Rémi Vaillancourt

MAT2784B Devoir 5 D 5.1

Q1 (3.29) $y'' + y = 3x^2 - 4\sin x \quad y(0)=0 \quad y'(0)=1$

$\lambda^2 + 1 = 0 \Rightarrow \lambda = \pm i$

donc $y_h(x) = c_1 \cos x + c_2 \sin x$ (sol gén. homogène)

on cherche $y_p(x) = c_1(x)\cos x + c_2(x)\sin x$

$y_{p_1}(x) = 3x^2 \quad y_{p_2}(x) = -4\sin x$

Sol. gén: $y(x) = y_h(x) + y_{p_1}(x) + y_{p_2}(x)$

$y_{p_1}(x) = ax^3 + bx^2 + cx + d$

$y'_{p_1}(x) = 3ax^2 + 2bx + c$

$y''_{p_1}(x) = 6ax + 2b$

$y''_{p_1}(x) + y_{p_1}(x) = 6ax + 2b + 1ax^3 + 1bx^2 + 1cx + 1d$

$3x^2 = 1ax^3 + 1bx^2 + (6a + c)x + (2b + d)$

degré 3: $0 = 1a \to a = 0$

degré 2: $3 = 1b \to b = 3$

degré 1: $0 = 6a + 1c \to c = 0$

degré 0: $0 = 2b + 1d \to d = -6$

$y_{p_1}(x) = 3x^2 - 6$

$y_{p_2}(x) = \alpha \cos x + \beta \sin x$ marche pas, on fait $\times x$

D5.2

$y_{p_2}(x) = \alpha x \cos x + \beta x \sin x$

$y_{p_2}'(x) = \alpha \cos x - \alpha x \sin x + \beta \sin x + \beta x \cos x$

$y_{p_2}''(x) = -\alpha \sin x - \alpha \sin x - \alpha x \cos x + \beta \cos x + \beta \cos x - \beta x \sin x$

$y_{p_2}''(x) + 1 y_{p_2}(x) = -2\alpha \sin x + 2\beta \cos x - \alpha x \cos x - \beta x \sin x + \alpha x \cos x + \beta x \sin x$

$-4 \sin x = -2\alpha \sin x + 2\beta \cos x$

$-4 = -2\alpha \rightarrow \alpha = 2 \quad \beta = 0$

$y_{p_2}(x) = 2x \cos x$

Sol générale : $y(x) = c_1 \cos x + c_2 \sin x + 3x^2 - 6 + 2x\cos x$

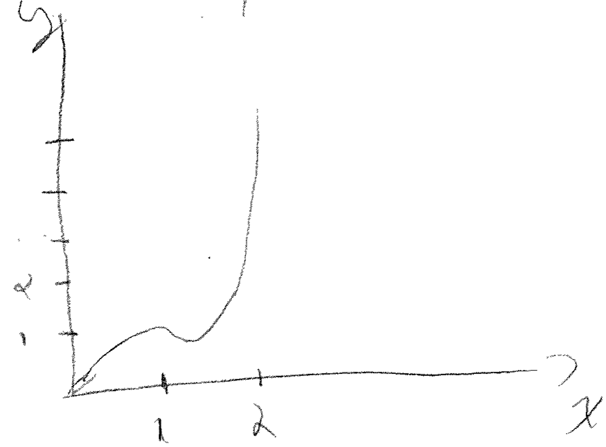
Sol particulière : $0 = c_1 \cdot 1 - 6 \rightarrow c_1 = 6$

$y'(x) = -c_1 \sin x + c_2 \cos x + 6x + 2\cos x - 2x \sin x$

$1 = c_2 + 2 \qquad c_2 = -1$

$y(x) = -\sin x + 3x^2 - 6 + 2x \cos x + 6 \cos x$

géométriquement :



D5.3

Q2 (3.32) $y'' + 6y' + 9y = \frac{e^{-3x}}{x^3}$

$\lambda^2 + 6\lambda + 9 = 0$ $\quad y_1(x) = e^{-3x}$

$(\lambda+3)^2 = 0 \quad \lambda_{1,2} = -3 \quad y_2(x) = xe^{-3x}$

Sol. gén homogène: $y_h(x) = c_1 e^{-3x} + c_2 x e^{-3x}$

on cherche $y_p(x) = c_1(x) e^{-3x} + c_2(x) x e^{-3x}$

$L_2 + 3L_1 \quad \begin{bmatrix} e^{-3x} & xe^{-3x} \\ -3e^{-3x} & e^{-3x} - 3xe^{-3x} \end{bmatrix} \begin{bmatrix} c_1'(x) \\ c_2'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{e^{-3x}}{x^3} \end{bmatrix}$

$\begin{bmatrix} e^{-3x} & xe^{-3x} \\ 0 & e^{-3x} \end{bmatrix} \begin{bmatrix} c_1'(x) \\ c_2'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{e^{-3x}}{x^3} \end{bmatrix}$

$e^{-3x} \cdot c_2'(x) = \frac{e^{-3x}}{x^3} \qquad c_2'(x) = \frac{1}{x^3}$

$c_1'(x) e^{-3x} + xe^{-3x} c_2'(x) = 0$

$c_1'(x) = -\frac{1}{x^2} \qquad \int -x^{-2} \quad x^{-1}$

$c_2(x) = \int \frac{1}{x^3} \, dx = \frac{-1}{2x^2} \qquad c_1(x) = \int \frac{-1}{x^2} dx = \frac{1}{x}$

Sol particulière: $y_p(x) = \frac{1}{x} e^{-3x} - \frac{1}{2x^2} e^{-3x}$

$= \frac{e^{-3x}}{2x}$

D5.3

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$c_2(x) = \int \frac{1}{x^3}\, dx = \frac{-1}{2x^2} \qquad c_1(x) = \int \frac{-1}{x^2}\, dx = \frac{1}{x}$

Sol particulière: $y_p(x) = \frac{1}{x} e^{-3x} - \frac{1}{2x^{\circ}} e^{-3x}$

$= \frac{a\, e^{-3x}}{2x}$

$\sin x . \cancel{\cos x} - \ln|\sec x + \tan x| \cos x . - \cancel{\cos x} \sin x$

DS.5

Q3 (suite) $y_p(x) = (\sin x - \ln|\sec x + \tan x|) \cos x - \cos x \sin x$

Sol gén: $y(x) = c_1 \cos x + c_2 \sin x + (\sin x - \ln|\sec x + \tan x|) \cos x$

$- \cos x \sin x \qquad + \sin^2 x - \cos^2 x$

Sol. particulière: $1 = c_1$ ✓

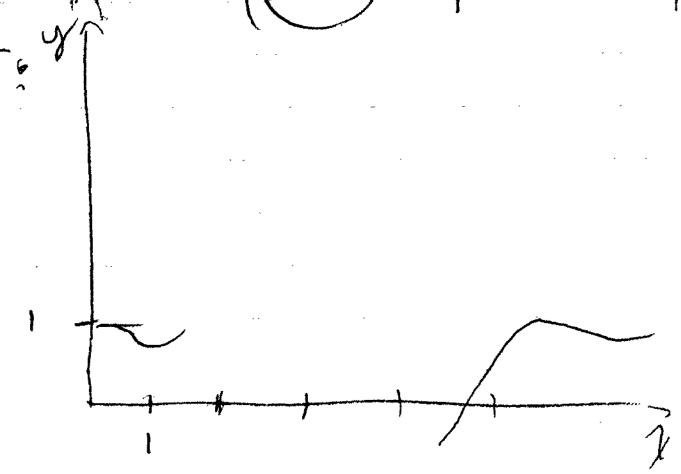
$y'(x) = -c_1 \sin x + c_2 \cos x - (\sin x - \ln|\sec x + \tan x|) \sin x$

$+ \cos x \left( \cos x - \frac{(\sec x \tan x + \sec^2 x)}{\sec x + \tan x} \right) + \sin^2 x - \cos^2 x$

$0 = c_2 - 1 \qquad c_2 = 1$ ✓

$y(x) = \cos x + \sin x + (\sin x - \ln|\sec x + \tan x|) \cos x - \cos x \sin x$

géométriquement:



ou $y(x) = \cos x + \sin x - \cos x \ln|\sec x + \tan x|$ ✓

Q4 (3.39) $2x^2 y'' + x y' - 3y = 2x^{-3}$

$2m(m-1)x^m + x^m m - 3x^m = 0$

$2m^2 - 2m + m - 3 = 0$

$m^2 - \frac{m}{2} - \frac{3}{2} = 0 \qquad m_1 = 3/2$

$m_2 = -1$

D 5.6

$y_h(x) = c_1 x^{3/2} + \frac{c_2}{x}$ $\qquad y_p(x) = ?$

$$\begin{bmatrix} x^{3/2} & 1/x \\ \frac{3}{2}x^{1/2} & -x^{-2} \end{bmatrix} \begin{bmatrix} c_1'(x) \\ c_2'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ 1/x^5 \end{bmatrix}$$

→ car $ay''$, $a \neq 1$ on doit faire $r(x)/a = \frac{1}{x^5}$

$x^{3/2} c_1'(x) + \frac{c_2'(x)}{x} = 0$ $\qquad c_2'(x) = -x^{5/2} c_1'(x)$

$\frac{3}{2} x^{1/2} c_1'(x) - x^{-2} c_2'(x) = 1/x^5$ $\qquad c_2'(x) = -x^{5/2} \frac{2}{5x^{11/2}}$

$\frac{3}{2} x^{1/2} c_1'(x) + x^{1/2} c_1'(x) = 1/x^5$ $\qquad c_2'(x) = \frac{-2}{5x^3}$

$\frac{5}{2} x^{1/2} c_1'(x) = \frac{1}{x^5}$ $\qquad c_2(x) = \frac{1}{5x^2}$

$c_1'(x) = \frac{2}{5x^{11/2}}$

$c_1(x) = \frac{-4}{45x^{9/2}}$

Solgén: $y(x) = c_1 x^{3/2} + \frac{c_2}{x} + \frac{-4}{45x^{9/2}} x^{3/2} + \frac{1}{5x^2 x}$

$y(x) = c_1 x^{3/2} + \frac{c_2}{x} + \frac{-4}{45x^3} + \frac{1}{5x^3}$

$y(x) = c_1 x^{3/2} + \frac{c_2}{x} + \frac{1}{9x^3}$

Sol spéc: $0 = c_1 + c_2 + \frac{1}{9} \to c_1 + c_2 = \frac{-1}{9}$

$y'(x) = \frac{3}{2} c_1 x^{1/2} - \frac{c_2}{x^2} - \frac{1}{3x^4}$ $\qquad c_1 = \frac{\begin{vmatrix} -1/9 & 1 \\ \frac{10}{3} & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 3/2 & -1 \end{vmatrix}} \quad \frac{58}{45}$

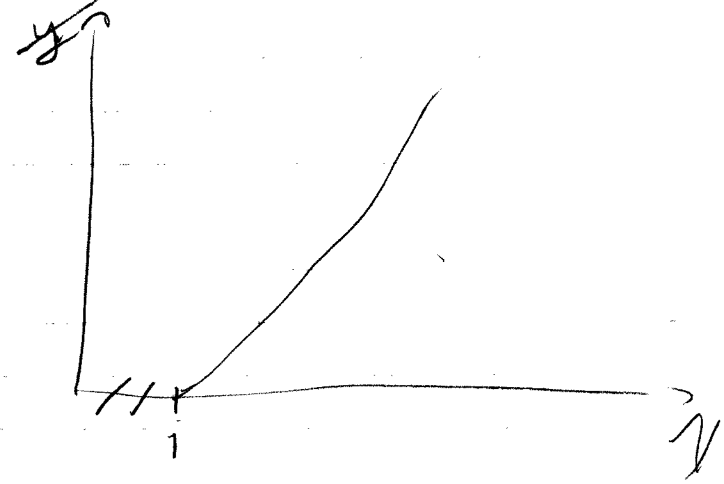
$3 = \frac{3}{2} c_1 - c_2 - \frac{1}{3}$ (Kramer)

$\frac{3}{2} c_1 - c_2 = \frac{10}{3}$

Q4 (suite) $c_2 = \dfrac{\begin{vmatrix} 1 & -1/4 \\ 3/2 & 10/3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 3/2 & -1 \end{vmatrix}} = -\dfrac{7}{5}$

donc $y(x) = \dfrac{58}{45} x^{3/2} - \dfrac{7}{5x} + \dfrac{1}{9x^3}$

géométriquement :




Q5 (4.1) $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ trouver $y' = Ay$

$A - \lambda Id = \begin{bmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{bmatrix}$ $\quad \det|A - \lambda Id| = 0$

$-\lambda(-3-\lambda) + 2 = 0$

$\lambda_1 = -1 \qquad \lambda^2 + 3\lambda + 2 = 0$

$\lambda_2 = -2 \qquad (\lambda+1)(\lambda+2) = 0$

pour 1 : $\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad u_1 + u_2 = 0$

$-2u_1 - 2u_2 = 0$

$u_1 = -u_2$

on pose $u_1 = 1 \quad u_2 = -1 \quad u_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

pour 2 : $\begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $2v_1 + v_2 = 0$ 05.8

$v_2 = -2v_1$

on pose $v_1 = 1 \to v_2 = -2$

$u_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ donc $u_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $u_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Q6 (4.11) $A - \lambda Id = \begin{bmatrix} 5-\lambda & -1 \\ 3 & 1-\lambda \end{bmatrix}$ $\det |A - \lambda Id| = 0$

$\lambda_1 = 2$

$\lambda_2 = 4$

$(5-\lambda)(1-\lambda) + 3 = 0$

$\lambda^2 - 6\lambda + 8 = 0$

pour 1 : $\begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $3u_1 - u_2 = 0$

$3u_1 = u_2$ on pose $u_1 = 1$

$u_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ pour 2 : $\begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$u_1 - u_2 = 0$

$u_1 = u_2$ $u_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$u_1 = 1$

$u_2 = 1$

Sol générale $= c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{2x} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4x} = y(x)$

$\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} c_1 \\ 3c_1 \end{bmatrix} + \begin{bmatrix} c_2 \\ c_2 \end{bmatrix}$ $c_1 = \dfrac{\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}}{\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}} = -\dfrac{3}{2}$ $c_2 = \dfrac{\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}}{\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}} = \dfrac{7}{2}$

$2 = c_1 + c_2$

$-1 = 3c_1 + c_2$

donc solution particulière :

$$y(x) = -\frac{3}{2}\begin{bmatrix}1\\3\end{bmatrix}e^{2x} + \frac{7}{2}\begin{bmatrix}1\\1\end{bmatrix}e^{4x}$$

$$y(x) = \begin{bmatrix}-3/2\\-9/2\end{bmatrix}e^{2x} + \begin{bmatrix}7/2\\7/2\end{bmatrix}e^{4x}$$

9.11 (Q7)

| $x$ | 0,0 | 0,2 | 0,4 | 0,6 | 0,8 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | 1,2214 | 1,49182 | 1,82212 | 2,22554 |

| $i$ | $x_i$ | $y_i$ | $\Delta y_i$ | $\Delta^2 y_i$ | $\Delta^3 y_i$ | $\Delta^4 y_i$ |
|---|---|---|---|---|---|---|
| 0 | 0,0 | $\boxed{1}$ | | | | |
| | | | $\boxed{0,2214}$ | | | |
| 1 | 0,2 | 1,2214 | | $\boxed{0,04902}$ | | |
| | | | 0,27042 | | $\boxed{0,01086}$ | |
| 2 | 0,4 | 1,49182 | | 0,05988 | | $\boxed{0,00238}$ |
| | | | 0,3303 | | 0,01324 | |
| 3 | 0,6 | 1,82212 | | 0,07312 | | |
| | | | 0,40342 | | | |
| 4 | 0,8 | 2,22554 | | | | |

$$p_4(r) = 1 + r(0,2214) + \frac{r(r-1)}{2}(0,04902) + \frac{r(r-1)(r-2)}{6}(0,01086)$$
$$+ \frac{r(r-1)(r-2)(r-3)}{24}(0,00238)$$

où $r = \dfrac{(0,05-0)}{0,2} = 0,25$

$$p_4(0,25) = 1,0512588$$

$$f(0,05) = 1,05\underline{12588}$$

Q8 (9.13) D 5:10

| $x$ | $f(x)$ | $f'(x)$ |
|---|---|---|
| 8,3 | 17,56492 | 3,116256 |
| 8,6 | 18,50515 | 3,151762 |

| $z$ | $f(z)$ | $f'(x)$ | $f''(x)$ | $f^{(3)}(x)$ |
|---|---|---|---|---|
| 8,3 | 17,56492 | 3,116256 | | |
| 8,3 | 17,56492 | 3,1341 | 0,05948 | -0,002 |
| 8,6 | 18,50515 | 3,151762 | 0,0588~~73~~ | |
| 8,6 | 18,50515 | | | |

$$p(x) = 17,56492 + (x-8,3)(3,116256) + (x-8,3)^2\, 0,05948$$
$$+ (x-8,3)^2 (x-8,6)(-0,002\cancel{03})$$