MAT 2784 B Dev n° 1 02.1

1.30 $(2xy^2 - 3y^3)dx + (7-3xy^2)dy = 0$ RÉMI VAILLANCOURT

$M dx + N dy = 0$ 07.01.25

$M_y = 4xy - 9y^2 \neq N_x = -3y^2$

$$\frac{M_y - N_x}{M} = \frac{4xy - 9y^2 + 3y^2}{2xy^2 - 3y^3} = \frac{4xy - 6y^2}{y(2xy - 3y^2)} = \frac{2(2xy - 3y^2)}{y(2xy - 3y^2)} = \frac{2}{y} = f(y)$$

$\mu_y = e^{-2\int 1/y\, dy} = e^{-2\ln y} = e^{\ln y^{-2}} = y^{-2} = \frac{1}{y^2}$

$du = \frac{1}{y^2}(2xy^2 - 3y^3)dx + \frac{1}{y^2}(7 - 3xy^2)dy$

$du = (2x - 3y)dx + \left(\frac{7}{y^2} - 3x\right)dy$

$du = M\,dx + N\,dy = 0.$ $M_y = -3 = N_x = -3$

$u(x,y) = 0$

$u(x,y) = \int (2x - 3y)\,dx + T(y)$

$= \frac{2x^2}{2} - 3yx + T(y) = x^2 - 3yx + T(y)$

$u_y(x,y) = -3x + T'(y)$

$= \frac{7}{y^2} - 3x$

$T'(y) = 7/y^2$

$T(y) = -7/y$

$C = x^2 - 3yx - \frac{7}{y}$ $\Leftarrow$ solution générale

D2.2

1.40 $y' - y\tan x = \frac{1}{\cos^3 x}$, $y(0)=0$.

forme: $y' + f(x)y = r(x)$

$u(x) = e^{\int -\tan x\, dx} = e^{-\ln|\sec x|} = e^{\ln|\sec x|^{-1}}$

$u(x) = (\sec x)^{-1} = \frac{1}{\sec x} = \cos x$.

$[u(x)y]' = u(x)\, r(x)$

$[\cos(x)y]' = \cos x \cdot \frac{1}{\cos^3 x} = \frac{1}{\cos^2 x}$

$\int [\cos(x)y]' = \int \frac{1}{\cos^2 x}\, dx + C$

$\cos(x)y = \tan(x) + C$

$y = \frac{\tan(x) + C}{\cos(x)}$ ⇐ solution générale

Pour $y(0)=0$.

$0 = \frac{\tan(0) + C}{\cos(0)}$

$C=0$.

Solution unique: $y = \frac{\tan x}{\cos x}$.

D2.3

1.44 $y = \arctan x + c$

$[y - \arctan x]' = [c]'$

$$\frac{dy}{dx} - \frac{1}{1+x^2} = 0$$

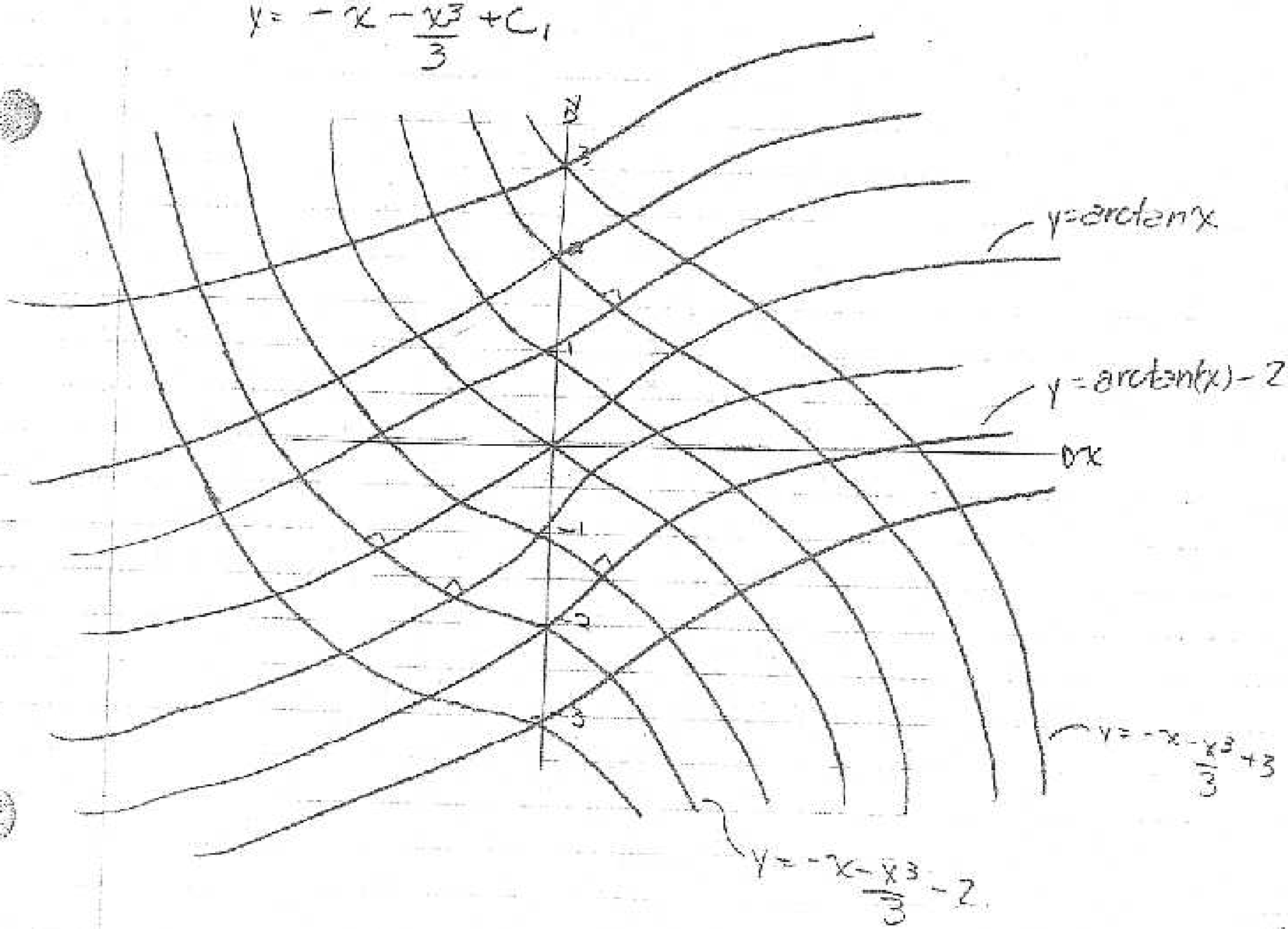
$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$-1/a \Rightarrow$ pente d'une courbe orthogonale

$$-\frac{dx}{dy} = \frac{1}{1+x^2}$$

$$\int dy = -\int (1+x^2)\,dx$$

$$y = -x - \frac{x^3}{3} + C_1$$



2.2 $y'' + 2y' + y = 0$

$Ly = y'' + 2y' + y$

$y = e^{\lambda x}$

$y' = \lambda e^{\lambda x}$

$y'' = \lambda^2 e^{\lambda x}$

$Ly = \lambda^2 e^{\lambda x} + 2\lambda e^{\lambda x} - e^{\lambda x} = 0$

$e^{\lambda x} (\lambda^2 + 2\lambda + 1) = 0$

$\lambda^2 + 2\lambda + 1 = 0$

$(\lambda + 1)(\lambda + 1)$

$\lambda_1 = -1$

$\lambda_2 = -1$

$\Delta = a^2 - 4b = 2^2 - 4(1) = 0 \Leftarrow$ cas III

$\lambda = \lambda_1 = \lambda_2 = \frac{-2}{2} = -1$

$y_1(x) = e^{\lambda x}$

$y_1(x) = e^{-x}$

$y_2(x) = x e^{\lambda x}$

$y_2(x) = x e^{-x}$

$\frac{y_1(x)}{y_2(x)} = x \neq$ const. $\therefore$ indep.

solution générale: $y(x) = c_1 e^{-x} + c_2 x e^{-x}$

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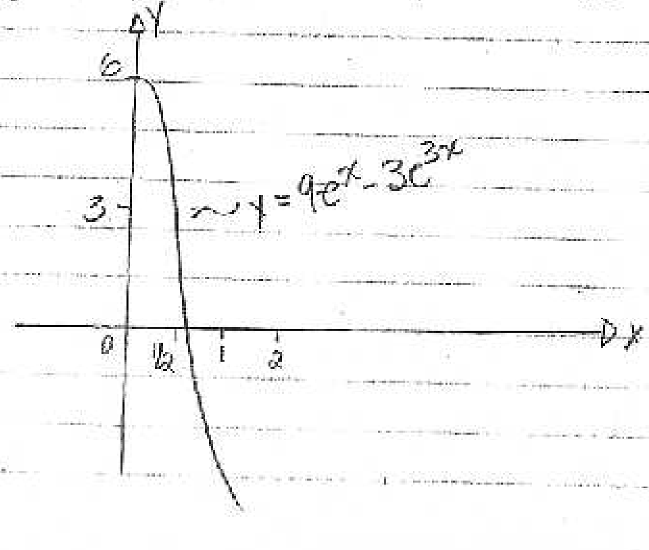
graphique pour 2.6 (pour $x \geq x_0 = 0$)



2.3 $y'' - 9y' + 20y = 0$

$Ly = y'' - 9y' + 20y = 0$

$y = e^{\lambda x}$

$y' = \lambda e^{\lambda x}$

$y'' = \lambda^2 e^{\lambda x}$

$Ly = \lambda^2 e^{\lambda x} - 9\lambda e^{\lambda x} + 20 e^{\lambda x} = 0$

$e^{\lambda x}(\lambda^2 - 9\lambda + 20) = 0$

$\lambda^2 - 9\lambda + 20 = 0$

$(\lambda - 5)(\lambda - 4) = 0$

$\lambda_1 = 4 \qquad y_1(x) = e^{4x} \qquad \dfrac{y_1(x)}{y_2(x)} = e^{x} \neq \text{const}$

$\lambda_2 = 5 \qquad y_2(x) = e^{5x} \qquad \therefore$ indépendent

Solution générale: $y = C_1 e^{4x} + C_2 e^{5x}$

2.6 $y'' - 4y' + 3y = 0, \quad y(0) = 6, \quad y'(0) = 0.$

$Ly = \lambda^2 e^{\lambda x} - 4\lambda e^{\lambda x} + 3e^{\lambda x} = 0$

$e^{\lambda x}(\lambda^2 - 4\lambda + 3) = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$(\lambda - 1)(\lambda - 3) = 0$

$\lambda_1 = 1 \qquad y_1(x) = e^{x} \qquad \dfrac{y_1(x)}{y_2(x)} = e^{-2x} \neq \text{const.}$

$\lambda_2 = 3 \qquad y_2(x) = e^{3x} \qquad \therefore$ indépen.

Solution générale: $y = C_1 e^{x} + C_2 e^{3x}$

$y(x) = C_1 e^{x} + C_2 e^{3x} \qquad y(0) = C_1 + C_2 = 6 \qquad C_1 = 6 - C_2$

$y'(x) = C_1 e^{x} + 3C_2 e^{3x} \qquad y'(0) = C_1 + 3C_2 = 0 \qquad 6 - C_2 + 3C_2 = 0$

$C_2 = -3$

$C_1 = 9$

Solution unique: $y = 9e^{x} - 3e^{3x}$

8.17 avec résultats de 8.6 $g(x) = \left(1 + \frac{1}{x}\right)^{1/2}$ $f(x) = x^3 - x - 1$

Steffensen: $S_0 = x_0$ $S_{n+1} = S_n - \frac{(z_1 - S_n)^2}{z_2 - 2z_1 + S_n}$

$z_1 = g(S_n)$

$z_2 = g(z_1)$

n	x_{n+1}	S_n
0	1	1
1	1,414213562	1,328769225
2	1,306562965	1,324718376
3	1,32867109	1,324717957
4	1,323869976	NAN
5	1,324900445	NAN

$g(S_0) = x_1$

$g(S_1) = 1,323848983 =$

$g(A) = 1,324904966$

$g(S_2) = 1,324717867 = B$

$g(B) = 1,324717977$

$g(S_3) = 1,324717957 = C$

$g(C) = 1,324717957$

$p = 1,324717957$

$g'(x) = \frac{-1}{2x^2} \cdot \frac{1}{\sqrt{1 + 1/x}}$

$g'(p) = -0,215079855 \neq 0$

↳ donc, la convergence est linéaire (d'ordre 1)

donc Steffensen converge d'ordre 2

8.20 $f(x) = x - \tan x$ Newton modifiée $x_0 = 1$.

$f(x) = x - \tan x \qquad f(0) = 0 - 0 = 0$

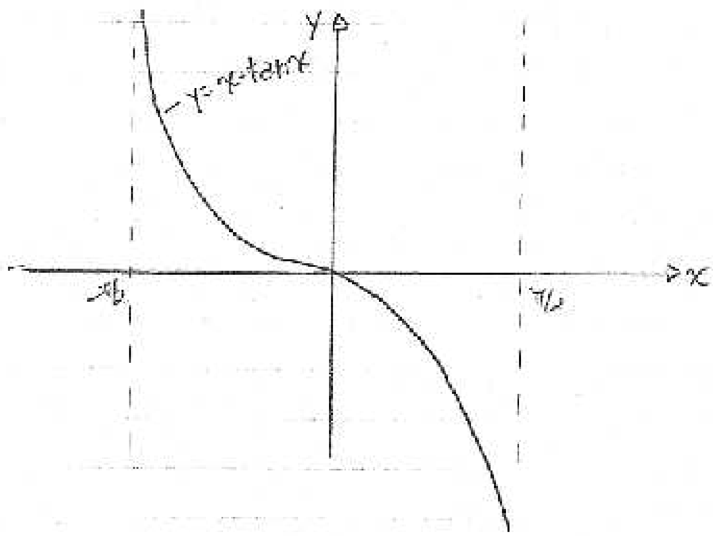
$f'(x) = 1 - \dfrac{1}{\cos^2 x} \qquad f'(0) = 1 - 1 = 0$

$f''(x) = \dfrac{-2\sin x}{\cos^3 x} \qquad f''(0) = 0$

$f'''(x) = -2\left[\dfrac{\cos^4 x + 3\sin^2 x \cos^2 x}{\cos^6 x}\right] \qquad f'''(0) = -2 \neq 0$

donc, Newton modifiée devient $x_{n+1} = x_n - 3\dfrac{f(x_n)}{f'(x_n)}$

parce qu'il y a une racine triple et une convergence d'ordre 2.



n	x_{n+1}
0	1
1	0,310571
2	0,008100...
3	0,000000
4	—

⇒ La racine trouvée est de 0.