MAT 2784 B DEV. n° 1 D1.1

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1.7. $\frac{dy}{\cancel{dt}} \sin x - y \cos x \frac{dx}{\cancel{dt}} = 0$

$$\int \frac{dy}{y} = \int \frac{\cos x}{\sin x}\, dx + c$$

$$= \int \operatorname{cotan}(x)\, dx + c$$

$$\ln|y| = \ln|\sin x| + c$$

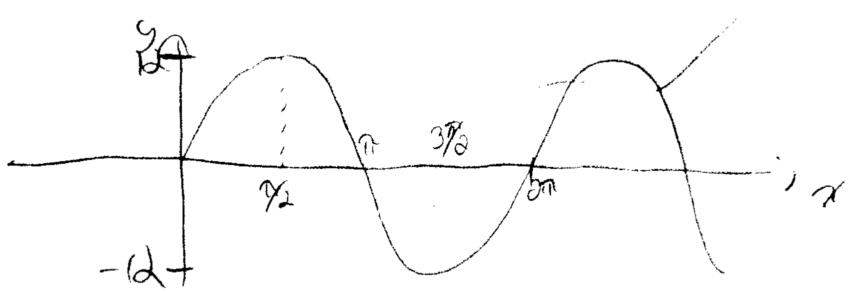
$$|y| = e^{\ln|\sin x| + c}$$

$$|y| = k|\sin x|$$

Avec la condition initiale : $y(\pi/2) = 12 = k \sin x$

$$12 = k \sin(\pi/2)$$
$$k = 1$$

$$|y(x)| = 12|\sin x|$$



1.3 $(1+x^2)\, dy = \cos^2 y\, dx$

$$\int \frac{dy}{\cos^2 y} = \int \frac{dx}{1+x^2} + c$$

$$\tan y = \arctan x + c$$

1.34. $\dfrac{dy}{dx} + \dfrac{2x}{x^2+1} y = x$

$f(x) = \dfrac{2x}{x^2+1}$ $\quad \mu(x) = e^{\int f(x)dx}$

$= e^{\int \frac{2x}{x^2+1} dx}$

$= e^{2\int \frac{x}{x^2+1} dx}$

On intègre $\int x \cdot \dfrac{1}{x^2+1} dx$ par parties.

$\underset{u}{x} \quad \underset{v'}{\dfrac{1}{x^2+1}}$ $\quad u' = dx$

$v = \arctan(x)$

$$\int \frac{x}{x^2+1} dx = x\arctan(x) - \int \arctan(x)\, dx$$

$$= x\arctan(x) - \left(x\arctan(x) - \frac{1}{2}\ln(1+x^2)\right)$$

$$= \frac{1}{2}\ln(1+x^2)$$

Alors, $\mu(x) = e^{2\int \frac{x}{x^2+1} dx}$

$$= e^{2\left(\frac{1}{2}\ln(1+x^2)\right)} = e^{\ln(1+x^2)} = 1+x^2.$$

$$(1+x^2)\left(y' + \frac{2x}{x^2+1}y\right) = (1+x^2)(x)$$

$$\int \left((1+x^2)y\right)' dx = \int (x+x^3)\,dx + c$$

$$(1+x^2)y = \frac{x^2}{2} + \frac{x^4}{4} + c$$

$$y = \frac{x^2}{2+2x^2} + \frac{x^4}{4+4x^2} + \frac{c}{1+x^2}$$

$$= \frac{2x^2 + x^4 + 4c}{4+4x^2}.$$

01.3

1.35. $x\ln(x)y' + y = 2\ln(x)$

$$y' + \frac{y}{x\ln(x)} = \frac{2\cancel{\ln(x)}}{x\cancel{\ln(x)}}$$

$f(x) = \dfrac{1}{x\ln(x)}$ $\quad \mu(x) = e^{\int f(x)dx}$

$$\mu(x) = e^{\int \frac{1}{x\ln(x)} dx} = e^{\ln|\ln(x)|} = \ln(x).$$

$$\ln(x)\left(y' + \frac{y}{x\ln(x)}\right) = \ln(x)\left(\frac{2}{x}\right)$$

$$\int (\ln(x)\,y)'dx = \int \ln(x)\left(\frac{2}{x}\right)dx + c$$

$$\ln(x)\,y = 2\int \frac{\ln(x)}{x}dx + c$$

$$= \cancel{2}\cdot\frac{(\ln(x))^2}{\cancel{2}} + c$$

$$y = \ln(x) + \frac{c}{\ln(x)}$$

1.38. $xy' - 2y = 2x^4$ avec $y(12) = 11$

$$y' - \frac{2}{x}y = 2x^3$$

$$f(x) = -\frac{2}{x} \qquad \mu(x) = e^{\int f(x)dx} = e^{-2\int \frac{1}{x}dx} = e^{-2\ln(x)}$$

$$= e^{\ln(x)^{-2}} = x^{-2} = \frac{1}{x^2}.$$

$$\left(\frac{1}{x^2}\right)\left(y' - \frac{2}{x}y\right) = \left(\frac{1}{x^2}\right)\left(2x^3\right)$$

$$\int \left(\frac{y}{x^2}\right)' dx = \int 2x\, dx + C$$

$$\frac{y}{x^2} = x^2 + C$$

$$y = x^4 + x^2 C$$

$$y(12) = (12)^4 + (12)^2 C = 11$$

$$11 = 20736 + 144C$$

$$\frac{-20725}{144} = C$$

$$y = x^4 - \frac{20725}{144}x^2.$$

1.40 $y' - y\tan x = \frac{1}{\cos^3 x}$ $y(0)=0$.

$f(x) = -\tan(x)$ $\mu(x) = e^{\int f(x)dx}$

$= e^{\int \tan(x) dx}$

$= e^{+\ln|\cos(x)|}$

$= \cos(x)$

$\cos(x)(y' - y\tan(x)) = \frac{\cancel{\cos(x)}}{\cos^{\cancel{3}2}(x)}$

$\int (\cos(x) y)' dx = \int \frac{1}{\cos^2 x} dx + c$

$\cos(x) y = \tan(x) + c$

$y = \frac{\tan(x)}{\cos(x)} + \frac{c}{\cos(x)}$

$y(0) = 0 = \frac{\tan(0) + c}{\cos(0)}$

$0 = \tan(0) + c$

$c = 0.$

$y(x) = \frac{\tan(x)}{\cos(x)}$

8.6. $f(x) = x^3 - x - 1 = 0$ à 10^{-2} près, $[1,2]$ $x_0 = 1$

$$x^3 = x+1$$
$$x^2 = \frac{x+1}{x}$$
$$x = \sqrt{\frac{x+1}{x}}$$
$$g(x) = \sqrt{\frac{x+1}{x}}$$

① $g(1) = \sqrt{\frac{2}{1}} = \sqrt{2}$ $\quad g(2) = \sqrt{\frac{3}{2}}$ $\quad$ $g(1)$ et $g(2)$ sont dans $[1,2]$

② $g(x) = \sqrt{\frac{x+1}{x}}$ $\quad g'(x) = \frac{1}{2}\left(\frac{x+1}{x}\right)^{-1/2} \cdot \frac{1}{x} - \frac{(x+1)}{x^2}$

$$= \frac{1}{2}\left(\frac{x+1}{x}\right)^{-1/2} \cdot \frac{-1}{x^2}$$
$$= \frac{1}{2}\sqrt{\frac{1}{\left(\frac{x+1}{x}\right)}} \cdot \frac{-1}{x^2}$$
$$= \frac{-1}{2x^2}\sqrt{\frac{1}{\left(\frac{x+1}{x}\right)}}$$

③ $g'(1) = \left|\frac{-1}{2} \cdot \sqrt{\frac{1}{2}}\right| = 0,35355339$

$g'(2) = \left|\frac{-1}{8} \cdot \frac{1}{\sqrt{3/2}}\right| = 0,10$

$0 < K < 1$

P 1.7

On peut donc commencer les itérations

$g(x) = \sqrt{\dfrac{x+1}{x}}$

$g(x_0) = g(1) = \sqrt{\dfrac{2}{1}} = 1,4142136 = x_1$

$g(x_1) = 1,30656297 = x_2$

$g(x_2) = 1,3286711 = x_3$

$g(x_3) = 1,3286998 = x_4$

$g(x_4) = 1,32490045 = x_5$

D'où, 1,32 est la réponse.

8.7. $\sqrt{12}$ à 10^{-4}

Essayons par la méthode de la position fausse.

$f(x) = x^2 - 12 = 0$

$a_0 = 3 \quad b_0 = 4$

$f(a_0) \cdot f(b_0) = f(-3) \cdot f(4) = -12 < 0$

1) $x_1 = \frac{a_0 f(b_0) - b_0 f(a_0)}{f(b_0) - f(a_0)}$

$= \frac{3 \cdot 4) - (4 \cdot -3)}{(4) - (-3)} = \frac{12 - (-12)}{7} = \frac{24}{7} = 3,42857142x$

2) $a_0 = a_1 = 3$ et $b_1 = 24/7 = x_1$

$x_2 = \frac{(3 \cdot -0,2449) - (24/7 \cdot -3)}{(-0,2449 + 3)} = 3,466667012$

3) $a_2 = x_2 = 3,466667012 \quad b_1 = b_2 = 24/7.$

$x_3 = 3,4640884$

4) $a_3 = x_3$ et $b_3 = b_2 = 24/7$

$x_4 = 3,46410168$

5) $a_4 = a_3 = x_3 \quad b_4 = x_4$

$x_5 = 3,46410162$

Bon, nous voyons que la quatrième décimale n'a pas changé et donc, nous pouvons terminer les itérations!

$\sqrt{12} \simeq 3,4641.$