begin{array}{cc|c} 1 & -1 & \alpha - 1 \\ 3 & -3 & \alpha + 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & \alpha - 1 \\ 0 & 0 & -2\alpha + 4 \end{array}\right]$

$A\vec{b} - \vec{b} = \vec{a} - \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad -2\alpha + 4 = 0 \qquad \vec{a} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$\alpha = 2$

$(A - I)\vec{b} = \begin{bmatrix} \alpha - 1 \\ \alpha + 1 \end{bmatrix}$

③ $\begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \qquad b_1 - b_2 = 1 \qquad 3b_1 - 3b_2 = 3 \qquad \vec{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

ⓑ $y_p(x) = e^{x} \left(\begin{bmatrix} 2 \\ 2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$

④ $y(x) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{x} + c_2 \begin{bmatrix} 1/3 \\ 1 \end{bmatrix} e^{-x} + x e^{x} \begin{bmatrix} 2 \\ 2 \end{bmatrix} + e^{x} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

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D6.5

#4.13 $y' = Ay$ $\quad A = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$ $\quad y_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

① $\det \begin{bmatrix} 1-\lambda & \sqrt{3} \\ \sqrt{3} & -1-\lambda \end{bmatrix} = \lambda^2 - 4 \quad \lambda_1 = 2 \quad \lambda_2 = -2$

② ⓐ $(A - 2I)u = \begin{bmatrix} -1 & \sqrt{3} \\ \sqrt{3} & -3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0 \quad u = \begin{bmatrix} 1 \\ 1/\sqrt{3} \end{bmatrix}$

ⓑ $(A + 2I)v = \begin{bmatrix} 3 & \sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \quad v = \begin{bmatrix} 1/\sqrt{3} \\ -1 \end{bmatrix}$

③ $y(x) = c_1 \begin{bmatrix} 1 \\ 1/\sqrt{3} \end{bmatrix} e^{2x} + c_2 \begin{bmatrix} 1/\sqrt{3} \\ -1 \end{bmatrix} e^{-2x}$

④ $\begin{bmatrix} 1 \\ 0 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1/\sqrt{3} \end{bmatrix} + c_2 \begin{bmatrix} \frac{1}{\sqrt{3}} \\ -1 \end{bmatrix}$

$c_1 + \frac{c_2}{\sqrt{3}} = 1 \qquad \frac{c_1}{\sqrt{3}} - c_2 = 0$

$c_1 + (c_1/\sqrt{3})/\sqrt{3} = 1 \qquad \frac{c_1}{\sqrt{3}} = c_2$

$c_1 + \frac{c_1}{3} = 1 \qquad (3/4)/\sqrt{3} = c_2$

$\frac{4}{3} c_1 = 1 \qquad \frac{3}{4\sqrt{3}} = \frac{\sqrt{3}}{4} = c_2$

$c_1 = \frac{3}{4}$

⑤ Solution unique:

$y = \frac{3}{4} \begin{bmatrix} 1 \\ 1/\sqrt{3} \end{bmatrix} e^{2x} + \frac{\sqrt{3}}{4} \begin{bmatrix} 1/\sqrt{3} \\ -1 \end{bmatrix} e^{-2x}$

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D6.6

#5.2 $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n\,3^{n+3}}\, x^n$

① $\displaystyle \frac{1}{R} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{2\cdot 2^n}{(n+1)\cdot 81\cdot 3^n} \cdot \frac{27n\,3^n}{2^n} \right|$

$\displaystyle = \lim_{n\to\infty} \left| \frac{2}{(n+1)81} \cdot 27n \right| = \frac{2}{3} \lim_{n\to\infty} \left|\frac{n}{n+1}\right|$

$\displaystyle = \frac{2}{3} \lim_{n\to\infty} \left| \frac{1}{1+1/n} \right| = \frac{2}{3}$

$R = \frac{3}{2}$ centré en 0

Intervalle: $\left(-\frac{3}{2}, \frac{3}{2}\right)$

② $\displaystyle\sum_{n=2}^{\infty} \frac{n 2^n}{n\,3^{n+3}}\, x^{n-1} = \sum_{n=2}^{\infty} \frac{2^n}{3^{n+3}}\, x^{n-1}$

2^{m+1+1}

On pose $m = n-1$

$\displaystyle\sum_{m=1}^{\infty} \frac{2^{m+1}}{3^{m+4}}\, x^m$ $\displaystyle \frac{1}{R} = \lim_{m\to\infty} \left| \frac{4\cdot 2^m}{243\cdot 3^m} \cdot \frac{81\cdot 3^m}{2\cdot 2^m} \right|$

$\displaystyle = \frac{2}{3} \lim_{m\to\infty} \left| \frac{2^m\cdot 3^m}{3^m\cdot 2^m} \right| = \frac{2}{3}$

$R = \frac{3}{2}$ centré en 0

Intervalle: $\left(-\frac{3}{2}, \frac{3}{2}\right)$

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D6.7

#5.3 $\sum_{n=2}^{\infty} \frac{\ln(n)}{n} x^n$

① $\frac{1}{R} = \lim_{n\to\infty} \left| \frac{\ln(n+1)}{n+1} \cdot \frac{n}{\ln(n)} \right| = \lim_{n\to\infty} \left| \frac{n \ln(n+1)}{(n+1)\ln(n)} \right| = 1$

$R = 1$ centré en 0

Intervalle : $(-1, 1)$

② $\sum_{n=3}^{\infty} \frac{n \ln(n)}{n} x^{n-1} = \sum_{n=3}^{\infty} \ln(n) x^{n-1}$ On pose $m = n-1$

$\sum_{m=2}^{\infty} \ln(m+1) x^m$

$\frac{1}{R} = \lim_{m\to\infty} \left| \frac{\ln(m+2)}{\ln(m+1)} \right| = \lim_{m\to\infty} \left| \frac{m+1}{m+2} \right| = \lim \left| \frac{1}{1} \right| = 1$

$R = 1$ centré en 0

Intervalle : $(-1, 1)$

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#10.1 (DN.5) $f'(x_0) = \frac{1}{2h}\left[f(x_0+h) - f(x_0-h) - \frac{h^2}{6} f^{(3)}(\xi) \right]$ D6.8

$h = 0{,}1$

① $f'(1{,}2) = \frac{1}{0{,}2}\left[f(1{,}3) - f(1{,}1) \right] = 5\,(0{,}27253179303401 - 0{,}33287108369808)$

$= -0{,}30169645332035$

② $f(x) = \cosh(x) - \sinh(x)$

$f'(x) = \sinh(x) - \cosh(x)$

$f'(1{,}2) = \sinh(1{,}2) - \cosh(1{,}2)$

$= \frac{e^{1{,}2} - e^{-1{,}2}}{2} - \frac{e^{1{,}2} + e^{-1{,}2}}{2} = -e^{-1{,}2} = -0{,}3011942119$

③ $E = dfn - dfe$

$= -0{,}30169645332035 + 0{,}3011942119$

$= -0{,}000050224142$

④ $|E| \leq \frac{h^2}{6} \left| f^{(3)}(x) \right|$ $f^{(3)}(x) = -e^{-x}$

$0{,}000050224142 \leq \frac{(0{,}1)^2}{6} \left| -e^{-1{,}1} \right|$

$0{,}000050224142 \leq 0{,}0005547851395$

10.2. $f(x) = x^2 e^{-x}$ $\quad h = 0{,}4,\ \underset{=0{,}2}{h/2}$ et $\underset{=0{,}1}{h/4}$ D6.9

$f'(1{,}4)$ pour DNS

$$N_1(h) = N(h) = \frac{1}{2h}\left[f(x_0+h) - f(x_0-h)\right]$$

$$N(0{,}4) = \frac{1}{0{,}8}\left[f(1{,}8) - f(1{,}0)\right] = 0{,}209611$$

$$N(0{,}2) = \frac{1}{0{,}4}\left[f(1{,}6) - f(1{,}2)\right] = 0{,}207839$$

$$N(0{,}1) = \frac{1}{0{,}2}\left[f(1{,}5) - f(1{,}3)\right] = 0{,}207321$$

$$N_2(h) = N_1\left(\frac{h}{2}\right) + \frac{N_1(h/2) - N_1(h)}{3}$$

$$N_2(0{,}4) = N_1(0{,}2) + \frac{N_1(0{,}2) - N_1(0{,}4)}{3}$$

$$= 0{,}207248$$

$$N_2(0{,}2) = N_1(0{,}1) + \frac{N_1(0{,}1) - N_1(0{,}2)}{3}$$

$$= 0{,}207148$$

$$N_3(h) = N_2\left(\frac{h}{2}\right) + \frac{N_2(h/2) - N_2(h)}{15}$$

$$N_3(0.4) = \frac{N_2(0.2) + N_2(0.2) - N_2(0.4)}{15}$$

$$= 0{,}207141$$