2.8 $y'' + 2y' + 2y = 0 \qquad y(0) = 2 \quad y'(0) = -3$

P3.1

MAT 2784B

DEV. 3

09.02.10

RÉMI VAILLANCOURT

① $\lambda^2 + 2\lambda + 2$

$\hookrightarrow \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-2 \pm \sqrt{4-8}}{2}$

$= \dfrac{-2 \pm \sqrt{-4}}{2}$

$= \dfrac{-2 \pm i\sqrt{4}}{2}$

$\alpha = -1 \quad \beta = 1 \qquad = -1 \pm i \rightarrow \lambda_1 = -1 + i$

$\lambda_2 = -1 - i$

② $y(x) = c_1 e^{\alpha x} \cos(\beta x) + c_2 e^{\alpha x} \sin(\beta x)$

$y(x) = c_1 e^{-x} \cos(x) + c_2 e^{-x} \sin(x)$

③ $y(0) = c_1 = 2$

$y'(x) = -c_1 e^{-x} \cos(x) - c_1 e^{-x} \sin(x) - c_2 e^{-x} \sin(x) + c_2 e^{-x} \cos(x)$

$y'(x) = -c_1 e^{-x} (\cos(x) - \sin(x)) - c_2 e^{-x} (\sin(x) - \cos(x))$

$y'(0) = -c_1 + c_2 = -3$

$-c_1 + c_2 = -3$

$-2 + c_2 = -3$

$c_2 = -1$

④ $y(x) = 2e^{-x} \cos(x) - e^{-x} \sin(x)$

2.10 $y'' + 16y = 0 \quad y(0) = 0 \quad y'(0) = 1$

Amplitude A et période P?

① $16 = \omega^2$

$\lambda^2 + \omega^2 = 0$

$\lambda^2 + 0\lambda + 16$

② $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-0 \pm \sqrt{0^2 - 4 \cdot 1 \cdot 16}}{2}$

$= \dfrac{\pm \sqrt{-64}}{2} = \dfrac{\pm i\sqrt{64}}{2} = \pm 4i$

$\alpha = 0 \quad \beta = 4$

③ $y(x) = C_1 e^{\alpha x} \cos(\beta x) + C_2 e^{\alpha x} \sin(\beta x)$

$= C_1 \cos(4x) + C_2 \sin(4x)$

$y(0) = C_1 \cos(0) + C_2 \sin(0) = 0$

$= C_1 = 0$

$y'(x) = -C_1 \sin(4x) \cdot 4 + C_2 \cos(4x) \cdot 4$

$y'(0) = C_2 \cos(0) \cdot 4 = 1$

$= 4C_2 = 1$

$C_2 = 1/4$

④ $y(x) = \dfrac{\sin(4x)}{4}$

⑤ Amplitude $= \sqrt{C_1^2 + C_2^2} = \sqrt{0 + (\frac{1}{4})^2} = \sqrt{(\frac{1}{4})^2} = \frac{1}{4}$

Période $= \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

D 3.2

2.12 $\quad y'' + 6y' + 9y = 0 \qquad y(0) = 0 \qquad y'(0) = 2$ $\qquad$ D3.3

① $\lambda^2 + 6\lambda + 9 = 0$

② $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot 9}}{2} = \dfrac{-6 \pm \sqrt{0}}{2} = -3$

③ $y(x) = C_1 e^{-\alpha x} + C_2 x e^{-\alpha x}$

$= C_1 e^{-3x} + C_2 x e^{-3x}$

$y(0) = C_1 = 0$

$y'(x) = -3C_1 e^{-3x} + C_2 e^{-3x} - 3C_2 x e^{-3x}$

$y'(0) = 0 + C_2 - 0 = 2$

$C_2 = 2$

④ $y(x) = 2x e^{-3x}$

⑤ $y'(x) = 2e^{-3x} - 6x e^{-3x}$

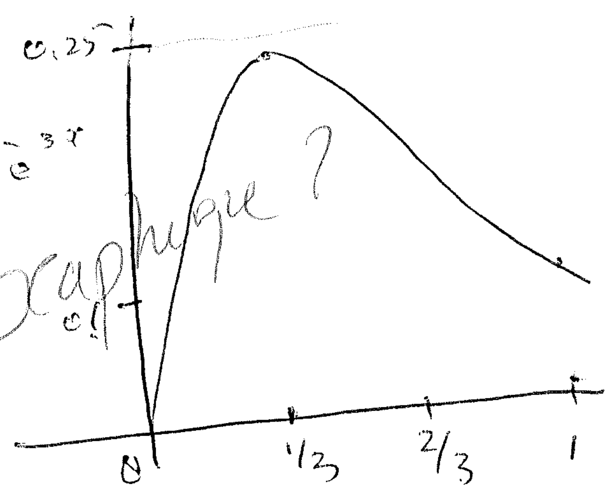
$0 = 2e^{-3x} - 6x e^{-3x}$ $\qquad$ pt critique

$6x e^{-3x} = 2e^{-3x}$

$6x = 2 \qquad \rightarrow x = 2/6 = 1/3$

Maximum à $y(\frac{1}{3})$

Pourquoi?



Graphique ?

$y''(x) = -6e^{-3x} - 6e^{-3x} + 18x e^{-3x}$

$= e^{-3x}(-12 + 18x)$

$y''(1/3) = e^{-1}(-12 + 6)$

$= -6e^{-1} < 0$

Donc en $x = 1/3$

$y(x)$ a un maximum : $y(\frac{1}{3}) = \frac{2}{3} \cdot e^{-1}$

Attention

#2.13 $x^2y'' + 3xy' + y = 0$ -3

D3.4

① $m^2 + (3-1)m + 1 = 0$

② $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-2 \pm \sqrt{4-4\cdot1\cdot1}}{2} = \dfrac{-2 \pm \sqrt{0}}{2} = -1$

③ $y(x) = c_1 x^m + c_2 \ln(x) x^m$

$= c_1 x^{-1} + c_2 \ln(x) x^{-1}$

$x^2y'' + 3xy' - 3 = 0$

$m^2 + (3-1)m - 3 = 0$

$m^2 + 2m - 3 = 0$ $\quad (m+3)(m-1) = 0$

$m_1 = -3$

$m_2 = +1$

$y(x) = c_1 x^{+1} + c_2 x^{-3}$

2.18 $\quad x^2y'' + 5xy' + 3y = 0 \qquad y(1) = 1 \quad y'(1) = -5$ $\qquad$ b3.5

① $m^2 + (5-1)m + 3 = 0$

② $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-4 \pm \sqrt{16-4\cdot1\cdot3}}{2} = \dfrac{-4 \pm \sqrt{4}}{2}$

$= -2 \pm 1 \rightarrow \begin{array}{l} x_1 = -1 \\ x_2 = -3 \end{array}$

③ $y(x) = C_1 x^{m_1} + C_2 x^{m_2}$

$\quad = C_1 x^{-1} + C_2 x^{-3}$

$y(1) = C_1 + C_2 = 1$

$y'(x) = -C_1 x^{-2} - 3C_2 x^{-4}$

$y'(1) = -C_1 - 3C_2 = -5$

④ ⓐ $C_1 + C_2 = 1 \quad \rightarrow \quad C_1 = 1 - C_2$

ⓑ $-C_1 - 3C_2 = -5 \quad \rightarrow \quad -(1 - C_2) - 3C_2 = -5$

$-1 + C_2 - 3C_2 = -5$

$-2C_2 = -4$

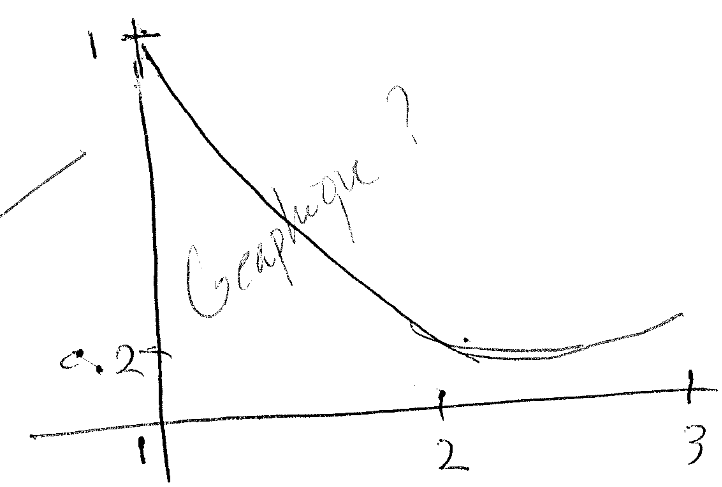
$C_2 = 2$

ⓐ $C_1 + C_2 = 1$

$C_1 + 2 = 1$

$C_1 = 1 - 2 = -1$

⑤ $y(x) = -x^{-1} + 2x^{-3}$



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2.19 $x^2 y'' - xy' + y = 0 \qquad y(1) = 1 \qquad y'(1) = 0$ D3.6

① $m^2 + (-1-1)m + 1 = 0$

② $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{2 \pm \sqrt{4-4\cdot 1\cdot 1}}{2} = \dfrac{2 \pm \sqrt{0}}{2} = 1$

③ $y(x) = c_1 x^m + c_2 \ln(x) x^m$

$\quad = c_1 x + c_2 \ln(x) x$

$y(1) = c_1 = 1$

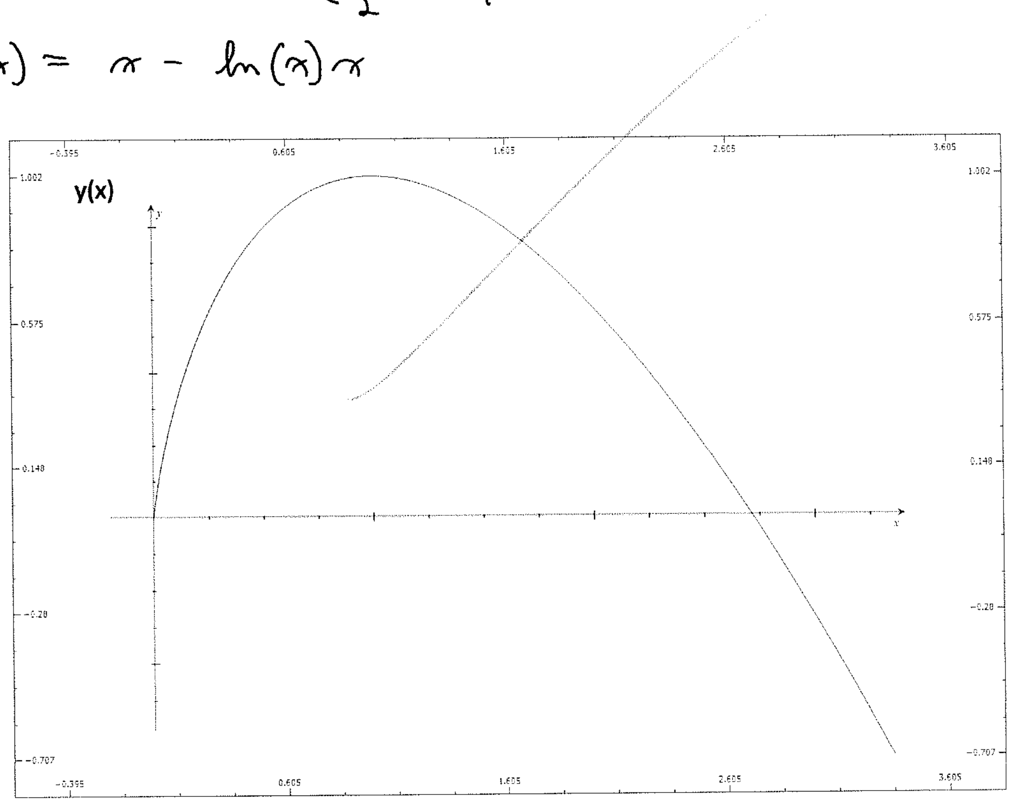
$y'(x) = c_1 + c_2 + c_2 \ln(x)$

$y'(1) = c_1 + c_2 = 0$

$1 + c_2 = 0$

$c_2 = -1$

④ $y(x) = x - \ln(x) x$



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D 3.7

8.14 $x_0 = 1 \quad x_1 = 0{,}5 \qquad f(x) = e^{-x} - \tan(x)$

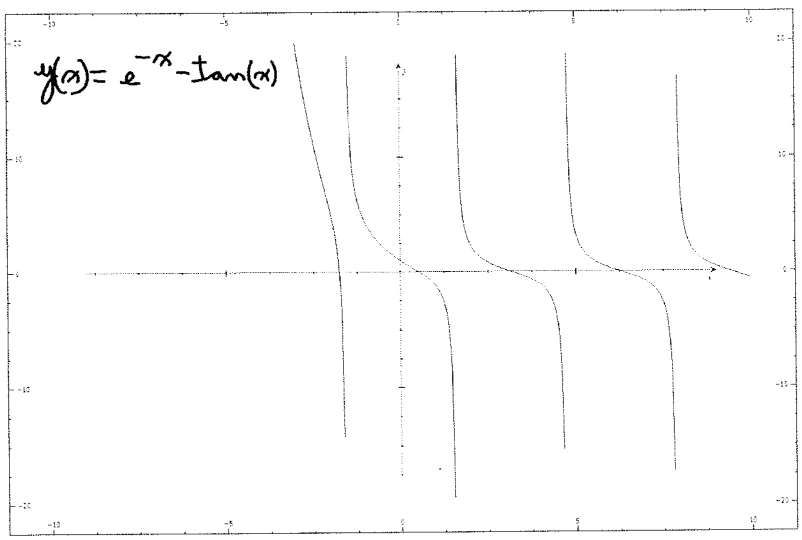
① $$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$$

n	x_n
0	1
1	0,5
2	0,5240959627
3	0,5314457103
4	0,5313907551
5	0,5313408567

② $f'(x) = -e^{-x} - \sec^2(x)$

$f'(p) \approx -1{,}9332 \neq 0$

Donc 0,531390 est une racine simple.
La méthode de la sécante converge d'ordre 1.618.



8.17 $g(x) = 1 + \sin^2(x)$ $\quad x_0 = 1$ MAT 2784B D3. 8

① $y_1 = g(x_0) = 1 + \sin^2(1) = 1{,}708073418$

$y_2 = g(y_1) = 1 + \sin^2(1{,}708073418) = 1{,}981273081$

$$x_1 = x_0 - \frac{(y_1 - x_n)^2}{y_2 - 2y_1 + x_n}$$

②

Itération	x_n
0	1
1	2,152904629
2	1,873464044
3	1,897028966
4	1,897194298
5	1,897194306

$p = 1{,}897194306$

③ $g'(x) = 2\cos(x)\sin(x)$ $\quad g'(p) \approx -0{,}607409 \neq 0$

Donc $g(x)$ converge d'ordre 1 et Steffensen accélère la convergence à ordre 2.