SOLUTIONS

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Devoir #8: Mat 2784 B

Numéros: 6.13 ; 6.29 ; 6,34 ; 6.42 ; 6.48 ; 6,52 ; 11,14 ; 11,16

#6.13:
$$f(t) = u(t-1)\cosh(t)$$
$$= u(t-1)\cosh((t-1)+1)$$
$$= u(t-1)\left[\cosh(t-1)\cosh(1) + \sinh(t-1)\sinh(1)\right]$$
$$= \left[\cosh(1)\, u(t-1)\cosh(t-1)\right] + \left[\sinh(1)\, u(t-1)\sinh(t-1)\right]$$
$$F(s) = \cosh(1)\, e^{-s}\frac{s}{s^2-1} + \sinh(1)\, e^{-s}\cdot\frac{1}{s^2-1}$$

#6.29:
$$F(s) = \frac{e^{-3s}}{s^2(s-1)} = e^{-3s}\left(\frac{1}{s^2(s-1)}\right) \Rightarrow e^{-3s}\left(\frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s-1)}\right)$$

$$1 = As(s-1) + B(s-1) + Cs^2$$
$$1 = As^2 - As + Bs - B + Cs^2$$
$$1 = s^2(A+C) + s(B-A) - B$$

$B = -1$ $\quad B - A = 0$ $\quad A = B = -1$

$A + C = 0$ $\quad A = -C$ $\quad C = 1$

$$F(s) = e^{-3s}\left(\frac{-1}{s} - \frac{1}{s^2} + \frac{1}{(s-1)}\right)$$
$$\mathcal{L}^{-1}(F(s)) = u(t-3)\left(-1 - (t-3) + e^{(t-3)}\right)$$
$$f(t) = u(t-3)\left(2 - t + e^{(t-3)}\right)$$
$$f(t) = \begin{cases} 0 & \text{si } 0 \le t < 3 \\ 2 - t + e^{t-3} & \text{si } t > 3 \end{cases}$$

#6.34: $f(t) = t \sin 3t$

soit $f(t) = \sin 3t$

$$\mathcal{L}\{t\, f(t)\} = -F'(s) \qquad \mathcal{L}(\sin 3t) = \frac{3}{s^2+9}$$

$$\mathcal{L}(t \sin 3t) = -\left(\frac{3}{s^2+9}\right)'$$

$$F(s) = -3\left(\frac{-2s}{(s^2+9)^2}\right) = \frac{6s}{(s^2+9)^2}$$

#6.42 $y'' + y = \sin 3t \qquad y(0) = 0 \quad y'(0) = 0$

soit $\mathcal{L}(y)(s) = Y(s)$

$$\mathcal{L}(y'')(s) + \mathcal{L}(y)(s) = \mathcal{L}(\sin 3t) \qquad \mathcal{L}(\sin 3t) = \frac{3}{s^2+9}$$

$$\mathcal{L}(y'')(s) = s^2 \mathcal{L}(y) - s\,y(0) - y'(0)$$
$$= s^2 Y(s) - s\,y(0) - y'(0)$$

$$s^2 Y(s) - s\,\cancel{y(0)}^{0} - \cancel{y'(0)}^{0} + Y(s) = \frac{3}{s^2+9}$$

$$s^2 Y(s) + Y(s) = \frac{3}{s^2+9}$$

$$Y(s)\,(s^2+1) = \frac{3}{s^2+9}$$

$$Y(s) = \frac{3}{(s^2+1)(s^2+9)}$$

$$\frac{3}{(s^2+1)(s^2+9)} = \frac{As+B}{(s^2+1)} + \frac{Cs+D}{(s^2+9)}$$

$$3 = (As+B)(s^2+9) + (Cs+D)(s^2+1)$$

$$3 = As^3 + 9As + Bs^2 + 9B + Cs^3 + Cs + Ds^2 + D$$

$$3 = s^3(A+C) + s^2(B+D) + s(9A+C) + (9B+D)$$

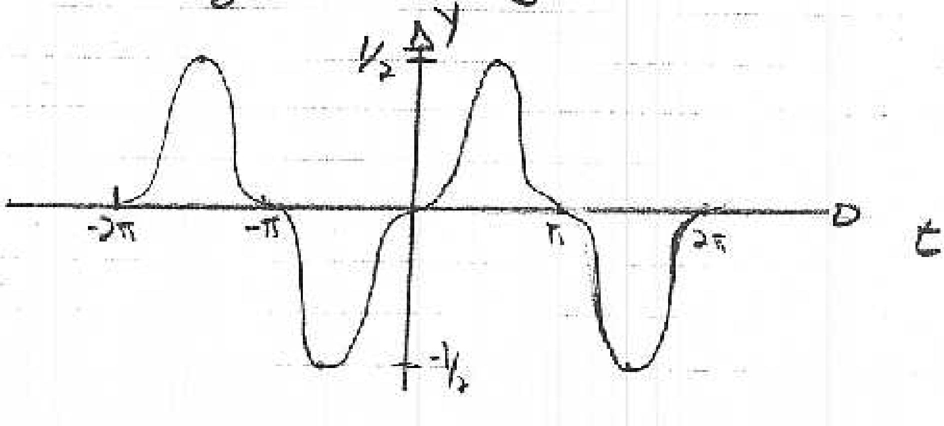
$$A + C = 0 \rightarrow A = -C$$
$$B + D = 0 \rightarrow B = -D$$

$$9A + C = 0 \qquad 9B + D = 3$$
$$-9C + C = 0 \qquad -9D + D = 3$$
$$C = 0 \qquad -8D = 3$$
$$A = 0 \qquad D = -3/8$$
$$B = 3/8$$

$$\frac{3}{(s^2+1)(s^2+9)} = \frac{3/8}{(s^2+1)} - \frac{3/8}{(s^2+9)}$$

$$\mathcal{L}\left(\frac{3}{(s^2+1)(s^2+9)}\right) = \mathcal{L}\left(\frac{3/8}{s^2+1}\right) - \mathcal{L}\left(\frac{3/8}{s^2+9}\right)$$
$$= \frac{3}{8}\mathcal{L}\left(\frac{1}{s^2+1}\right) - \frac{3}{8}\mathcal{L}\left(\frac{1}{s^2+9}\right)$$
$$= \frac{3}{8}\sin t - \frac{3}{8}\left(\frac{1}{3}\sin 3t\right)$$

$$y(t) = \frac{3}{8}\sin t - \frac{1}{8}\sin 3t$$



#6.48 $y'' + 4y = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & t \geq 1 \end{cases}$ $\quad y(0)=0 \quad y'(0)=-1$

$\mathcal{L}(y) = Y(s) \qquad \mathcal{L}(g) = G(s)$

$\mathcal{L}(y''+4y) = s^2 Y(s) - s y(0) - y'(0) + 4Y(s)$

$= (s^2+4)Y_s + 1 = G(s)$

$Y(s) = \dfrac{G(s) - 1}{s^2+4}$

$g(t) = 1 - u(t-1)1 + u(t-1)0$

$= 1 - u(t-1)1$

$G(s) = \dfrac{1}{s} - \dfrac{e^{-s}}{s}$

$Y(s) = \dfrac{\left(\frac{1}{s} - \frac{e^{-s}}{s}\right) - 1}{s^2+4} = \dfrac{1}{s(s^2+4)} - \dfrac{e^{-s}}{s(s^2+4)} - \dfrac{1}{s^2+4}$

$\dfrac{1}{s(s^2+4)} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+4} = \dfrac{As^2+4A+Bs^2+Cs}{s(s^2+4)}$

$\underline{C=0} \qquad 4A=1 \quad \underline{A=1/4} \qquad A+B=0$

$\underline{B=-1/4}$

$Y(s) = \dfrac{1}{4s} - \dfrac{s}{4(s^2+4)} - e^{-s}\left[\dfrac{1}{4s} - \dfrac{s}{4(s^2+4)}\right] - \dfrac{1}{s^2+4}$

$Y(t) = \dfrac{1}{4} - \dfrac{1}{4}\cos 2t - u(t-1)\left[\dfrac{1}{4} - \dfrac{1}{4}\cos(2(t-1))\right] - \dfrac{1}{2}\sin 2t$

$Y(t) = \begin{cases} 1/4 - 1/4\cos 2t - 1/2 \sin 2t & \text{si } 0 \leq t < 1 \\ -1/4\cos 2t + \frac{1}{4}\cos(2t-2) - 1/2\sin 2t & \text{si } t > 1 \end{cases}$ Voir graphique à la fin

D 8.5

#6.52 $y'' + 3y' + 2y = 1 - u(t-1) \quad y(0)=0 \quad y'(0)=1$

$$\left[s^2 Y(s) - s\cancel{y(0)}^{0} - y'(0)\right] + 3\left[s\,Y(s) - \cancel{y(0)}^{0}\right] + 2\left[Y(s)\right] = \frac{1}{s} - \frac{e^{-s}}{s}$$

$$Y(s)\left[s^2 + 3s + 2\right] - 1 = \frac{1}{s} - \frac{e^{-s}}{s}$$

$$Y(s) = \left(\frac{1}{s} - \frac{e^{-s}}{s} + 1\right)\left(\frac{1}{(s+2)(s+1)}\right)$$

$$Y(s) = \frac{1}{s(s+2)(s+1)} - \frac{e^{-s}}{s(s+2)(s+1)} + \frac{1}{(s+2)(s+1)}$$

$$\frac{1}{s(s+2)(s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$
$$= A(s+2)(s+1) + Bs(s+2) + Cs(s+1)$$
$$= (A+B+C)s^2 + (3A+2B+C)s + 2A = 1$$

$\underline{A = 1/2}$ $\quad A + B + C = 0 \quad\quad 3A + 2B + C = 0$

$B = -C - 1/2 \quad\quad 3/2 + 2(-C-1) + C = 0$

$\underline{B = -1} \quad\quad \underline{C = 1/2}$

$$\frac{1}{(s+2)(s+1)} = \frac{A}{s+1} + \frac{B}{s+2} = \frac{As + 2A + Bs + B}{(s+1)(s+2)}$$

$A + B = 0 \quad\quad 2A + B = 1$

$A = -B \quad\quad -2B + B = 1$

$\underline{A = 1} \quad\quad \underline{B = -1}$

$$Y(s) = \left[\frac{1/2}{s} - \frac{1}{s+1} + \frac{1/2}{s+2}\right] - e^{-s}\left[\frac{1/2}{s} - \frac{1}{s+1} + \frac{1/2}{s+2}\right] + \frac{1}{s+1} - \frac{1}{s+2}$$

$$y(t) = \left[\frac{1}{2} - \cancel{e^{-t}} + \frac{1}{2}e^{-2t}\right] - u(t-1)\left[\frac{1}{2} - e^{-(t-1)} + \frac{1}{2}e^{-2(t-1)}\right] + e^{-t} - e^{-2t}$$

$$y(t) = \frac{1}{2} - \frac{e^{-2t}}{2} - u(t-1)\left[\frac{1}{2} - e^{-(t-1)} + \frac{1}{2}e^{-2(t-1)}\right]$$

$$y(t) = \begin{cases} \frac{1}{2} - e^{-2t}/2 & \text{si } 0 \leq t < 1 \\ -\frac{e^{-2t}}{2} - e^{1-t} + \frac{1}{2}e^{-2(t-1)} & \text{si } t > 1 \end{cases}$$

voir graphique à la fin

~~#11.14~~: $(-1, 2)$ $(0, 0)$ $(1, 1)$ $(2, 2)$

i	1	2	3	4
x_i	-1	0	1	2
y_i	2	0	1	2

$f(x) = a_0 + a_1 x + a_2 x$

équations normales

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 \\ 1 & 0 & 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 \\ 1 & 0 & 1 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 2 & 6 \\ 2 & 6 & 8 \\ 6 & 8 & 18 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 11 \end{bmatrix} \quad \text{où } Na = b$$

Cholesky $G \cdot G^T = N$

$$\begin{bmatrix} g_{11} & 0 & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & g_{33} \end{bmatrix}\begin{bmatrix} g_{11} & g_{21} & g_{31} \\ 0 & g_{22} & g_{32} \\ 0 & 0 & g_{33} \end{bmatrix} = \begin{bmatrix} 4 & 2 & 6 \\ 2 & 6 & 8 \\ 6 & 8 & 18 \end{bmatrix}$$

$g_{11}^2 = 4$ $\quad g_{11} = 2 \ \underline{\geq 0}$

$g_{11}g_{21} = 2$ $\quad g_{21} = 1$

$g_{11}g_{31} = 6$ $\quad g_{31} = 3$

$g_{21}^2 + g_{22}^2 = 6$ $\quad g_{22} = \sqrt{5} \ \underline{\geq 0}$

$g_{21}g_{31} + g_{22}g_{32} = 8$ $\quad g_{32} = \sqrt{5}$

$g_{31}^2 + g_{32}^2 + g_{33}^2 = 18$

$\hookrightarrow g_{33} = 2 \ \underline{\geq 0}$

#11.14 (suite) :

$$Gw = b$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & \sqrt{5} & 0 \\ 3 & \sqrt{5} & 2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 11 \end{bmatrix}$$

$$\underline{w_1 = 5/2}$$

$$w_1 + \sqrt{5}w_2 = 3 \qquad \underline{w_2 = 1/2\sqrt{5}}$$

$$3w_1 + \sqrt{5}w_2 + 2w_3 = 11 \qquad \underline{w_3 = 3/2}$$

$$G^T a = w$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & \sqrt{5} & \sqrt{5} \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 5/2 \\ 1/2\sqrt{5} \\ 3/2 \end{bmatrix}$$

$$2a_2 = 3/2 \qquad a_2 = 3/4 = 0,75$$

$$\sqrt{5}a_1 + \sqrt{5}a_2 = \frac{1}{2\sqrt{5}} \qquad a_1 = -0,65$$

$$2a_0 + a_1 + 3a_2 = 5/2 \qquad a_0 = 0,45$$

$$\Rightarrow f(x) = 0,45 - 0,65x + 0,75x^2$$

08.8

#11.16:

x_i	-1	-0.5	0	0.25	0.5	0.75	1
y_i	e^{-1}	$e^{-1/2}$	1	$e^{1/4}$	$e^{1/2}$	$e^{3/4}$	e

$$\begin{array}{c} 1 \\ x \\ \frac{1}{2}(3x^2-1) \end{array}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -0.5 & 0 & 0.25 & 0.5 & 0.75 & 1 \\ 1 & -1/8 & -0.5 & -13/32 & -1/8 & 11/32 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -0.5 & -1/8 \\ 1 & 0 & -0.5 \\ 1 & 0.25 & -13/32 \\ 1 & 0.5 & -1/8 \\ 1 & 0.75 & 11/32 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} =$$

$$\begin{bmatrix} 7 & 1 & 19/16 \\ 1 & 25/8 & 5/32 \\ 19/16 & 5/32 & 1313/512 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -0.5 & 0 & 0.25 & 0.5 & 0.75 & 1 \\ 1 & -1/8 & -0.5 & -13/32 & -1/8 & 11/32 & 1 \end{bmatrix}\begin{bmatrix} e^{-1} \\ e^{-1/2} \\ 1 \\ e^{1/4} \\ e^{1/2} \\ e^{3/4} \\ e \end{bmatrix}$$

$$\begin{bmatrix} 7 & 1 & 19/16 \\ 1 & 25/8 & 5/32 \\ 19/16 & 5/32 & 1313/512 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 9.742438633 \\ 4.780254059 \\ 2.510338209 \end{bmatrix} \qquad Na = b$$

$$GG^T = N$$

$$\begin{bmatrix} g_{11} & 0 & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & g_{33} \end{bmatrix}\begin{bmatrix} g_{11} & g_{21} & g_{31} \\ 0 & g_{22} & g_{32} \\ 0 & 0 & g_{33} \end{bmatrix} = \begin{bmatrix} 7 & 1 & 19/16 \\ 1 & 25/8 & 5/32 \\ 19/16 & 5/32 & 1313/512 \end{bmatrix}$$

$g_{11}^2 = 7$ $\qquad g_{11} = \sqrt{7}$ $\quad \geq 0$

$g_{11}g_{21} = 1$ $\qquad g_{21} = 1/\sqrt{7}$

$g_{11}g_{31} = 19/16$ $\qquad g_{31} = 19/16\sqrt{7}$

$g_{21}^2 + g_{22}^2 = 25/8$ $\qquad g_{22} = \sqrt{167/56}$ $\quad \geq 0$

$g_{21}g_{31} + g_{22}g_{32} = 5/32$ $\qquad g_{32} = -0.007754859$

$g_{31}^2 + g_{32}^2 + g_{33}^2 = 1313/512$ $\qquad g_{33} = 1.537186418$

$$\begin{bmatrix} \sqrt{7} & 0 & 0 \\ 1/\sqrt{7} & \sqrt{167/56} & 0 \\ 19/16\sqrt{7} & -0{,}0077554859 & 1{,}537186418 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9{,}742438633 \\ 4{,}780254059 \\ 2{,}510338209 \end{bmatrix}$$

$\sqrt{7}\ x_1 = 9{,}742438633$ $\qquad$ $\underline{x_1 = 3{,}682295684}$

$1/\sqrt{7}\ x_1 + \sqrt{167/56}\ x_2 = 4{,}780254059$ $\qquad$ $\underline{x_2 = 1{,}96218673}$

$19/16\sqrt{7}\ x_1 - 0{,}0077554859 \cdot x_2 + 1{,}537186418\, x_3 = 2{,}510338209$

$\underline{x_3 = 0{,}567804129}$

$$\begin{bmatrix} \sqrt{7} & 1/\sqrt{7} & 19/16\sqrt{7} \\ 0 & \sqrt{169/56} & -0{,}0077554859 \\ 0 & 0 & 1{,}537186418 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 3{,}682295684 \\ 1{,}96218673 \\ 0{,}567804129 \end{bmatrix}$$

$1{,}537186418\, a_2 = 0{,}567804129$ $\qquad$ $a_2 = 0{,}3693788355$

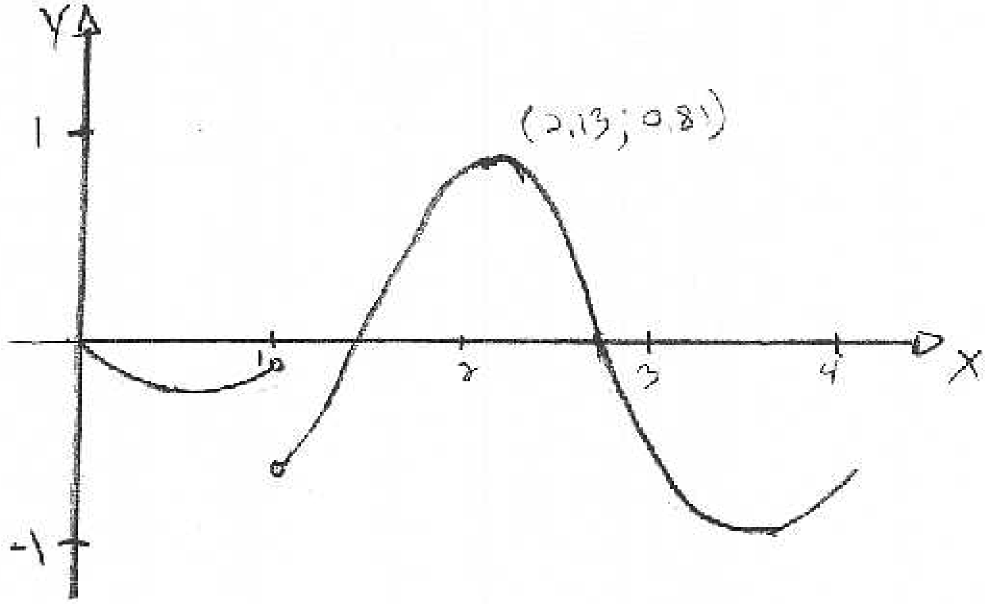
$a_1 = 1{,}137914685$

$a_0 = 1{,}166555226$

$f(x) = 1{,}166555226 + 1{,}137914685\, x + 0{,}3693788355\, \frac{(3x^2-1)}{2}$

D 8.10

#6.48



#6.52