D5.1

SOLUTIONS

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MAT 2784 8

Devoir 5

1 → $\boxed{\#3.34}$

$$y'' - 2y' + y = \frac{e^x}{x}$$

$$Ly = y'' - 2y' + y = 0$$
$$\Rightarrow \lambda^2 - 2\lambda + 1 = 0$$
$$\Rightarrow \lambda_{1,2} = 1$$

$y_1(x) = e^x \qquad y_2(x) = u(x)\, y_1(x)$
$= x e^x$

$$\boxed{y_h(x) = Ae^x + Bxe^x}$$

$$y_p(x) = c_1(x)\, e^x + c_2(x)\, x e^x$$

$$\begin{vmatrix} e^x & xe^x \\ e^x & e^x + xe^x \end{vmatrix} \begin{vmatrix} c_1' \\ c_2' \end{vmatrix} = \begin{vmatrix} 0 \\ e^x/x \end{vmatrix} \quad L_2 - L_1$$

$$\Rightarrow \begin{vmatrix} e^x & xe^x \\ 0 & e^x \end{vmatrix} \begin{vmatrix} c_1' \\ c_2' \end{vmatrix} = \begin{vmatrix} 0 \\ e^x/x \end{vmatrix}$$

$$e^x c_2' = \frac{e^x}{x} \Rightarrow c_2' = \frac{1}{x} \Rightarrow \boxed{c_2 = \ln x}$$

$$e^x c_1' + xe^x c_2' = 0 \Rightarrow c_1' = -x\, c_2' \qquad \text{où } c_2' = \frac{1}{x}$$
$$= -x \cdot \frac{1}{x}$$
$$\Rightarrow c_1' = -1.$$
$$\Rightarrow \boxed{c_1 = -x}$$

$y_p(x) = -xe^x + xe^x \ln x$

$\Rightarrow \boxed{y_p(x) = (\ln x - 1)\, x e^x}$

Solution générale

$$\boxed{y_G(x) = Ae^x + Bxe^x + (\ln x - 1)\, xe^x}$$

2 $\rightarrow$ $\boxed{3.36}$

$y'' + y = \tan x \qquad y(0) = 1 \;,\quad y'(0) = 0$

$Ly = y'' + y = 0$

$\Rightarrow \lambda^2 + 1 = 0 \quad \Rightarrow \lambda_{1,2} = \pm i$

$y_1(x) = \cos x \quad , \; y_2(x) = \sin x$

$y_H(x) = A\cos x + B\sin x$

$y_p(x) = C_1(x)\cos x + C_2(x)\sin x$

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \begin{bmatrix} C_1' \\ C_2' \end{bmatrix} = \begin{bmatrix} 0 \\ \tan x \end{bmatrix}$$

$C_1' \cos x + C_2' \sin x = 0 \Rightarrow C_1' = -\dfrac{\sin x}{\cos x} C_2' \quad ①$

$-\sin x\, C_1' + \cos x\, C_2' = \tan x$

$\Leftrightarrow \dfrac{\sin^2 x}{\cos x} C_2' + \cos x\, C_2' = \tan x \Leftrightarrow C_2'\left(\dfrac{1}{\cos x}\right) = \tan x$

$\Rightarrow C_2' = \sin x \Rightarrow \boxed{C_2 = -\cos x}$

$\Rightarrow ① \quad C_1' = -\dfrac{\sin^2 x}{\cos x} = \cos x - \dfrac{1}{\cos x}$

$\Rightarrow \boxed{C_1 = \sin x - \ln|\sec x + \tan x|}$

$$y_p(x) = (\sin x - \ln|\sec x + \tan x|)\cos x - \cos x \sin x$$
$$= -\cos x \ln|\sec x + \tan x|$$

sol. générale

$$\boxed{y(x) = A\cos x + B\sin x + \cos x \ln|\sec x + \tan x|}$$

$$y'(x) = -A\sin x + B\cos x + \sin x \ln|\sec x + \tan x| - \frac{\cos x}{\sec x + \tan x}\left(2 + \frac{\sin x - 1}{\cos^2 x}\right)$$

$y(0) = 1 \Rightarrow A = 1$

$y'(0) = 0 \Rightarrow B = 1$

solution unique

$$\boxed{y(x) = \cos x + \sin x - \cos x \ln|\sec x + \tan x|}$$

3 $\longrightarrow$ $\boxed{3.38}$

$$2x^2 y'' + xy' - 3y = x^{-2} \qquad y(1) = 0 \qquad y'(1) = 2$$

$y = x^m$

eq. caract. $2m^2 - m - 3 = 0 \Rightarrow m_1 = \frac{3}{2}$, $m_2 = -1$

$Ly = 0 \qquad y_h(x) = C_1 x^{3/2} + C_2 x^{-1}$

$$y_p(x) = C_1(x)\, x^{3/2} + C_2(x)\, x^{-1}$$

$$\begin{bmatrix} x^{3/2} & x^{-1} \\ \frac{3}{2}x^{1/2} & -x^{-2} \end{bmatrix}\begin{bmatrix} C_1' \\ C_2' \end{bmatrix} = \begin{bmatrix} 0 \\ 1/2x^4 \end{bmatrix} \quad l_2 - \frac{3}{2}x^{-1}\, l_1$$

$$\Rightarrow \begin{bmatrix} x^{3/2} & x^{-1} \\ 0 & -\frac{5}{2}x^{-2} \end{bmatrix}\begin{bmatrix} C_1' \\ C_2' \end{bmatrix} = \begin{bmatrix} 0 \\ 1/2x^4 \end{bmatrix}$$

$$-\frac{5}{2}x^{-2}\, C_2' = \frac{1}{2x^4} \Rightarrow C_2' = -\frac{1}{2x^4} \cdot \frac{2}{5}x^2$$

$$\Rightarrow C_2' = -\frac{1}{5x^2}$$

$$x^{3/2} C_1' + x^{-1} C_2' = 0 \Rightarrow C_1' = -x^{-1} x^{-3/2} C_2' = \frac{1}{5} x^{-9/2}$$

$$C_1 = \frac{1}{5}\int x^{-9/2}\,dx = \frac{1}{5}\left(\frac{-2}{7}\right)x^{-7/2} = \frac{-2}{35}x^{-7/2}$$

$$C_2 = -\frac{1}{5}\int x^{-2}\,dx = \frac{1}{5}x^{-1}$$

Sol. part.

$$y_p(x) = x^{3/2}\left(\frac{-2}{35}\right)x^{-7/2} + x^{-1}\left(\frac{1}{5}\right)x^{-1}$$

$$y_p(x) = \frac{-2}{35}x^{-2} + \frac{1}{5}x^{-2}$$

$$\Rightarrow \boxed{y_p(x) = \frac{1}{7}x^{-2}}$$

Sol. gen

$$y(x) = A x^{3/2} + B x^{-1} + \frac{1}{7}x^{-2}$$

$$y'(x) = \frac{3}{2}A x^{1/2} - B x^{-2} - \frac{2}{7}x^{-3}$$

$$y(1) = A + B + \frac{1}{7} = 0$$
$$y'(1) = \frac{3}{2}A - B - \frac{2}{7} = 2 \qquad \Rightarrow A = \frac{6}{7} \qquad B = -1$$

Sol. unique

$$\boxed{y(x) = \frac{6}{7}x^{3/2} - x^{-1} + \frac{1}{7}x^{-2}}$$

D5.5

4 $\longrightarrow$ $\boxed{4.4}$

$$y' = Ay \qquad A = \begin{bmatrix} -1 & 1 & 4 \\ -2 & 2 & 4 \\ -1 & 0 & 4 \end{bmatrix}$$

$$|A - \lambda I| = \begin{bmatrix} -1-\lambda & 1 & 4 \\ -2 & 2-\lambda & 4 \\ -1 & 0 & 4-\lambda \end{bmatrix} = (-1-\lambda)\begin{bmatrix} 2-\lambda & 4 \\ 0 & 4-\lambda \end{bmatrix} - 1 \begin{bmatrix} -2 & 4 \\ -1 & 4-\lambda \end{bmatrix}$$
$$+ 4 \begin{bmatrix} -2 & 2-\lambda \\ -1 & 0 \end{bmatrix}$$

$$= (-1-\lambda)(2-\lambda)(4-\lambda) - (-8+2\lambda+4) + 4(2-\lambda)$$
$$= (-1-\lambda)(8-2\lambda-4\lambda+\lambda^2) + 8 - 2\lambda - 4 + 8 - 4\lambda$$
$$= -8 + 6\lambda - \lambda^2 - 8\lambda + 6\lambda^2 - \lambda^3 + 16 - 6\lambda - 4$$
$$= -\lambda^3 + 5\lambda^2 - 8\lambda + 4$$
$$= (\lambda - 1)(-\lambda^2 + 4\lambda - 4)$$

$\lambda_1 = 1$
$\lambda_{2,3} = 2$

<u>Premier vecteur propre</u> $(\lambda_1 = 1)$

$[A - I]\vec{u} = 0$ où $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} -2 & 1 & 4 \\ -2 & 1 & 4 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$|A - I| = \begin{bmatrix} -2 & 1 & 4 \\ -2 & 1 & 4 \\ -1 & 0 & 3 \end{bmatrix} \begin{matrix} \ell_1 - \ell_3 \\ \\ \\ \end{matrix} \sim \begin{bmatrix} -1 & 1 & 1 \\ -2 & 1 & 4 \\ -1 & 0 & 3 \end{bmatrix} \begin{matrix} \\ \ell_2 - 2\ell_1 \\ \ell_3 - \ell_1 \end{matrix} \sim \begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & -1 & 2 \end{bmatrix}_{\ell_3 - \ell_2} \sim \begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$[A - I]\vec{u} = 0 \Rightarrow$ Si $u_3 = 1 \Rightarrow u_2 = 2$
et $u_1 = 3$

alors $\vec{u} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

<u>Deuxième vecteur propre</u> $(\lambda_2 = 2)$

$[A - 2I]\vec{v} = 0$ où $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} -3 & 1 & 4 \\ -2 & 0 & 4 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -3v_1 + v_2 + 4v_3 = 0 & (1) \\ -2v_1 + 4v_3 = 0 & (2) \\ -v_1 + 2v_3 = 0 & (3) \end{cases} \Rightarrow v_1 = 2v_3$$

Posons $v_3 = 1$

$\Rightarrow v_1 = 2$ et $v_2 = 2$

alors $\vec{v} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$

Troisième vecteur propre.

$[A - 2I]\vec{w} = \vec{v}$

$$\begin{bmatrix} -3 & 1 & 4 \\ -2 & 0 & 4 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{cases} -3w_1 + w_2 + 4w_3 = 2 \\ -2w_1 + 4w_3 = 2 \\ -w_1 + 2w_3 = 1 \end{cases} \Rightarrow w_1 = 2w_3 - 1$$

Posons $w_3 = 1$

$\Rightarrow w_1 = 1$ et $w_2 = 1$

alors $\vec{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$y_1(x) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} e^{x}$ $\qquad y_2(x) = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} e^{2x}$

$$y_3(x) = (x\vec{v} + \vec{w})e^{2x} = \left(x\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right) e^{2x}$$

alors

$$\boxed{y(x) = c_1 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} e^{x} + c_2 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} e^{2x} + c_3 \left(x\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right) e^{2x}}$$ sol gen.

5

$\boxed{4.6}$

$$y' = Ay + f(x)$$

$$A = \begin{bmatrix} -3 & -2 \\ -1 & 0 \end{bmatrix} \quad , \quad f(x) = \begin{bmatrix} 2e^{-x} \\ -e^{-x} \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -3-\lambda & -2 \\ -1 & -\lambda \end{vmatrix} = 3\lambda + \lambda^2 + 2 = (\lambda + 2)(\lambda + 1)$$

$$\lambda_1 = -2 \quad , \quad \lambda_2 = -1$$

<u>1^{er} vecteur propre</u> $\lambda_1 = -2$

$$[A + 2I]\,\vec{u} = 0 \quad \text{où} \quad \vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} -u_1 - 2u_2 = 0 \\ u_1 + 2u_2 = 0 \end{cases} \quad \text{Posons } u_1 = 2 \Rightarrow u_2 = -1$$

$$\text{alors} \quad \vec{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

<u>2^{eme} vecteur propre</u> $\lambda_2 = -1$

$$[A + I]\,\vec{v} = 0 \quad \text{où} \quad \vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} -2v_1 - 2v_2 = 0 \\ v_1 + v_2 = 0 \end{cases} \quad \text{Posons } v_1 = 1 \Rightarrow v_2 = -1$$

$$\text{alors} \quad \vec{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$\lambda_1 \neq \lambda_2 \therefore$ on a 2 sol. indép : $y_1(x) = \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-2x}$, $y_2(x) = \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-x}$

$$y_h(x) = c_1 e^{-2x} \begin{bmatrix} 2 \\ -1 \end{bmatrix} + c_2 e^{-x} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$f(x) = 2e^{-x} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + e^{-x} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} e^{-x} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} e^{-x}$$

$$y_p(x) = \begin{bmatrix} a \\ b \end{bmatrix} x e^{-x} + \begin{bmatrix} c \\ d \end{bmatrix} e^{-x}$$

$$y_p'(x) = \begin{bmatrix} a \\ b \end{bmatrix} e^{-x} - \begin{bmatrix} a \\ b \end{bmatrix} x e^{-x} - \begin{bmatrix} c \\ d \end{bmatrix} e^{-x}$$

$$= \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} x e^{-x} + \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} e^{x} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-x}$$

$$-\begin{bmatrix} a \\ b \end{bmatrix} x e^{-x} = \begin{bmatrix} -3a - 2b \\ a \end{bmatrix} x e^{-x}$$

ON PREND :

$$\Rightarrow \begin{matrix} -a = -3a - 2b \\ -b = a \end{matrix} \Rightarrow \begin{matrix} 2a = -2b \\ a = -b \end{matrix} \Rightarrow a = b = 0$$

$$\begin{bmatrix} -c \\ -d \end{bmatrix} e^{-x} = \begin{bmatrix} +3 & +2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} e^{x} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{x}$$

$$\begin{bmatrix} -c \\ -d \end{bmatrix} = \begin{bmatrix} +3c + 2d \\ -c \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

(1) $-4c + 2d = 2$

(2) $-c - d = -1$

(1) + 4(2) : $-6d = -2 \Rightarrow d = 1/3$

$c = d - 1 = \frac{1}{3} - \frac{3}{3} \Rightarrow c = -2/3$

Alors $y_p(x) = \begin{bmatrix} -2/3 \\ 1/3 \end{bmatrix} e^{-x}$

et la sol. générale :

$$y(x) = c\, e^{-2x} \begin{bmatrix} 2 \\ -1 \end{bmatrix} + c_2\, e^{-x} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2/3 \\ 1/3 \end{bmatrix} e^{-x}$$

(6) $\longrightarrow$

$\boxed{4.1.1}$

$y' = Ay \quad y(0) = y_0$

$A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \quad , \quad y_0 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\det(A - \lambda I) = \begin{vmatrix} 5-\lambda & -1 \\ 3 & 1-\lambda \end{vmatrix} = (5-\lambda)(1-\lambda) + 3$

$= 5 - 5\lambda - 1\lambda + \lambda^2 + 3$

$= \lambda^2 - 6\lambda + 8$

$\lambda_1 = 2 \quad , \quad \lambda_2 = 4$

Premier vecteur propre $\lambda_1 = 2$

$(A - 2I)\vec{u} = 0 \quad$ où $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \quad 3u_1 - u_2 = 0 \quad$ posons $u_1 = 1 \Rightarrow u_2 = 3$

alors $\vec{u} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

2ème vecteur propre $\lambda_2 = 4$

$(A - 4I)\vec{v} = 0 \quad$ où $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \quad v_1 - v_2 = 0 \quad$ Posons $v_1 = 1 \Rightarrow v_2 = 1$

alors $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$y(x) = C_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{2x} + C_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4x}$

$y(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \Rightarrow C_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\Rightarrow \begin{cases} C_1 + C_2 = 2 \\ 3C_1 + C_2 = -1 \end{cases} \Rightarrow \begin{cases} C_1 = -3/2 \\ C_2 = 7/2 \end{cases}$

D 5.10

Sol. part.

$$\boxed{y(x) = -3/2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{2x} + 7/2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4x}}$$

7 $\longrightarrow$

$\boxed{10.1}$

$$f'(x_0) = \frac{1}{2h}\left[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)\right] + \frac{h^2}{3} f^{(3)}(\xi)$$

x	$\cosh(x) - \sinh(x)$
1.0	0,367879
1.1	0,332871
1.2	0,301194
1.3	0,272531
1.4	0,246596

a) $df_n = f'(1,2) = \dfrac{-3(0,301194) + 4(0,272531) - (0,246596)}{2 \times 0,1}$

$$\simeq -0,30027$$

b) $f(x) = \cosh x - \sinh x = \dfrac{e^x + e^{-x}}{2} - \dfrac{e^x - e^{-x}}{2} = e^{-x}$

$f'(x) = -e^{-x}$

$df_e = f'(1,2) = -e^{-1,2} = -0,30119$

c) L'erreur

$\varepsilon = df_n - df_e = -0,30027 + 0,30119$
$= 0,00092$

d) Vérifier que $|\varepsilon| \leq \max\limits_{1,2 \leq \xi \leq 1,4} \dfrac{h^2}{3} |f^{(3)}(\xi)|$

$f''(x) = e^{-x}$

$f'''(x) = -e^{-x}$

$$\frac{h^2}{3} |f^{(3)}(\xi)| = \frac{0,1^2}{3} \cdot e^{-1,2} = 0,001004$$

$|E| = 0{,}00052 \leq 0{,}001004$

8 $\longrightarrow$ $\boxed{10.10}$

10.10 $\int_0^2 \frac{1}{x+4}\,dx$ à 10^{-5} près avec la méthode des pts milieux

h et n = ?

$|E| < \frac{(b-a)h^2}{24} f''(\xi)$

$f' = -(x+4)^{-2} \cdot 1 = -(x+4)^{-2}$
$f'' = 2(x+4)^{-3} \cdot 1 = 2(x+4)^{-3}$
$\max(f'') = f''(0) = 0{,}03125 = 1/32$

$10^{-5} < \frac{2}{24} \cdot \frac{1}{32} \cdot h^2$

$h = \sqrt{\frac{32 \cdot 24 \cdot 10^{-5}}{2}}$ $0{,}06197$

$h = \frac{b-a}{n} = \frac{2}{n}$ $\boxed{n = 32{,}27 = 33 \quad h = \frac{2}{33} = 0{,}0606}$