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#1.22 $\frac{2x}{y^3}\,dx + \frac{(y^2-3x^2)}{y^4}\,dy = 0$ $\qquad y(1)=1$

① $M = \frac{2x}{y^3}$ $\qquad N = \frac{(y^2-3x^2)}{y^4}$

$M_y = \frac{-6x}{y^4}$ $\qquad N_x = \frac{-6x}{y^4}$

$M_y = N_x \to$ Exacte

② $u'_x = \frac{2x}{y^3} = M$

$u = \int \frac{2x}{y^3}\,dx + T(y) = \frac{x^2}{y^3} + T(y)$

$u'_y = \frac{d}{dy}\frac{x^2}{y^3} + T'(y)$

$= \frac{-3x^2}{y^4} + T'(y) = \frac{y^2}{y^4} + \frac{-3x^2}{y^4} = N$

$T'(y) = \frac{y^2}{y^4} = \frac{1}{y^2}$

$T(y) = \int \frac{1}{y^2}\,dy = \cancel{\frac{+2}{y^3}} \quad -\frac{1}{y}$

③ $u(x,y) = \cancel{\frac{x^2-2}{y^3}}$ $\qquad \frac{x^2}{y^3} - \frac{1}{y} = C$ sol. gen.

$\cancel{\frac{x^2-2}{y^3} = C}$ $\qquad 1 - 1 = C = 0$

④ $y(1)=1$ $\qquad \cancel{\frac{1^2-2}{1} = C}$

$\cancel{-1 = C}$

$\boxed{\text{Solution: (unique) } \frac{x^2-2}{y^3} = -1}$ $\qquad \frac{x^2}{y^3} - \frac{1}{y} = 0$

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#1.23 $\quad (ye^x + 2e^x + y^2)\,dx + (e^x + 2xy)\,dy = 0 \qquad y(0) = 6$

① $M = ye^x + 2e^x + y^2 \qquad N = e^x + 2xy$

$M_y = e^x + 2y \qquad N_x = e^x + 2y$

$M_y = N_x \Rightarrow$ Exacte

② $u'_y = e^x + 2xy = N$

$u = ye^x + xy^2 + T(x)$

$u'_x = ye^x + y^2 + T'(x) = ye^x + 2e^x + y^2 = M$

$T'(x) = 2e^x$

$T(x) = \int 2e^x\,dx = 2e^x$

③ $u(x,y) = ye^x + xy^2 + 2e^x$

$ye^x + xy^2 + 2e^x = C$

④ $y(0) = 6 \qquad 6e^0 + 0(6)^2 + 2e^0 = C$

$6 + 0 + 2 = C$

$8 = C$

Solution: $ye^x + xy^2 + 2e^x = 8$

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#1.30 $(2xy^2 - 3y^3)dx + (7 - 3xy^2)dy = 0$

① $M = 2xy^2 - 3y^3$ $\quad N = 7 - 3xy^2$

$M_y = 4xy - 9y^2$ $\quad N_x = -3y^2$

$M_y \neq N_x \Rightarrow$ Pas Exacte

② $g(y) = \dfrac{4xy - 9y^2 + 3y^2}{2xy^2 - 3y^3} = \dfrac{2(xy - 3y^2)}{y(xy - 3y^2)} = \dfrac{2}{y}$

③ $\mu(y) = e^{-\int g(y)\,dy} = e^{-2\ln|y|\,dy} = e^{\ln|y^{-2}|} = \dfrac{1}{y^2}$

④ $(2x - 3y)dx + \left(\dfrac{7}{y^2} - 3x\right)dy = 0$

$M = 2x - 3y$ $\quad N = \dfrac{7}{y^2} - 3x$

$M_y = -3$ $\quad N_x = -3$

$M_y = N_x \rightarrow$ Exacte

⑤ $u = \int (2x - 3y)\,dx + T(y)$

$= x^2 - 3xy + T(y)$

$u_y = -3x + T'(y) = \dfrac{7}{y^2} - 3x = N$

$T'(y) = \dfrac{7}{y^2}$ $\quad T(y) = 7\int \dfrac{1}{y^2}\,dy = \dfrac{-7}{y}$

$u(x,y) = x^2 - 3xy - \dfrac{7}{y}$

Solution générale : $x^2 - 3xy - \dfrac{7}{y} = C$

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#1.45 $x^2 - y^2 = c^2$

① $2x - 2yy' = 0$

$$y' = \frac{-2x}{-2y} = x/y = m$$

② $m = \frac{b}{a} = \frac{x}{y} = y'$ $\qquad y'_{orth} = -\frac{a}{b} = -\frac{y}{x}$

③ $y\,dx + x\,dy = 0$

$M_y = 1 = N_x \quad \Rightarrow$ Exacte

$u_x = y$

$u(x,y) = \int y\,dx = yx + T(y)$

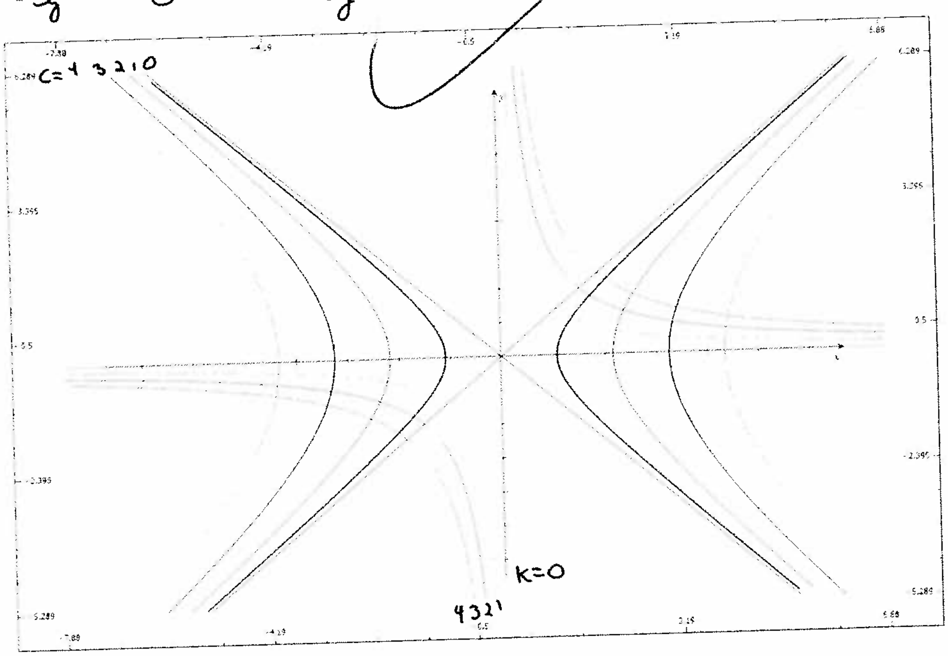
$u_y = x + T'(y) = x$

$T'(y) = 0$

$T(y) = c$

$u(x,y) = yx + C$

④ $x^2 - y^2 = c^2 \qquad yx = K$



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2.2 $y'' + 2y' + y = 0$

① $\lambda^2 + 2\lambda + 1 = (\lambda+1)(\lambda+1) = (\lambda+1)^2$

$\hookrightarrow y_1 = e^{-x} \qquad y_2 = x e^{-x}$

② $y = C_1 e^{-x} + C_2 x e^{-x}$

2.6 $y'' - 4y' + 3y = 0 \qquad y(0) = 6 \qquad y'(0) = 0$

① $\lambda^2 - 4\lambda + 3$ ② $\lambda_{1,2} = \dfrac{-a \pm \sqrt{a^2 - 4b}}{2}$

$= (\lambda - 3)(\lambda - 1)$ $\quad = \dfrac{4 \pm 2}{2} \rightarrow \lambda_1 = 3$, $\hookrightarrow \lambda_2 = 1$

③ $y_1(x) = e^{3x} \qquad \dfrac{e^{3x}}{e^{x}} = e^{2x} \neq$ const.

$y_2(x) = e^{x}$

$\hookrightarrow$ Donc y_1 et y_2 sont linéairement indépendant

④ $y = C_1 e^{3x} + C_2 e^{x} \qquad y(0) = C_1 + C_2 = 6 \qquad C_2 = 6 - C_1$

$y'(x) = 3C_1 e^{3x} + C_2 e^{x} \qquad y'(0) = 3C_1 + C_2 = 0$

⑤ $\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix} \Rightarrow Ac = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$

$\text{Det } A = 1 - 3 = -2 \rightarrow c$ est unique

⑥ $C_1 = \dfrac{1}{-2} \begin{vmatrix} 6 & 1 \\ 0 & 1 \end{vmatrix} = \dfrac{6}{-2} = -3$

$C_2 = \dfrac{1}{-2} \begin{vmatrix} 1 & 6 \\ 3 & 0 \end{vmatrix} = \dfrac{-18}{-2} = 9$

⑦ Solution unique : $y = -3e^{3x} + 9e^{x}$

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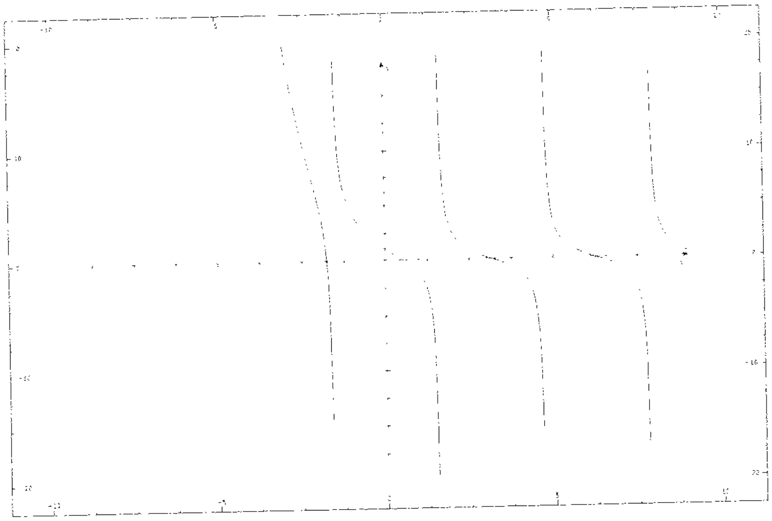
8.11 $f(x) = e^{-x} - \tan(x)$ racine de $f(x)=0$ à 6 décimales
$x_0 = 1$



① $f'(x) = -e^{-x} - \sec^2(x)$

② $x_{n+1} = x - \dfrac{e^{-x} - \tan(x)}{-e^{-x} - \sec^2(x)}$

Iteration	x_n	Erreur
0	1,0000000000	—
1	0,6864214614	0,3135785386
2	0,5411300974	0,1452913640
3	0,5314160869	0,0097140105
4	0,5313908568	0,0000252288
5	0,5313908567	0,0000000001

$p = 0,531390856$ à 10^{-9} près

③ $f(p) = 0$

$f'(p) = -e^{-p} - \sec^2(p) \approx -3,0467 \neq 0$

$f''(x) = e^{-x} - \left(\dfrac{2\sin(x)}{\cos(x)^3}\right) \approx 0,1195$

$f'(p) \neq 0$ et $f''(x)$ existe donc la méthode de newton converge au moins d'ordre 2.

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8.12 $f(x) = 2x - \tan(x)$ $\quad x_0 = 1 \quad x_1 = 0{,}5$

① $x_{n+1} = x_n - \dfrac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$

Itération	x_{n+1}	Erreur
0	1	—
1	0,5	—
2	20,9271922	20,4271922
3	0,2864772525	20,6417495
4	0,1534458174	0,1320314351
5	−0,00705133884	0,1604971558
6	0,0000537095 4664	0,0071050479
7	−0,00000000018342253	0,000005379788 89
8	0,00000000 00	0,000000001

racine : $x = 0{,}000000$

② $f'(x) = 2 - \sec^2(x)$

$f'(0) = 2 - \sec^2(0) = 2 - 1 = 1 \neq 0$

0 est une racine simple, l'ordre de convergence devrait être 1,618 (page 159).