SOLUTIONS

D7.1

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Devoir #7: Mat 2784

#5.29 $f(x) = \begin{cases} 0 & -1 < x < 0 \\ 1 & 0 < x < 1 \end{cases}$

$$f(x) = \sum_{n=0}^{\infty} a_m P_m(x) \quad ; \quad -1 < x < 1$$

$P_0(x) = 1$
$P_1(x) = x$
$P_2(x) = \frac{1}{2}(3x^2$

alors $a_m = \frac{2m+1}{2} \int_{-1}^{1} f(x) P_m(x)\, dx$

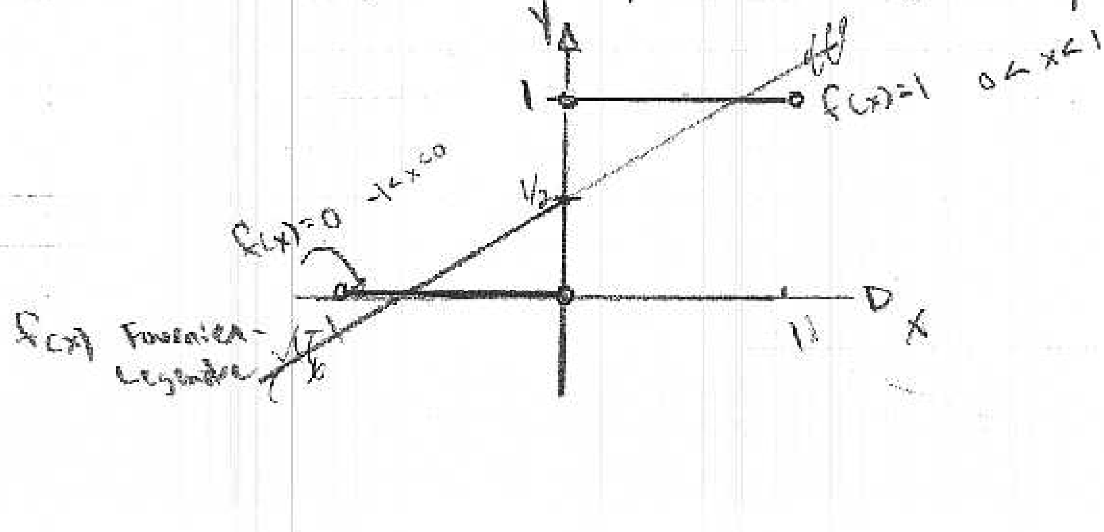
$$a_0 = \frac{2(0)+1}{2} \int_{-1}^{1} f(x) P_m(x) dx = \frac{1}{2}\left[\int_{-1}^{0} 0\, dx + \int_0^1 dx\right] = \frac{1}{2} x \Big|_0^1 = 1/2$$

$$a_1 = \frac{2(1)+1}{2} \int_{-1}^{1} f(x) P_m(x) dx = \frac{3}{2}\left[\int_{-1}^{0} 0\, x\, dx + \int_0^1 x\, dx\right] = \frac{3}{2} \frac{x^2}{2}\Big|_0^1 = 3/4$$

$$a_2 = \frac{2(2)+1}{2} \int_{-1}^{1} f(x) P_m(x)\, dx = \frac{5}{2}\left[\int_{-1}^{0} 0 \cdot \frac{1}{2}(3x^2-1)\, dx + \int_0^1 \frac{1}{2}(3x^2-1) dx\right] = \frac{5}{2}\left[\frac{x^3}{2} - \frac{x}{2}\right]_0^1 = 0$$

$$f(x) \approx \frac{1}{2} P_0(x) + \frac{3}{4} P_1(x) + 0 P_2(x)$$

$$f(x) \approx \frac{1}{2} P_0(x) + \frac{3}{4} P_1(x) \approx \frac{1}{2} + \frac{3x}{4}$$



#5.32 $I = \int_{0,3}^{1,7} e^{-x^2} dx$ quadrature gausienne à 3pts

$$\int_a^b f(x)\,dx = \frac{b-a}{2}\int_{-1}^{1} f\left(\frac{b-a}{2}t + \frac{b+a}{2}\right)dt \qquad \begin{array}{l} a = 0,3 \\ b = 1,7 \end{array}$$

$$= \frac{1,7-0,3}{2} \cdot \int_{-1}^{1} e^{-\left[\frac{(1,7-0,3)t + 1,7+0,3}{2}\right]^2} dt$$

$$= \frac{1,4}{2}\int_{-1}^{1} e^{-[0,7t+1]^2} dt \qquad f(t) = e^{-(0,7t+1)^2}$$

$$= 0,7\left[\frac{5}{9} f(-\sqrt{3/5}) + \frac{8}{9} f(0) + \frac{5}{9} f(\sqrt{3/5})\right]$$

$$= 0,7\left(\frac{5}{9} e^{-(0,7\cdot -\sqrt{3/5} + 1)^2} + \frac{8}{9} e^{-(0,7\cdot 0+1)^2} + \frac{5}{9} e^{-(0,7\cdot\frac{\sqrt{3}}{5} + 1)^2}\right)$$

$$= 0,7\left(0,810945 \cdot \frac{5}{9} + \frac{8}{9} e^{-1} + \frac{5}{9} \cdot 0,092696\right)$$

$$= 0,7(0,829022)$$

$$= 0,580316$$

$$\underline{I = 0,580316}$$

#6.2 $f(t) = t^2 + at + b$

$$F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

$$F(t^2 + at + b) = \int_0^\infty e^{-st} t^2 dt + \int_0^\infty e^{-st} at\, dt + \int_0^\infty e^{-st}\, b\, dt$$

$$= \mathcal{L}(t^2)(s) + a\mathcal{L}(t)(s) + b\mathcal{L}(1)(s)$$

$$= \frac{2!}{s^3} + \frac{a}{s^2} + \frac{b}{s}$$

#6.8 $f(t) = 2e^{-2t} \sin t$

$$\mathcal{L}(\sin t)(s) = \frac{1}{s^2+1}$$

$$F(s) = \frac{2}{(s+2)^2+1}$$

#6.17 $F(s) = \dfrac{2}{s^2+3}$

$$= \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{s^2 + \sqrt{3}^2}$$

$$f(t) = \frac{2}{\sqrt{3}} \sin(\sqrt{3}\, t)$$

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#6.21 $F(s) = \dfrac{1}{s^2+s-20}$

$= \dfrac{1}{(s+5)(s-4)}$ $\quad a=-5 \quad b=4$

$f(t) = \dfrac{1}{-5-4}\left(e^{-5t} - e^{4t}\right)$

$\therefore f(t) = \dfrac{-1}{9}\left(e^{-5t} - e^{4t}\right)$

#11.9:

$$\begin{bmatrix} 16 & -4 & 4 \\ -4 & 10 & -1 \\ 4 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 3 \\ 1 \end{bmatrix} \qquad Ax = b$$

$$\begin{bmatrix} g_{11} & 0 & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \begin{bmatrix} g_{11} & g_{21} & g_{31} \\ 0 & g_{22} & g_{32} \\ 0 & 0 & g_{33} \end{bmatrix} = \begin{bmatrix} 16 & -4 & 4 \\ -4 & 10 & -1 \\ 4 & -1 & 5 \end{bmatrix}$$

$$\begin{array}{ll} g_{11}^2 = 16 & g_{11} = 4 \quad > 0 \\ g_{11} \cdot g_{21} = -4 & g_{21} = -1 \\ g_{11} g_{31} = 4 & g_{31} = 1 \\ g_{21}^2 + g_{22}^2 = 10 & g_{22} = 3 \quad > 0 \\ g_{21} \cdot g_{31} + g_{22} \cdot g_{32} = -1 & g_{32} = 0 \\ g_{31}^2 + g_{32}^2 + g_{33}^2 = 5 & g_{33} = 2 \quad > 0 \end{array}$$

$$\therefore \quad G = \begin{bmatrix} 4 & 0 & 0 \\ -1 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\det A = \det G \cdot \det G^T = \det(G)^2 = (4 \cdot 3 \cdot 2)^2 > 0$$

$$Gy = b \qquad \begin{bmatrix} 4 & 0 & 0 \\ -1 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{array}{ll} 4y_1 = -12 & y_1 = -3 \\ -y_1 + 3y_2 = 3 & y_2 = 0 \\ y_1 + 2y_3 = 1 & y_3 = 2 \end{array} \qquad y = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$$

$$G^T x = y \qquad \begin{bmatrix} 4 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{array}{ll} 2x_3 = 2 & x_3 = 1 \\ 3x_2 = 0 & x_2 = 0 \\ 4x_1 - x_2 + x_3 = -3 & x_1 = -1 \end{array} \qquad x = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

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#11.11

$$\begin{aligned} 6x_1 + x_2 - x_3 &= 3 & x_1^{(0)} &= 1 \\ -x_1 + x_2 + 7x_3 &= -17 & x_2^{(0)} &= 1 \\ x_1 + 5x_2 + x_3 &= 0 & x_3^{(0)} &= 1 \end{aligned}$$

$$\begin{aligned} 6x_1 + x_2 - x_3 &= 3 \\ x_1 + 5x_2 + x_3 &= 0 \\ -x_1 + x_2 + 7x_3 &= -17 \end{aligned}$$

$$\therefore \left.\begin{aligned} x_1^{(n+1)} &= 1/6 \,(3 - x_2^{n} + x_3^{n}) \\ x_2^{(n+1)} &= 1/5 \,(0 - x_1^{(n+1)} - x_3^{n}) \\ x_3^{(n+1)} &= 1/7 \,(-17 + x_1^{(n+1)} - x_2^{n+1}) \end{aligned}\right\} \quad \begin{aligned} x_1^{(0)} &= 1 \\ x_2^{(0)} &= 1 \\ x_3^{(0)} &= 1 \end{aligned}$$

avec $n=0$ on a :

$$x_1^{(1)} = \frac{1}{6}(3 - 1 + 1) = 3/6 = 1/2$$

$$x_2^{(1)} = 1/5\,(-1/2 - 1) = \frac{-3}{10}$$

$$x_3^{(1)} = 1/7\,(-17 + 1/2 + 3/10) = \frac{1}{7}\left(\frac{-81}{5}\right) = -81/35$$

avec $n=1$ on a :

$$\begin{aligned} x_1^2 &= 1/6\,(3 + 3/10 - 81/35) = 23/140 \\ x_2^2 &= 1/5\,(0 - 23/140 + 81/35) = 43/100 \\ x_3^2 &= 1/7\,(-17 + 23/140 - 43/100) = -6043/2450 \end{aligned}$$

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avec $n=2$

$$\begin{aligned} x_1^3 &= 1/6\,(3 - 43/100 - 6043/2450) = \cancel{0{,}0165646}\ 0.0172449 \\ x_2^3 &= 1/5\,(0 - 0{,}0165646 + 6043/2450) = 0{,}489993 \\ x_3^3 &= 1/7\,(-17 + 0{,}0165646 - 0{,}489993) = -2{,}496204 \end{aligned}$$

Alors $x^n \underset{\text{si } n \to \infty}{\longrightarrow} \begin{bmatrix} 0 \\ 0{,}5 \\ -2{,}5 \end{bmatrix}$